

roughly

$$R(V) = E = O(V^2)$$

$E$  close to  $V$  - sparse

$E$  close to  $V^2$  - dense

$10^6$  node  
 $2 \times 10^6$  edge  
graph

Matrix has  $10^{12}$  entries  
only  $2 \times 10^6$  are non-zero

$$\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

for each  $u \in V$

for each  $v \in Adj(u)$

$O(1)$ .

How many times is an edge  <sup>$(x,y)$</sup>  processed  
by the alg?

2 times

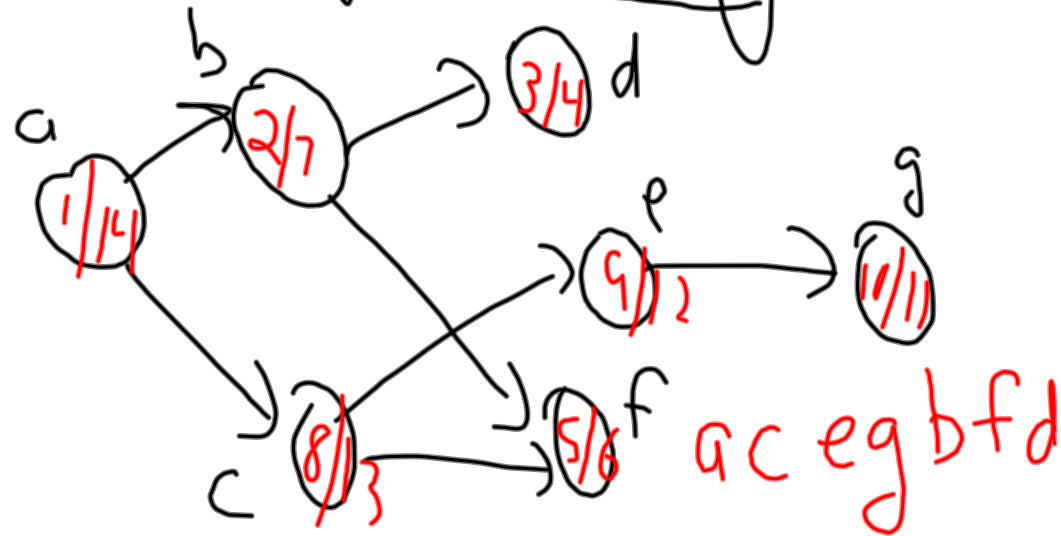
$O(V+E)$



$v$  is a desc $\rightarrow$  of  $u$  iff

$$d(u) < d(v) < f(v) < f(u).$$

# Topological Sorting



DAG - directed acyclic graph

Top. sort find an ordering s.t.

if  $(x, y)$  is an edge,  $x$  comes before  $y$ .

a b c d e g f

## Top Sort

- DFS
- Output nodes by decreasing finishing times

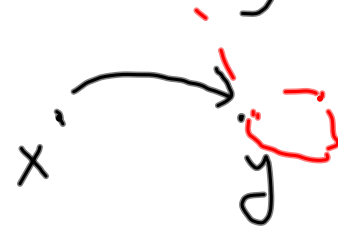
$O(V+E)$  time

Thm Alg is correct

(edge  $(x, y) \Rightarrow f(x) > f(y)$ )

Pf

case 1)  $d(x) < d(y)$



w.p.t.  $d(x) < d(y) < f(y) < f(x)$

case 2)  $d(x) > d(y)$

$\Rightarrow f(y) < d(x)$

$\Rightarrow d(y) < f(x) < d(x) < f(y)$