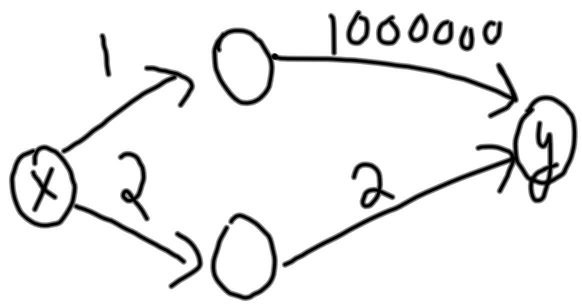
Greedy Algorithms

- easy to design
- not always correct

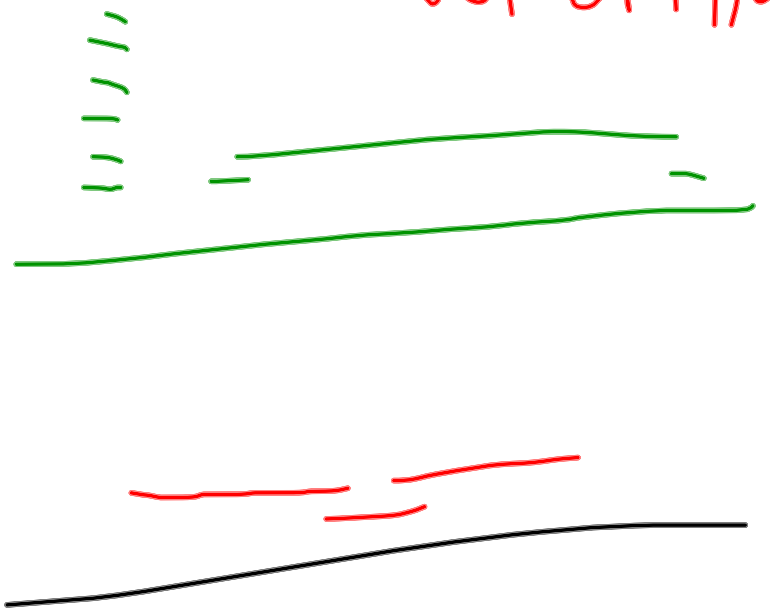
greedy alg: Choose next "step" based only on "simple" calculations of the input



Algs:

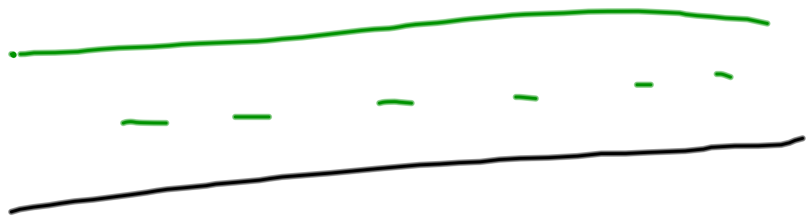
Shortest segment first:

NOT OPTIMAL

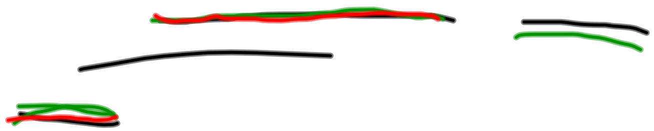


Earliest segment (by start time)

NOT OPTIMAL



Least # of overlaps



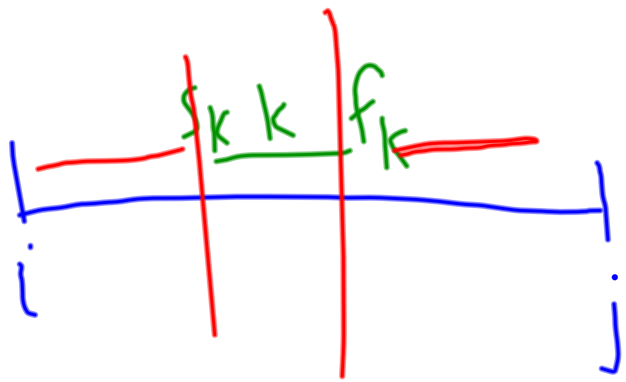
Earliest segment (by finish time)

Prove a greedy alg is opt.

- Opt. substructure ✓
- greedy choice properties
  → there exist an optimal solution that contains the choice made in step 1

$m[i,j]$ = # of intervals in an opt. sol'n from time i to time j

$$m[i,j] = \max_{\substack{k \text{ intervals} \\ \text{in } (i,j)}} \left\{ m[i,s_k] + m[f_k,j] + 1 \right\}$$



EFT

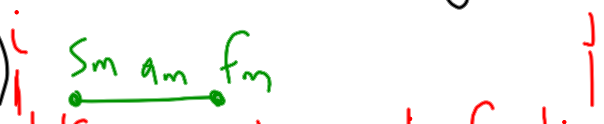
Let $a_m = (s_m, f_m)$ be the activity in (i, j) w/ earliest finishing time.

Then

greedy choice prop. → a) $\exists$ an optimal sol'n using a_m

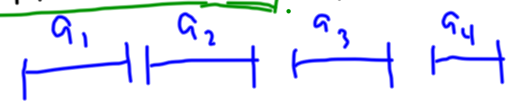
→ b) (i, s_m) is empty

Pf

b) 

would have earlier fin. time

a) suppose we have an opt. sol'n X w/o a_m



or 

then $X - a_1 + a_m$ is also an optimal sol'n.

because a_m does not overlap w/ $a_2, a_3, \ldots$

Greedy

- identify opt. substructure
- cast problem as one choice
  + recurse on remaining problem(s)
- prove $\exists$ an opt. sol'n consistent
  w/ first choice

Robbery

Rob a house

Enter w/ knapsack. Fill the knapsack w/ most profitable items

Item	1	2	3
w_j wt.	10	20	30
v_j value	60	100	120
v_j/w_j	6	5	4

$B = 50$ knapsack capacity

Problem Variants

integral (0-1) - take all or nothing of an item

fractional - take fractional parts of items

Both fractional & integral have optimal substructure.

Choose items by $\frac{v_j}{w_j}$ maximized

$1, 2, \frac{2}{3}$ of 3

50 lbs.

\$240

Integral - greedy does not work