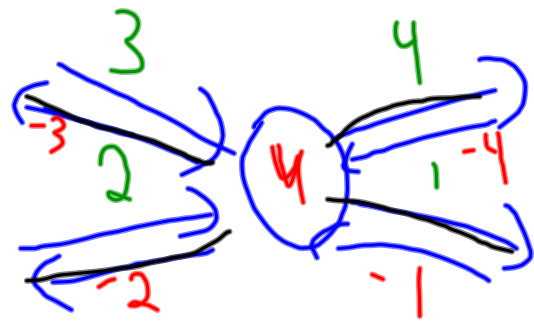
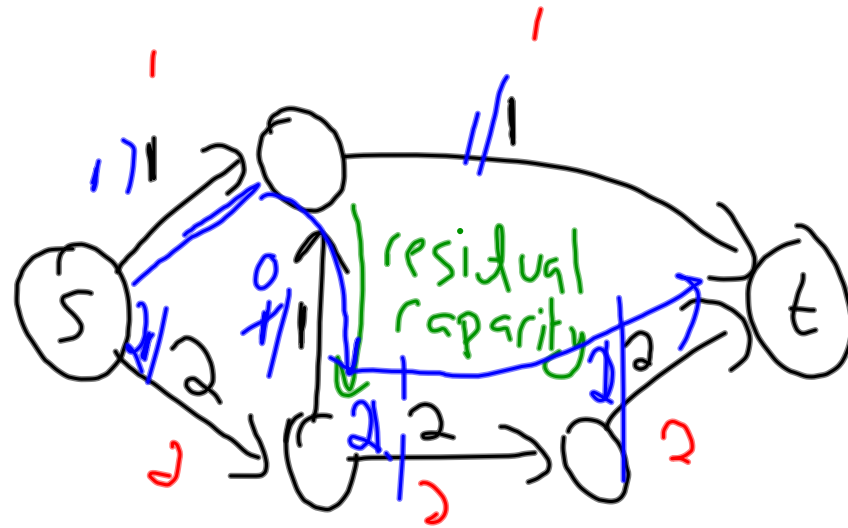


Folget specific notation

For each vertex, flow in
= flow out



$$-3 + -2 + 4 + 1 = 0$$



$$|f| = 2$$

$$|f| = 3$$

$$c_f(u,v) = c(u,v) - f(u,v)$$

(x to y)

$$\begin{array}{ccc} \textcircled{x} & \xrightarrow{c=5} & \textcircled{y} \\ & f=2 & \end{array} \quad \begin{array}{l} c_f = 5 - 2 \\ = 3 \end{array}$$

$$c_f(y,x) = c(y,x) - f(y,x)$$

$$0 - (-2) = 2$$

- 1) f max flow
- 2) G_f has no aug. paths
- 3) \exists a cut (S, T) w) $c(S, T) = |f|$.

$1 \Rightarrow 2$ $2 \Rightarrow \neg 1$

G_f has an aug path $\Rightarrow f$ not a max flow
 ✓

$2 \Rightarrow 3$



$3 \Rightarrow 1$

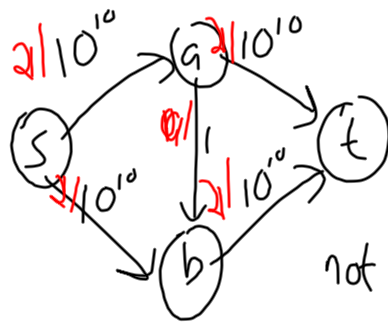
no edge from S to T in G_f
 in G , any edge $\overset{c(x,y)}{\text{from } S \text{ to } T}$
 has $f(x,y) = c(x,y)$
 $\therefore \sum_{\substack{x \in S \\ y \in T}} f(x,y) = \sum_{\substack{x \in S \\ y \in T}} c(x,y)$



duality

Running Time

1 iteration = $O(V+E)$
How many iterations?



At most
 $|F|$ iterations
 $|F| \leq \sum c(u,v)$
not polynomial
 $U = \max c(u,v)$

input $V+E + V \lg U$

time $U \cdot E$

you get a polytime alg by choosing
path intelligently
e.g. shortest augmenting path
max capacity path
...