Online Appendix for: "The Collateral Rule: Evidence from the Credit Default Swap Market"

Agostino Capponi* W. Allen Cheng† Stefano Giglio‡ Richard Haynes§

Appendices A and OA.A by Chuan Du

*Department of Industrial Engineering and Operations Research, Columbia University, New York, NY 10027, USA, ac3827@columbia.edu.

†Department of Industrial Engineering and Operations Research, Columbia University, New York, NY 10027, USA, wc2232@columbia.edu.

‡Corresponding author: Yale School of Management, NBER, and CEPR, 165 Whitney Avenue, New Haven, CT 06520. Tel. 203-432-3373. stefano.giglio@yale.edu.

§Division of Clearing and Risk, U.S. Commodity Futures Trading Commission, Washington, DC 20581, USA. rhaynes@cftc.gov.

OA.A Proofs, Comparative Statics, and Discussion of Assumptions of Theoretical Model

OA.A.1 Additional discussion of the assumptions

This section discusses in greater detail some of the assumptions in the model. The first inequality (A.2) states that the optimist's initial endowment is not large enough to purchase the entire supply of cash by issuing only fully collateralized CDS contracts.

To see this, note that the value of one unit of cash to the optimist when selling contract γ is given by:

$$1 + \frac{1}{\gamma}\left[E_0\left[\min\left\{s^{max} - s, \gamma\right\}\right] - E_1\left[\min\left\{s^{max} - s, \gamma\right\}\right]\right]$$

The total amount of funding the optimist can raise by selling riskless CDS contracts $\gamma = s^{max}$ (the contract with the maximum collateral) using 1 unit of collateral (i.e. cash) is:

$$\frac{1}{s^{max}} E_0\left[\min\left\{s^{max} - s, s^{max}\right\}\right]$$

It is assumed that the optimist's endowment is not large enough to finance the purchase of cash using riskless CDS contracts:

$$\begin{aligned}
n_1 + \frac{1}{s^{max}} E_0\left[\min\left\{s^{max} - s, s^{max}\right\}\right] &< 1 + \frac{1}{s^{max}}\left[E_0\left[\min\left\{s^{max} - s, s^{max}\right\}\right] - E_1\left[\min\left\{s^{max} - s, s^{max}\right\}\right]\right] \\
n_1 &< 1 - \frac{1}{s^{max}} E_1\left[\min\left\{s^{max} - s, s^{max}\right\}\right] \\
&= 1 - \frac{1}{s^{max}}\left[s^{max} - E_1\left[s\right]\right] = \frac{E_1\left[s\right]}{s^{max}}
\end{aligned}$$

Since $n_1 < \frac{E_1[s]}{s^{max}} \leq 1$, this in turn ensures that the optimist cannot simply purchase the entire stock of cash (normalized to 1) without borrowing from the pessimist.

OA.A.2 Proof of Proposition 2

Returning to the optimist's problem under the principal-agent formulation (A.4), one can substitute the pessimist's participation constraint into the objective function to reduce dimension of the choice variable to one:

$$\begin{aligned}
&\max_{\gamma\in\mathbb{R}_+} \frac{n_1}{\left(p-\frac{1}{\gamma}E_0\left[\min\left\{s^{max}-s,\gamma\right\}\right]\right)}\left[1-\frac{1}{\gamma}E_1\left[\min\left\{s^{max}-s,\gamma\right\}\right]\right] \\
\Leftrightarrow &\max_{\gamma\in\mathbb{R}_+} n_1 R_1^{CDS}(\gamma)
\end{aligned}$$

where $R_1^{CDS}(\gamma)$ is defined as the expected return to the optimist who buys one unit of cash and uses it to back the sale of the CDS contract:

$$R_1^{CDS}(\gamma) := \frac{1-\frac{1}{\gamma}E_1\left[\min\left\{s^{max}-s,\gamma\right\}\right]}{p-\frac{1}{\gamma}E_0\left[\min\left\{s^{max}-s,\gamma\right\}\right]} = \frac{\gamma-E_1\left[\min\left\{s^{max}-s,\gamma\right\}\right]}{p\gamma-E_0\left[\min\left\{s^{max}-s,\gamma\right\}\right]}$$

(the numerator is the expected $t=1$ payoff from purchasing the cash whilst simultaneously selling the CDS; the denominator is the down payment required to purchase the cash).

Note that since $E_i\left[\min\left\{s^{max}-s,\gamma\right\}\right] \equiv \gamma F_i\left(s^{max}-\gamma\right)+\int_{s^{max}-\gamma}^{s^{max}}\left(s^{max}-s\right)dF_i(s)$, it holds

$$\begin{aligned}
\frac{dE_i\left[\min\left\{s^{max}-s,\gamma\right\}\right]}{d\gamma} &= F_i\left(s^{max}-\gamma\right)-\gamma f_i\left(s^{max}-\gamma\right)-(-1)\left(s^{max}-\left(s^{max}-\gamma\right)\right)f_i\left(s^{max}-\gamma\right) \\
&= F_i\left(s^{max}-\gamma\right)
\end{aligned}$$

Therefore the derivative of $R_1^{CDS}(\gamma)$ with respect to λ can be expressed as:

$$\begin{aligned}
\frac{dR_1^{CDS}(\gamma)}{d\gamma} &= \frac{\left(1-F_1\left(s^{max}-\gamma\right)\right)\left(p\gamma-E_0\left[\min\left\{s^{max}-s,\gamma\right\}\right]\right)}{\left(p\gamma-E_0\left[\min\left\{s^{max}-s,\gamma\right\}\right]\right)^2} \\
&\quad -\frac{\left(p-F_0\left(s^{max}-\gamma\right)\right)\left(\gamma-E_1\left[\min\left\{s^{max}-s,\gamma\right\}\right]\right)}{\left(p\gamma-E_0\left[\min\left\{s^{max}-s,\gamma\right\}\right]\right)^2} \\
&= \frac{1}{\left(p\gamma-E_0\left[\min\left\{s^{max}-s,\gamma\right\}\right]\right)}\left[\left(1-F_1\left(s^{max}-\gamma\right)\right)-\left(p-F_0\left(s^{max}-\gamma\right)\right)R_1^{CDS}(\gamma)\right]
\end{aligned}$$

The first order condition for the optimist's problem simplifies to:

$$\left(1-F_1\left(s^{max}-\bar{s}\right)\right)=\left(p-F_0\left(s^{max}-\bar{s}\right)\right)\frac{\bar{s}-E_1\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]}{p\bar{s}-E_0\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]}$$

Re-arranging and simplifying notation (let $F_i := F_i\left(s^{max}-\bar{s}\right)$) gives:

$$\begin{aligned}
p &= \frac{\left(1-F_1\right)E_0\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]+F_0E_1\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]-\bar{s}F_0}{E_1\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]-\bar{s}F_1}\\
&= \frac{\left(1-F_1\right)\left[\bar{s}F_0+\int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_0\left(s\right)ds\right]+F_0\left[\bar{s}F_1+\int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_1\left(s\right)ds\right]-\bar{s}F_0}{\left[\bar{s}F_1+\int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_1\left(s\right)ds\right]-\bar{s}F_1}\\
&= F_0+\frac{\left(1-F_1\right)\int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_0\left(s\right)ds}{\int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_1\left(s\right)ds}\\
&= F_0+\frac{\left(1-F_1\right)\left(1-F_0\right)E_0\left[s^{max}-s|s\geq s^{max}-\bar{s}\right]}{\left(1-F_1\right)E_1\left[s^{max}-s|s\geq s^{max}-\bar{s}\right]}\\
&= F_0\left(s^{max}-\bar{s}\right)+\left(1-F_0\left(s^{max}-\bar{s}\right)\right)\frac{E_0\left[s^{max}-s|s\geq s^{max}-\bar{s}\right]}{E_1\left[s^{max}-s|s\geq s^{max}-\bar{s}\right]}\\
&=: p^{opt}\left(\bar{s}\right)
\end{aligned}$$

as required.

Lemma 1. *The price of cash (i.e. the collateral) is strictly increasing in the optimist's desired level of collateralization:* $\frac{dp^{opt}(\bar{s})}{d\bar{s}}>0$ *for* $\bar{s}\in\left(s^{min},s^{max}\right)$.

Proof. Using the fact that

$$\begin{aligned}
&\frac{d}{d\bar{s}}E_i\left[s^{max}-s|s\geq s^{max}-\bar{s}\right]\\
=&\frac{d}{d\bar{s}}\left(\frac{1}{1-F_i\left(s^{max}-\bar{s}\right)}\int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_1\left(s\right)ds\right)\\
=&\frac{\left(\bar{s}f_i\left(s^{max}-\bar{s}\right)\right)\left(1-F_i\left(s^{max}-\bar{s}\right)\right)-f_i\left(s^{max}-\bar{s}\right)\int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_1\left(s\right)ds}{\left(1-F_i\left(s^{max}-\bar{s}\right)\right)^2}\\
=&\frac{\bar{s}f_i\left(s^{max}-\bar{s}\right)-f_i\left(s^{max}-\bar{s}\right)E_i\left[s^{max}-s|s\geq s^{max}-\bar{s}\right]}{1-F_i\left(s^{max}-\bar{s}\right)}\\
=&\frac{f_i\left(s^{max}-\bar{s}\right)}{1-F_i\left(s^{max}-\bar{s}\right)}\left(\bar{s}-E_i\left[s^{max}-s|s\geq s^{max}-\bar{s}\right]\right)
\end{aligned}$$

Differentiating $p^{opt}(\bar{s})$ (and using short-hands: $f_i := f_i(s^{max} - \bar{s})$, $F_i := F_i(s^{max} - \bar{s})$, and $E_i := E_i[s^{max} - s | s \geq s^{max} - \bar{s}]$) yields:

$$\begin{aligned}
\frac{dp^{opt}(\bar{s})}{d\bar{s}} &= -f_0 + \frac{\left[f_0 E_0 + (1-F_0)\left(\frac{f_0}{1-F_0}(\bar{s} - E_0)\right)\right][E_1] - \left[\frac{f_1}{1-F_1}(\bar{s} - E_1)\right][(1-F_0)E_0]}{(E_1)^2} \\
&= -f_0 + \frac{\bar{s} f_0}{E_1} - \bar{s} f_1 \frac{1-F_0}{1-F_1}\frac{E_0}{(E_1)^2} + f_1 \frac{1-F_0}{1-F_1}\frac{E_0}{E_1} \\
&= f_0\left(\frac{\bar{s}}{E_1} - 1\right) + f_1 \frac{1-F_0}{1-F_1}\frac{E_0}{E_1}\left(1 - \frac{\bar{s}}{E_1}\right) \\
&= \underbrace{\left(\frac{\bar{s}}{E_1} - 1\right)}_{>0}\underbrace{\left(f_0 - f_1 \frac{\int_{s^{max}-\bar{s}}^{s^{max}} (s^{max} - s) f_0(s)\, ds}{\int_{s^{max}-\bar{s}}^{s^{max}} (s^{max} - s) f_1(s)\, ds}\right)}_{>0}
\end{aligned}$$

where the first inequality follows from $\bar{s} > E_1[s^{max} - s | s \geq s^{max} - \bar{s}] \quad \forall \bar{s} \in (s^{min}, s^{max})$, and the second inequality holds due to assumption A2:

$$\begin{aligned}
&f_0(s^{max} - \bar{s}) \int_{s^{max}-\bar{s}}^{s^{max}} (s^{max} - s) f_1(s)\, ds - f_1 \int_{s^{max}-\bar{s}}^{s^{max}} (s^{max} - s) f_0(s)\, ds \\
= &\int_{s^{max}-\bar{s}}^{s^{max}} (s^{max} - s)\left[f_0(s^{max} - \bar{s}) f_1(s) - f_1(s^{max} - \bar{s}) f_0(s)\right] ds \\
> &0 \quad \text{since by A2: } \frac{f_1(s)}{f_0(s)} > \frac{f_1(s^{max} - \bar{s})}{f_0(s^{max} - \bar{s})} \quad \forall s > (s^{max} - s)
\end{aligned}$$

Lemma 2. *The optimist's perceived interest rate on the CDS is strictly decreasing in the level of collateral* $\bar{s}$*:* $\frac{d\left(1 + r_1^{per}(\bar{s})\right)}{d\bar{s}} < 0$ *for* $\bar{s} \in (s^{min}, s^{max})$.

Recall from equation (A.3) that the perceived interest rate is defined as $1 + r_1^{per}(\bar{s}) := \frac{E_1[\min\{s^{max} - s, \bar{s}\}]}{E_0[\min\{s^{max} - s, \bar{s}\}]}$. Differentiating with respect to $\bar{s}$ (using the result $\frac{dE_i[\min\{s^{max} - s, \bar{s}\}]}{d\bar{s}} = F_i(s^{max} - \bar{s})$) yields:

$$\frac{d\left(1 + r_1^{per}(\bar{s})\right)}{d\bar{s}} = \frac{F_1(s^{max} - \bar{s}) E_0[\min\{s^{max} - s, \bar{s}\}] - F_0(s^{max} - \bar{s}) E_1[\min\{s^{max} - s, \bar{s}\}]}{(E_0[\min\{s^{max} - s, \bar{s}\}])^2}$$

Because the denominator in the above expression is always positive, to show that $\frac{d\left(1 + r_1^{per}(\bar{s})\right)}{d\bar{s}} < 0$

it is enough to show that for $\bar{s} \in (s^{min}, s^{max})$, it holds that $\frac{E_1[\min\{s^{max}-s,\bar{s}\}]}{E_0[\min\{s^{max}-s,\bar{s}\}]} > \frac{F_1(s^{max}-\bar{s})}{F_0(s^{max}-\bar{s})}$. Next, one proceed as follows:

$$\begin{aligned}
\frac{E_1\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]}{E_0\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]} &= \frac{\bar{s}F_1\left(s^{max}-\bar{s}\right)+\int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_1\left(s\right)ds}{\bar{s}F_0\left(s^{max}-\bar{s}\right)+\int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_0\left(s\right)ds} \\
&> \frac{\frac{F_1(s^{max}-\bar{s})}{F_0(s^{max}-\bar{s})}\bar{s}F_0\left(s^{max}-\bar{s}\right)+\int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_0\left(s\right)\frac{F_1(s)}{F_0(s)}ds}{\bar{s}F_0\left(s^{max}-\bar{s}\right)+\int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)dF_0\left(s\right)} \\
&> \frac{\frac{F_1(s^{max}-\bar{s})}{F_0(s^{max}-\bar{s})}\bar{s}F_0\left(s^{max}-\bar{s}\right)+\frac{F_1(s^{max}-\bar{s})}{F_0(s^{max}-\bar{s})}\int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_0\left(s\right)ds}{\bar{s}F_0\left(s^{max}-\bar{s}\right)+\int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)dF_0\left(s\right)} \\
&= \frac{F_1\left(s^{max}-\bar{s}\right)}{F_0\left(s^{max}-\bar{s}\right)}
\end{aligned}$$

where the first inequality follows from Assumption A2 ($f_0\left(s\right)\frac{F_1(s)}{F_0(s)} < f_1\left(s\right)$), and the second inequality follows from Assumption A2 ($\frac{d}{ds}\frac{F_0(s)}{F_1(s)} < 0$) . □

OA.A.3 Proof of Proposition 3

In Appendix A, it has been argued that the principal-agent equilibrium is given by the intersection of the optimality condition $p^{opt}\left(\bar{s}\right) = F_0\left(s^{max}-\bar{s}\right) + \left(1-F_0\left(s^{max}-\bar{s}\right)\right)\frac{E_0[s^{max}-s|s>s^{max}-\bar{s}]}{E_1[s^{max}-s|s>s^{max}-\bar{s}]}$ derived from the optimist's optimization problem, and the market clearing condition for cash: $p^{mc}\left(\bar{s}\right) = n_1 + \frac{1}{\bar{s}}E_0\left[\min\left\{s^{max}-s,\bar{s}\right\}\right]$. From the proof to Proposition 2, it has also been established that $p^{opt}\left(\bar{s}\right)$ is strictly increasing in $\bar{s}$ over $\bar{s} \in (s^{min}, s^{max})$. It remains to show that (i) $p^{mc}\left(\bar{s}\right)$ is strictly decreasing in $\bar{s}$ over $\bar{s} \in (s^{min}, s^{max})$; and (ii) the boundary conditions are such that an intersection exists: $p^{mc}\left(s^{min}\right) > p^{opt}\left(s^{max}\right)$ and $p^{mc}\left(s^{max}\right) < p^{opt}\left(s^{max}\right)$.

1. Show that $p^{mc}\left(\bar{s}\right)$ is strictly decreasing in $\bar{s}$ over $\bar{s} \in \left(s^{min}, s^{max}\right)$.

Given $p^{mc}(\bar{s}) = n_1 + \frac{1}{\bar{s}} E_0 [\min \{s^{max} - s, \bar{s}\}]$, differentiating with respect to $\bar{s}$ yields:

$$\begin{aligned}
\frac{dp^{mc}(\bar{s})}{d\bar{s}} &= \frac{F_0 (s^{max} - \bar{s}) \bar{s} - E_0 [\min \{s^{max} - s, \bar{s}\}]}{\bar{s}^2} \\
&= \frac{\bar{s} F_0 (s^{max} - \bar{s}) - \left(\bar{s} F_0 (s^{max} - \bar{s}) + \int_{s^{max} - \bar{s}}^{s^{max}} s dF_0 \right)}{\bar{s}^2} \\
&= -\frac{\int_{s^{max} - \bar{s}}^{s^{max}} s dF_0}{\bar{s}^2} < 0 \quad \forall \bar{s} \in \left(s^{min}, s^{max}\right)
\end{aligned}$$

2. Consider the boundary conditions for $p^{mc}(\bar{s})$ and $p^{opt}(\bar{s})$:

   (a) For $\bar{s} = s^{max}$, it holds:

$$\begin{aligned}
p^{mc}(s^{max}) &= n_1 + \left(\frac{s^{max} - E_0 [s]}{s^{max}} \right) \\
&< \frac{E_1 [s]}{s^{max}} + \left(\frac{s^{max} - E_0 [s]}{s^{max}} \right) \quad \text{by assumption A1} \\
&= 1 + \frac{E_0 [s^{max} - s] - E_1 [s^{max} - s]}{s^{max}} \\
&< 1 + \frac{E_0 [s^{max} - s] - E_1 [s^{max} - s]}{E_1 [s^{max} - s]} \\
&= \frac{E_0 [s^{max} - s]}{E_1 [s^{max} - s]} = p^{opt}(s^{max}).
\end{aligned}$$

   (b) For $\bar{s} = s^{min} \equiv 0$, using L'Hospital's rule, one gets:

$$\begin{aligned}
\lim_{\bar{s} \downarrow 0} p^{mc}(\bar{s}) &= n_1 + \lim_{\bar{s} \downarrow 0} \frac{\frac{d}{d\bar{s}} (E_0 [\min \{s^{max} - s, \bar{s}\}])}{\frac{d}{d\bar{s}} (\bar{s})} \\
&= n_1 + \lim_{\bar{s} \downarrow 0} \frac{F_0 (s^{max} - \bar{s})}{1} = n_1 + 1
\end{aligned}$$

and

$$
\begin{aligned}
\lim_{\bar{s}\downarrow 0} p^{opt}(\bar{s}) &= \lim_{\bar{s}\downarrow 0}\left[F_0\left(s^{max}-\bar{s}\right)+\left(1-F_0\left(s^{max}-\bar{s}\right)\right)\frac{E_0\left[s^{max}-s|s\geq s^{max}-\bar{s}\right]}{E_1\left[s^{max}-s|s\geq s^{max}-\bar{s}\right]}\right] \\
&= \lim_{\bar{s}\downarrow 0} F_0\left(s^{max}-\bar{s}\right)+\lim_{\bar{s}\downarrow 0}\frac{\int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_0(s)\,ds}{\frac{1}{1-F_1(s^{max}-\bar{s})}\int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_1(s)\,ds} \\
&= 1+\lim_{\bar{s}\downarrow 0}\left(1-F_1\left(s^{max}-\bar{s}\right)\right)\lim_{\bar{s}\downarrow 0}\frac{\frac{d}{d\bar{s}}\int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_0(s)\,ds}{\frac{d}{d\bar{s}}\int_{s^{max}-\bar{s}}^{s^{max}}\left(s^{max}-s\right)f_1(s)\,ds} \\
&= 1+0\times\lim_{\bar{s}\downarrow 0}\frac{\bar{s}f_0\left(s^{max}-\bar{s}\right)}{\bar{s}f_1\left(s^{max}-\bar{s}\right)}=1+0\times\frac{f_0\left(s^{max}\right)}{f_1\left(s^{max}\right)} \\
&= 1
\end{aligned}
$$

so

$$p^{mc}\left(s^{min}\right)=n_1+1>1=p^{opt}\left(s^{min}\right)$$

3. Because $p^{opt}(\gamma)$ and $p^{mc}(\gamma)$ are both continuous functions, by the intermediate value theorem they intersect at some interior point $\gamma^* \in (s^{min}, s^{max})$ and $p^* \in [1, \frac{E_0[s^{max}-s]}{E_1[s^{max}-s]})$. Since $p^{mc}(\bar{s})$ is strictly decreasing, and $p^{opt}(\bar{s})$ is strictly increasing, the intersection is unique.

OA.A.4 Proof of Proposition 1

The existence of a unique Principal-Agent equilibrium has been shown in the proof of propositions 2 and 3. In this section, it is shown that a collateral general equilibrium as defined in the main body exists and is equivalent to the principal-agent equilibrium.

1. Step 1: Simplifying observations for solving the collateral general equilibrium:

   (a) Without loss of generality, one can show that the equilibrium price of cash satisfies $\hat{p} \in \left[1, 1+\frac{[E_1[s]-E_0[s]]}{s^{max}}\right)$. Since cash guarantees a safe return of 1, its equilibrium price will never fall below 1. The optimist attach a higher value to cash, because by using cash as collateral in selling CDS contracts γ, an optimist also gains the difference in the expected delivery $\frac{1}{\gamma}\left[E_0\left[\min\left\{s^{max}-s,\gamma\right\}\right]-E_1\left[\min\left\{s^{max}-s,\gamma\right\}\right]\right]$[1]. This difference in beliefs is fully exploited when the CDS is fully collateralized at $\gamma=s^{max}$. So the maximum price an optimist is willing to pay for cash is equal to: $1+\frac{1}{s^{max}}\left[E_0\left[\min\left\{s^{max}-s,s^{max}\right\}\right]-E_1\left[\min\left\{s^{max}-s,s^{max}\right\}\right]\right]=1+\frac{[E_1[s]-E_0[s]]}{s^{max}}$, at which point the optimist would also weakly prefer to hold the illiquid asset instead. $\frac{[E_1[s]-E_0[s]]}{s^{max}}$ can be interpreted as the upper bound on the equilibrium collateral value for cash. For the rest of the proof, the focus is on the more interesting cases where $\hat{p}$ is strictly less than $1+\frac{[E_1[s]-E_0[s]]}{s^{max}}$.

(b) Note that each agent's optimization problem (A.3) is linear in the objective variables, thus their value functions will take the form $v_i n_i$, where v_i denotes the return on agent i's endowment n_i.

(c) Since the agents can always just hold their endowment of the illiquid asset or buy cash in order to sell the fully collateralized CDS contract $\gamma=s^{max}$, one must have:

$$v_i \geq \max\left\{1,\frac{1-\frac{1}{s^{max}}E_i\left[\min\left\{s^{max}-s,s^{max}\right\}\right]}{p-\frac{1}{s^{max}}E_0\left[\min\left\{s^{max}-s,s^{max}\right\}\right]}\right\}\quad \forall i=\{0,1\} \tag{A.1}$$

In the above expression, 1 represents the rate of return on the illiquid asset; and

$$\left(\frac{1-\frac{1}{s^{max}}E_i\left[\min\left\{s^{max}-s,s^{max}\right\}\right]}{p-\frac{1}{s^{max}}E_0\left[\min\left\{s^{max}-s,s^{max}\right\}\right]}\right)$$

represents the expected rate of return on buying cash to use as collateral in selling the CDS contract $\gamma=s^{max}$. For the latter, the expected payoff in the second period is 1 (from the cash) minus $\frac{1}{s^{max}}E_i\left[\min\left\{s^{max}-s,s^{max}\right\}\right]$ (the expected delivery on the CDS contract γ). The down-payment on this transaction is $\left(p-\frac{1}{s^{max}}E_0\left[\min\left\{s^{max}-s,s^{max}\right\}\right]\right)$, where p is the price paid for the unit of cash, and $\frac{1}{s^{max}}E_0\left[\min\left\{s^{max}-s,s^{max}\right\}\right]$ is

[1] Note that even though $\frac{1}{\gamma}\left[E_0\left[\min\left\{s^{max}-s,\gamma\right\}\right]-E_1\left[\min\left\{s^{max}-s,\gamma\right\}\right]\right]$ is maximized at $\gamma=s^{max}$. This is does not mean the optimist will always want to sell the fully collateralised CDS at any p. The $p^{opt}(\bar{s})$ curve plots the optimal collateral level $\bar{s}$ for the optimist at any given price p. (Derived from the interior solution to $\max_\gamma R$, where $R:=\frac{1-\frac{1}{\gamma}E_1[\min\{s^{max}-s,\gamma\}]}{p-\frac{1}{\gamma}E_0[\min\{s^{max}-s,\gamma\}]}$).

the amount raised from selling the CDS to the pessimist who values it the most.

(d) Summing over the two agents' budget constraints (A.1), and imposing the market clearing conditions in equilibrium (holdings of CDS contracts must cancel out and the sum of total cash holdings is normalized to 1) yields:

$$a_0 + a_1 + \hat{p} \times 1 = n_0 + n_1$$

Recall that by Assumption A1 $n_0 + n_1 > \frac{E_0[s^{max}-s]}{E_1[s^{max}-s]} = 1 + \frac{E_1[s]-E_0[s]}{E_1[s^{max}-s]} > 1 + \frac{[E_1[s]-E_0[s]]}{s^{max}}$, so $\hat{p} \in [1, 1 + \frac{[E_1[s]-E_0[s]]}{s^{max}}]$ implies that $a_0 + a_1 > 0$ (i.e. one or more agents must hold the illiquid asset in equilibrium).

(e) Since $p < 1 + \frac{[E_1[s]-E_0[s]]}{s^{max}}$, it follows from equation (A.1) that $v_1 > 1$. Therefore, the pessimist must be the one holding the illiquid asset in the collateral equilibrium, which gives us $v_0 = 1$.

(f) Lastly, without loss of generality, CDS contracts with $\gamma > s^{max}$ will not be used in equilibrium (such contracts tie down a larger amount of collateral without a compensating increase in price).

2. Agents' bid and ask prices for CDS contracts:

(a) An agent's bid price is the price that would make her indifferent between buying the CDS contract and simply receiving the equilibrium value per net worth v_i , so:

$$\begin{aligned} q_0^{bid}(\gamma) &= \frac{E_0\left[\min\left\{s^{max}-s,\gamma\right\}\right]}{v_0} = E_0\left[\min\left\{s^{max}-s,\gamma\right\}\right] \\ > q_1^{bid}(\gamma) &= \frac{E_1\left[\min\left\{s^{max}-s,\gamma\right\}\right]}{v_1} \end{aligned} \tag{A.2}$$

(b) An agent's ask price for the CDS contract γ is the price that would make the trader indifferent between taking a negative position in the CDS γ and simply receiving the equilibrium value v_i, so:

$$\begin{aligned} v_0 &= \frac{1-\frac{1}{\gamma}E_0\left[\min\left\{s^{max}-s,\gamma\right\}\right]}{p-\frac{1}{\gamma}q_0^{ask}\left(\gamma\right)} = 1 \\ v_1 &= \frac{1-\frac{1}{\gamma}E_1\left[\min\left\{s^{max}-s,\gamma\right\}\right]}{p-\frac{1}{\gamma}q_1^{ask}\left(\gamma\right)} > 1 \end{aligned} \tag{A.3}$$

(c) Market clearing for CDS contracts $(\hat{\mu}_1^+ + \hat{\mu}_0^+ = \hat{\mu}_1^- + \hat{\mu}_0^-)$ implies:

$$\min_i q_i^{ask}\left(\gamma\right) \geq q\left(\gamma\right) \geq \max_i q_i^{bid}\left(\gamma\right) \quad \forall\gamma$$

(Suppose $\max_i q_i^{bid}\left(\gamma\right) > q\left(\gamma\right)$, then some buyer wants to buy an infinite amount, but seller can only sell a finite amount due to the collateral constraint. It also cannot occur that $q\left(\gamma\right) > \max\left\{\min_i q_i^{ask}\left(\gamma\right), \max_i q_i^{bid}\left(\gamma\right)\right\}$, so one must have $\min_i q_i^{ask}\left(\gamma\right) \geq q\left(\gamma\right)$)

(d) A CDS contract is traded in positive quantities only if:

$$q_i^{ask}\left(\hat{\gamma}\right) = q\left(\hat{\gamma}\right) = q_j^{bid}\left(\hat{\gamma}\right) \quad \text{for some } \{i,j\} = \{0,1\}$$

(e) Claim the pessimist's ask prices are always higher than optimist's bid prices:

$$\begin{aligned} q_0^{ask}\left(\gamma\right) &= \gamma\left(p-1\right) + E_0\left[\min\left\{s^{max}-s,\gamma\right\}\right] \quad \text{from eqn (A.3)} \\ &> E_0\left[\min\left\{s^{max}-s,\gamma\right\}\right] \quad \text{since } p>1 \\ &= q_0^{bid}\left(\gamma\right) > q_1^{bid}\left(\gamma\right) \quad \text{from eqn (A.2)} \end{aligned}$$

so there are no traded CDS contracts in which the optimist buy and the pessimist sell.

(f) The equilibrium prices of CDS contracts are therefore:

$$\begin{cases} q(\hat{\gamma}) = q_0^{bid}(\hat{\gamma}) = q_1^{ask}(\hat{\gamma}) & \text{for each } \hat{\gamma} \text{ with positive trade} \\ q(\gamma) \in \left[\max_i q^{bid}(\gamma), \min_i q^{ask}(\gamma)\right] & \text{for each } \gamma \end{cases}$$

3. Characterize the equilibrium in CDS markets for a given price for cash $p \in [1, \frac{E_0[s^{max}-s]}{E_1[s^{max}-s]})$

- The optimist faces quasi-equilibrium prices for all CDS contracts (even those that are not positively traded in equilibrium):

$$\tilde{q}(\gamma) = q_0^{bid}(\gamma) = E_0[\min\{s^{max} - s, \gamma\}]$$

- Given these quasi-equilibrium prices, optimists solve the following optimization problem:

$$\begin{aligned} v_1 n_1 = \max_{c_1 \geq 0, \mu_1^-} \; & c_1 - \int_{\gamma \in B^{CDS}} \frac{1}{\gamma} E_1[\min\{s^{max} - s, \gamma\}] \, d\mu_1^- \\ \text{s.t. } & pc_1 - \int_{\gamma \in B^{CDS}} \frac{1}{\gamma} E_0[\min\{s^{max} - s, \gamma\}] \, d\mu_1^- = n_1 \quad \text{[budget constraint]} \\ & \int_{\gamma \in B^{CDS}} \frac{1}{\gamma} d\mu_1^- \leq c_1 \quad \text{[collateral constraint]} \end{aligned}$$

- Since $v_1 > 1$, the collateral constraint binds in equilibrium. Let λ_1 denote the Lagrangian multiplier for the collateral constraint. v_1 will correspond to the multiplier for the budget constraint

- The FOCs for c_1 and μ_1^- yields:

$$\begin{aligned} 1 + \lambda_1 &= v_1 p \\ v_1 \frac{1}{\gamma} E_0[\min\{s^{max} - s, \gamma\}] &\leq \frac{1}{\gamma} E_1[\min\{s^{max} - s, \gamma\}] + \lambda_1 \quad \text{with equality only if } \gamma \in supp\left(\mu_1^-\right) \end{aligned}$$

- Combining the FOCs yield:

$$
\begin{aligned}
&v_1 p = 1+\lambda_1 \\
&\qquad \geq 1 + v_1 \frac{1}{\gamma} E_0 \left[\min\left\{s^{max}-s,\gamma\right\}\right] - \frac{1}{\gamma} E_1 \left[\min\left\{s^{max}-s,\gamma\right\}\right] \\
&\Rightarrow v_1 \geq \frac{1-\frac{1}{\gamma}E_1\left[\min\left\{s^{max}-s,\gamma\right\}\right]}{p-\frac{1}{\gamma}E_0\left[\min\left\{s^{max}-s,\gamma\right\}\right]} =: R_1^{CDS}(\gamma) \quad \text{with equality only in } \gamma \in \operatorname{supp}\left(\mu_1^-\right)
\end{aligned}
$$

- As per the proof of Proposition 2, $R_1^{CDS}(\gamma)$ has a unique maximum characterized by $p^{opt}(\gamma = \tilde{s})$. So again the unique collateral-GE is pinned down by the intersection between $p^{opt}(\bar{s})$ and the market clearing condition for cash: $p^{mc}(\tilde{s}) = n_1 + \frac{1}{\tilde{s}} E_0 \left[\min\left\{s^{max}-s,\tilde{s}\right\}\right]$, s.t. the equilibrium collateral level $\hat{\gamma}$ and the price of cash $\hat{p}$ satisfies:

$$
\hat{p} = p^{opt}(\hat{\gamma}) = p^{mc}(\hat{\gamma})
$$

4. It follows that the unique general equilibrium is equivalent to the principal-agent equilibrium.

OA.A.5 Proof of Proposition 4

Let us consider the case where the optimist becomes 'more pessimistic'. Let $g_1(s) := \frac{f_1(s)}{1-F_1(s^{max}-\bar{s})} \quad \forall s \in [s^{max}-\bar{s}, s^{max}]$, $\forall \bar{s} \in [s^{min}, s^{max})$ and $\tilde{g}_1(s) := \frac{\tilde{f}_1(s)}{1-\tilde{F}_1(s^{max}-\bar{s})} \quad \forall s \in [s^{max}-\bar{s}, s^{max})$. Then by Assumption A2 $g(\cdot)$ and $\tilde{g}(\cdot)$ must also satisfy the monotone likelihood ratio condition: $\frac{d}{ds}\left(\frac{f_1}{\tilde{f}_1}\right) > 0 \quad \forall s \in S \Rightarrow \frac{d}{ds}\left(\frac{g_1}{\tilde{g}_1}\right) > 0 \quad \forall s \geq (s^{max}-\bar{s})$. This in turn implies $\tilde{E}_1\left[s|s > s^{max}-\bar{s}\right] < E_1\left[s|s \in s^{max}-\bar{s}\right] \forall \bar{s} \in (s^{min}, s^{max})$, so the upward sloping p^{opt} curve shifts down when the optimist becomes 'more pessimistic'. The converse of the proposition follows from the same logic.

OA.A.6 Proof of Proposition 5

The objective is to show that a sufficient (but not necessary) condition for the equilibrium collateral level to increase is for the pessimist to attach sufficiently larger probability weights to the default states.

Let $F_0(s)$ and $\tilde{F}_0(s)$ denote the initial and the new beliefs of the pessimist respectively. For brevity, let $\tilde{E}_0[x] \coloneqq E_{\tilde{F}_0}[x]$; $\tilde{p}^{opt}(\bar{s}) \coloneqq \tilde{F}_0(s^{max}-\bar{s}) + \left(1-\tilde{F}_0(s^{max}-\bar{s})\right)\frac{\tilde{E}_0[s^{max}-s|s>s^{max}-\bar{s}]}{E_1[s^{max}-s|s>s^{max}-\bar{s}]}$; and $\tilde{p}^{mc}(\bar{s}) \coloneqq n_1 + \frac{1}{\bar{s}}\tilde{E}_0[\min\{s^{max}-s,\bar{s}\}]$. Define, respectively, the initial and the new equilibrium collateral levels γ^* and γ^{**} implicitly as:

$$p^{opt}(\gamma^*) = p^{mc}(\gamma^*)$$

$$\tilde{p}^{opt}(\gamma^{**}) = \tilde{p}^{mc}(\gamma^{**})$$

Then given $\tilde{p}^{mc}$ is strictly decreasing, p^{opt} is strictly increasing, and the two curves intersects within (s^{min}, s^{max}) (see proofs for Propositions 2 and 3), it holds that $\gamma^{**} > \gamma^*$ iff:

$$\tilde{p}^{mc}(\gamma^*) > \tilde{p}^{opt}(\gamma^*)$$

$$\Leftrightarrow \tilde{p}^{mc}(\gamma^*) - p^{mc}(\gamma^*) > \tilde{p}^{opt}(\gamma^*) - p^{opt}(\gamma^*)$$

With a little bit of algebra, one can show that:

$$\begin{aligned}
&\tilde{p}^{mc}(\gamma^*) - p^{mc}(\gamma^*)\\
=&\frac{1}{\gamma^*}\left[\tilde{E}_0[\min\{s^{max}-s,\gamma^*\}] - E_0[\min\{s^{max}-s,\gamma^*\}]\right]\\
=&\frac{1}{\gamma^*}\left[\gamma^*\tilde{F}_0 + \int_{s^{max}-\gamma^*}^{s^{max}}(s^{max}-s)\tilde{f}_0(s)\,ds - \gamma^* F_0 - \int_{s^{max}-\gamma^*}^{s^{max}}(s^{max}-s)f_0(s)\,ds\right]\\
=&\left(\tilde{F}_0 - F_0\right) + \frac{1}{\gamma^*}\left[\int_{s^{max}-\gamma^*}^{s^{max}}(s^{max}-s)\tilde{f}_0(s)\,ds - \int_{s^{max}-\gamma^*}^{s^{max}}(s^{max}-s)f_0(s)\,ds\right]
\end{aligned}$$

and

$$
\begin{aligned}
&\tilde{p}^{opt}(\gamma^*) - p^{opt}(\gamma^*) \\
=&\tilde{F}_0 + \left(1-\tilde{F}_0\right)\frac{\tilde{E}_0\left[s^{max}-s|s>s^{max}-\gamma^*\right]}{E_1\left[s^{max}-s|s>s^{max}-\gamma^*\right]} \\
&\cdots - \left[F_0 + (1-F_0)\frac{E_0\left[s^{max}-s|s>s^{max}-\gamma^*\right]}{E_1\left[s^{max}-s|s>s^{max}-\gamma^*\right]}\right] \\
=&\left(\tilde{F}_0 - F_0\right) + \frac{\left[\int_{s^{max}-\gamma^*}^{s^{max}}(s^{max}-s)\tilde{f}_0(s)\,ds - \int_{s^{max}-\gamma^*}^{s^{max}}(s^{max}-s)f_0(s)\,ds\right]}{E_1\left[s^{max}-s|s>s^{max}-\gamma^*\right]}
\end{aligned}
$$

Taken together, one gets:

$$
\begin{aligned}
&\left[\tilde{p}^{mc}(\gamma^*) - p^{mc}(\gamma^*)\right] - \left[\tilde{p}^{opt}(\gamma^*) - p^{opt}(\gamma^*)\right] \\
=&\left[\frac{1}{\gamma^*} - \frac{1}{E_1\left[s^{max}-s|s>s^{max}-\gamma^*\right]}\right]\left[\int_{s^{max}-\gamma^*}^{s^{max}}(s^{max}-s)\tilde{f}_0(s)\,ds - \int_{s^{max}-\gamma^*}^{s^{max}}(s^{max}-s)f_0(s)\,ds\right] \\
=&\underbrace{\left[\frac{1}{E_1\left[s^{max}-s|s>s^{max}-\gamma^*\right]} - \frac{1}{\gamma^*}\right]}_{>0}\left[\int_{s^{max}-\gamma^*}^{s^{max}}(s^{max}-s)\left(f_0(s)-\tilde{f}_0(s)\right)ds\right]
\end{aligned}
$$

Therefore, whether the collateral requirement in the new equilibrium increases ($\gamma^{**} > \gamma^*$) depends on whether the second term $\int_{s^{max}-\hat{\gamma}}^{s^{max}}(s^{max}-s)\left(f_0(s)-\tilde{f}_0(s)\right)ds$ is positive, for which a sufficient condition is that $f_0(s) > \tilde{f}_0(s) \quad \forall s \in [s^{max}-\gamma^*, s^{max})$.

OA.B Part 39 CDS prices

End-of-day (EOD) prices within the Part 39 data set are provided by ICC in terms of points upfront. CDS prices historically have been quoted in terms of conventional or "break-even" spreads, defined as the annualized quarterly spread payment per unit of purchased protection that makes the market value of the position zero at initiation. Contracts thus were negotiated bilaterally over the counter and, depending on when they were traded, carried different spreads. The push for standardized CDS contracts, however, has drastically changed the landscape of CDS price quotes and traded contracts. In particular, the 2011 "CDS Big Bang" resulted in standardized CDSs having fixed coupons (usually 100 or 500 basis points). Thus, contract market values are often non-zero at outset. When trading standardized CDSs, the protection buyer makes an upfront payment to the protection seller at initiation (or vice versa). Price quotes are then in "points upfront" instead of break-even spreads. For instance, if a CDS contract were quoted at 0.97, the protection buyer would pay $1 - 0.97 = 3\%$ of the notional to the seller at contract initiation. Notice that this quote convention is analogous to bond quotes, where a higher price quote represents a lower payment for the buyer. Some data providers, such as Bloomberg, convert the quoted prices using a standardized model provided by the International Swaps and Derivatives Association (ISDA) and, by convention, record break-even spreads.

We note that quoted prices are model prices. Since CDSs trade relatively thinly, EOD transaction prices are not always available. ICC and Markit have a specific price discovery process tailored to the CDS market. Participants submit price quotes at the end of every business day and the clearinghouse creates periodic trade executions among participants via an auction process. The resulting prices are used for daily mark-to-market purposes.

OA.C Market events during our sample period

In this section, we briefly review the main world events that affected, directly or indirectly, CDS markets during our sample period May 2014-February 2019. In particular, during this period we find: (i) the plummet of oil prices in November 2014, when Saudi Arabia blocked OPEC from cutting oil production; (ii) the plunge in the Euro when the ECB chief Mario Draghi expressed unexpectedly dovish outlooks on monetary policy in January 2015; (iii) the 2015–2016 stock market sell-off, starting with the Chinese stock market burst ("Black Monday"), and followed by an unexpected devaluation in the Renminbi, which was further fueled by Greek Debt default; (iv) the unexpected negative interest rate policy announced by the Bank of Japan in January 2016; (v) the volatility spike when the Brexit referendum was announced in February 2016; (vi)

Donald Trump's election in November 2016 which, immediately following the announcement, created extreme volatility spikes in global markets and led the total trading volumes in CDS markets to double on the election night; (vii) OPEC's decision to cut oil production, followed by non-OPEC countries, led to hikes in oil prices in November 2016, especially because this was the first time after financial crisis; (viii) the Venezuela's delayed payments on its sovereign debt and bonds issued by state oil giant Petroleos de Venezuela in November 2017 constituted a failure to pay "credit event", and led to extreme volatility in CDS prices which sky-rocketed in a period of 6-7 days; (ix) the current COVID 19 pandemic, which caused the US stock market to hit the circuit breaker mechanism four times in ten days in March 2020, led to the announcement of a zero-percent interest rate policy and a $700 billion quantitative easing (QE) program, and led to a spike in unemployment rates which exceeded levels of 15%.

Our sample period also covers the (widely expected) interest rate hike by the Federal Reserve in December 2015, the first increase in nearly a decade, the Bitcoin's record price surge in the year 2017, and the (expected) Bank of England's decisions to raise interest in November 2017 and August 2018, respectively first and second time after the global crisis, despite the ongoing uncertainty over the future of the UK economy.

OA.D Time-series test of the VaR rule using realized returns

We consider the Z statistic:

$$Z := \frac{1}{NT} \sum_{t=1}^{T} \sum_{n=1}^{N} \mathbb{I}\{\Psi_{M,t}(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)\}, \tag{D.4}$$

where $\mathbb{I}\{\cdot\}$ is the indicator function. The indicator takes value 1 when realized $M-$day losses exceed the initial margin requirement; this is typically referred to as an *exceedance*. The statistic Z is the empirical frequency at which exceedances occur, averaged over time and across market participants. We have, for quite general correlation structures:

$$Z \xrightarrow{\mathbb{P}} \alpha,$$

by the law of large numbers. For M and α specified in the null hypothesis, we can test H_0 using Z as the test statistic.

We compute standard errors for the test using binomial probabilities. To proceed, we assume that exceedances are perfectly correlated when underlying losses overlap (for robustness against autocorrelation), and also assume that exceedances are uncorrelated across accounts. In particular, standard errors are computed as

$$S.E. = \sqrt{\frac{\alpha(1-\alpha)\zeta_M}{NT}}.$$

In the above equation, the term $\zeta_M := 2M - 1$ adjusts for our assumption that exceedances are perfectly correlated when underlying losses overlap. For $\alpha = 1\%$, $NT = 23,310$ and $M = 5$, we obtain a standard error of 0.19%. We remark that our standard errors are likely to be overly conservative. As a robustness check, we also compute one-day returns and autocorrelation estimates. For each account, we find autocorrelation estimates on the orders of 10^{-4} for the first five lags. Thus, autocorrelation would likely have a smaller impact on actual standard errors compared to our assumption of perfect correlation.

Finally, to explicitly account for potential cross-sectional correlation in the returns on margins, we perform the test separately account by account, finding that the null hypothesis is rejected in every case.

OA.E Time-series test of the VaR rule using counterfactual returns

We compute our test statistic using an extended version of Eq. (D.4):

$$Z' = \frac{1}{NTU} \sum_{t=1}^{T} \sum_{u=1}^{U} \sum_{n=1}^{N} \mathbb{I}\{\Psi_{5,u} MtM(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)\},$$

where $\Psi_{5,u} MtM(\mathbf{X}_t^n)$ is constructed as in Duffie et al. (2015) (see Appendix B for additional details), and U is the number of evaluation dates, that is, dates for which we observe the portfolio. For each portfolio $\mathbf{X}_t^n$, we estimate the frequency at which losses exceed portfolio margins. Under the null hypothesis of a 5-day 99% VaR margining rule, Z' should converge to 1% in probability.

The test can be simply implemented as a regression of observed exceedances onto a constant, with double-clustering as in Petersen (2009) by time and by account (there is no need to use the binomial model as exceedances are observed in the data, so the variance of the residuals is nonzero).

OA.F Cross-sectional test of the VaR rule

The margining rule H_0 implies $\mathbb{P}(\Psi_{M,t}(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)) = \alpha$ for all n, which further implies

$$H_0' : \mathbb{P}(\Psi_{M,t}(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)) = \mathbb{P}(\Psi_{M,t}(\mathbf{X}_t^{n'}) < -IM_t(\mathbf{X}_t^{n'})),$$

for all $n \neq n'$. The statistics to consider are then

$$Z_n := \frac{1}{T} \sum_{t=1}^{T} \mathbb{I}\{\Psi_{M,t}(\mathbf{X}_t^n) + IM_t(\mathbf{X}_t^n) < 0\} \xrightarrow{\mathbb{P}} \alpha.$$

We describe here how to implement a test for equality (H_0'). The most straightforward test for equality of the frequencies of exceedances across accounts is the $G-$test (i.e. the two-way likelihood ratio test). Because the confidence level is expected to be large (the expected frequency of exceedances is low), the typical χ^2-test for homogeneity is not appropriate (Hoey (2012)). As exceedances are expected to occur with low probability, we instead use the $G-$test to test the null hypothesis.

The test statistic is computed as:

$$G := 2\sum_{n=1}^{N} O_n \log \frac{O_n}{E_n},$$

where O_n is the observed number of exceedances for clearing member n, and E_n is the expected number of exceedances for account n. The probability of observing an exceedance, needed for calculating E_n, is estimated by pooling observations across accounts. In particular,

$$E_n := TU \times Z = \frac{1}{N}\sum_{t=1}^{T}\sum_{u=1}^{U}\sum_{n'=1}^{N} \mathbb{I}\{\Psi_{5,u} MtM(\mathbf{X}_t^{n'}) < -IM_t(\mathbf{X}_t^{n'})\},$$

and

$$O_n := \sum_{t=1}^{T}\sum_{u=1}^{U} \mathbb{I}\{\Psi_{5,u} MtM(\mathbf{X}_t^{n}) < -IM_t(\mathbf{X}_t^{n})\}.$$

Under the null that frequencies are the same for each account, $G \xrightarrow{d} \chi^2_{N-1}$.

We also derive an extension of this test that explicitly accounts for potential autocorrelation of the exceedances. For a fixed portfolio, we first count the number of exceedances, and then divide it by the number of evaluation dates. This gives an estimate for the probability of an exceedance occurring for that portfolio. We then sum the exceedance probabilities for portfolios associated with each fixed account, and use the rounded up integer as the estimate of observed exceedances for that account. We enter this estimate into the contingency table used for the $G-$test. Formally, we estimate the probability of an exceedance for an account/day combination

(n,t) as

$$\hat{p}_{n,t} = \frac{1}{T}\sum_{u=1}^{T} \mathbb{I}\{\Psi_{5,u} MtM(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)\}.$$

The number of (estimated) observed exceedances is then

$$\hat{O}_n := \left\lceil \sum_{t=1}^{T} \hat{p}_{n,t} \right\rceil.$$

The estimate $\hat{O}_n$ replaces O_n in our computation of the G statistic (Eq. (OA.F)).[2] The estimated observations are thus more robust to autocorrelation compared to treating each observation as an individual count, which may inflate the sample size.

OA.G Robustness: Details

In this section, we provide more details about the robustness tests of Section 4.4.

Value-at-Risk and maximum shortfall used in the panel analyses (Tables 4 and 8) were based on P&L generated from our entire sample of credit spreads (that is, on the entire historical distribution of returns for each portfolio held at time t by member n). Because our data set covered the financial crisis, the risk measures captured extreme movements and may thus be viewed as overly conservative for estimating portfolio losses. In this section we consider using only the last 1000 days (approximately 4 years) of credit spreads data to generate P&L, as in Duffie et al. (2015). Using these newly estimated counterfactual P&L, we compute Value-at-Risk and maximum shortfall. We replicate our panel analyses and report the results in Tables OA.3 and OA.4.

Comparing Table OA.3 to Table 4, we see there is a decrease in explanatory power of Value-at-Risk (VaR) (column (1)). This is likely due the exclusion of the financial crisis period in our simulation, resulting in both lower level and variability of Value-at-Risk. Aggregate short notional (AS) still retains its strong explanatory power (columns (3) and (4)), and columns

[2] The ceiling operation is performed to ensure that the contingency table only contains integer entries. We also performed the test with unrounded data, yielding similar, if not stronger, results.

(7) and (8) show that the DSV model still captures a significant portion of variation in initial margins, and outperforms Value-at-Risk in terms of explanatory power (columns (1) and (2)). Our conclusions remain consistent with our previous results.

Comparing Table OA.4 to Table OA.3, we observe again that there is a non-negligible increase in explanatory power compared to models that include only portfolio variables (Table OA.3). This confirms that market variables can capture a dimension of initial margins not explained by portfolio variables, especially for the VIX.

Table OA.1: Initial margins and market variables summary statistics

Summary Statistic	In. Margins ($IM_{n,t}$)	*Market variables, in basis points (bps)* Overnight Index Swap Spread (OIS_t)	LIBOR-OIS spread ($LOIS_t$)	CBOE VIX (VIX_t)
Pooled mean (over all n and t): $\mu\left(x_{n,t}\right)$	654.0	100.6	25.5	1,606.1
Std. deviation (over all n and t): $\sigma\left(x_{n,t}\right)$	427.4	81.8	16.6	750.0

Summary Statistic	In. Margins ($IM_{n,t}$)	*Market variables, in basis points (bps)* CDS spread ($CDS_{n,t}$)	Avg. CDS spread ($ACDS_t$)
Pooled mean (over all n and t): $\mu\left(x_{n,t}\right)$	654.0	73.6	73.6
Std. deviation (over all n and t): $\sigma\left(x_{n,t}\right)$	427.4	33.8	19.8
Time-series variation of cross-sectional averages: $\sigma\left(\bar{x}_t\right)$	108.4	19.8	
Mean cross-sectional dispersion: $\mu\left(\sigma_t\left(x_{n,t}\right)\right)$	421.0	26.4	
Cross-sectional dispersion of time-series averages: $\sigma\left(\bar{x}_n\right)$	360.7	21.4	
Mean time-series dispersion: $\mu\left(\sigma_n\left(x_{n,t}\right)\right)$	225.4	24.4	

Note: The table displays summary statistics of our key market variables and initial margins, in basis points and millions of USD, respectively. Definitions of market variables are reported in Table 2. In addition to the overall mean and standard deviations (dispersions), we report panel statistics that describe properties of variables both across accounts and time, the calculations of which are reviewed in Table 3. Panel summaries are not reported for market variables that do not vary across accounts.

Table OA.2: Check for multicollinearity

Estimates (R^2)	*Dependent variable:*	
	SD	ES
VaR	0.306***	1.422***
(OLS)	(97.1%)	(95.0%)
VaR	0.295***	1.364***
(Two-way Panel)	(97.7%)	(96.8%)
Observations	23,310	23,310

Note: We regress both expected shortfall and standard deviation on Value-at-Risk, and report the results. The first row corresponds to estimates from (pooled) OLS regression, and the second row corresponds to estimates after accounting for time and account fixed effects. Coefficient estimates are all significant at the 1% level. R^2s are in parentheses.

Table OA.3: Regression results for explaining initial margins with portfolio variables

	Dependent variable:											
	Initial margins (IM) - Daily Frequency											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Value-at-Risk (VaR)	4.284***	2.394***	1.917**	1.529***					0.037	0.285	0.946	0.919***
	(0.738)	(0.467)	(0.769)	(0.332)					(0.588)	(0.499)	(0.607)	(0.316)
Maximum shortfall (MS)			−0.072	−0.036	0.420***	0.333***						
			(0.142)	(0.079)	(0.132)	(0.063)						
Aggregate short notional (AS)			0.030***	0.020***	0.034***	0.022***						
			(0.003)	(0.004)	(0.004)	(0.004)						
Duffie et al. model (DSV)							1.085***	0.685***	1.079***	0.641***		
							(0.131)	(0.089)	(0.144)	(0.147)		
Modified DSV model (MDSV)											0.880***	0.563***
											(0.107)	(0.108)
Number of Observations	23310	23310	23310	23310	23310	23310	23310	23310	23310	23310	23310	23310
Adjusted R^2	0.400	0.772	0.700	0.847	0.678	0.837	0.595	0.817	0.595	0.818	0.689	0.844
Account Fixed Effect	*N*	*Y*	*N*	*Y*	*N*	*Y*	*N*	*Y*	*N*	*Y*	*N*	*Y*
Time Fixed Effect	*N*	*Y*	*N*	*Y*	*N*	*Y*	*N*	*Y*	*N*	*Y*	*N*	*Y*

Note: *p<0.1; **p<0.05; ***p<0.01
Two-Way Clustered Standard Errors (by Time and Account)

Note: Same as Table 4, but computing risk measures only using the last 1000 days of simulated return on margins and omitting the NGR explanatory variable.

Table OA.4: Regression results for initial margins using portfolio and market variables with last 1000 days of P&L

	Dependent variable:					
	Initial margins (IM) - Daily Frequency					
	(1)	(2)	(3)	(4)	(5)	(6)
Value-at-Risk (VaR) 1%	1.040*	0.733**	1.014*	0.622**	2.193***	1.442***
	(0.577)	(0.331)	(0.607)	(0.303)	(0.689)	(0.303)
Modified DSV Model (MDSV)	0.861***	0.614***	0.864***	0.631***		
	(0.102)	(0.094)	(0.102)	(0.087)		
Maximum Shortfall (MS)					−0.171	−0.051
					(0.135)	(0.071)
Aggregate Short Notional (AS)					0.029***	0.022***
					(0.003)	(0.003)
CBOE Volatility Index (VIX)	0.013	0.040**	0.006	0.020	0.037**	0.052***
	(0.016)	(0.018)	(0.017)	(0.014)	(0.015)	(0.017)
Member CDS Spread (DCDS)	2.683**	0.824	2.680**	0.830	2.713**	0.987
	(1.229)	(0.765)	(1.206)	(0.739)	(1.228)	(0.756)
Average CDS Spread (ACDS)	−0.111	−0.699			−0.424	−0.836
	(1.210)	(1.158)			(1.207)	(1.147)
LIBOR-OIS Spread (LOIS)			0.487	1.004		
			(0.994)	(1.114)		
Number of Observations	23310	23310	23310	23310	23310	23310
Adjusted R^2	0.719	0.837	0.719	0.837	0.733	0.842
Account Fixed Effect	*N*	*Y*	*N*	*Y*	*N*	*Y*
Time Fixed Effect	*N*	*N*	*N*	*N*	*N*	*N*

Note: *p<0.1; **p<0.05; ***p<0.01
Two-Way Clustered Standard Errors (by Time and Account)

Note: Same as Table 8, but computing risk measures only using the last 1000 days of simulated return on margins.

References

Duffie, D., Scheicher, M., and Vuillemey, G. (2015). Central clearing and collateral demand. *Journal of Financial Economics*, 116(2):237–256. OA.E, OA.G

Hoey, J. (2012). The two-way likelihood ratio (g) test and comparison to two-way chi squared test. *arXiv preprint arXiv:1206.4881*. OA.F

Petersen, M. A. (2009). Estimating standard errors in finance panel data sets: Comparing approaches. *Review of Financial Studies*, 22(1):435–480. OA.E