SAT is NP-complete.

3-SAT : SAT but exactly 3 literals / clause (literal = var. or negation of variable)

$$\phi = (x_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge (x_4 \vee x_1 \vee x_2) \wedge (x_2 \vee x_5 \vee \bar{x}_3) \wedge (x_1 \vee x_5 \vee \bar{x}_6)$$

n vars.
m clauses

3SAT is a special case of SAT

1-SAT $x_1 \wedge x_2 \wedge \bar{x}_3 \wedge x_4 \ldots$

is easy

2-SAT $(x_1 \vee x_2) \wedge (x_1 \vee \bar{x}_2) \wedge (x_3 \vee x_4) \wedge \ldots$

is poly time.

Show 3-SAT is NP-complete

1. 3-SAT $\in$ NP
2. Choose SAT to reduce from
3. Given a instance of SAT, show how to create an instance of 3-SAT

Describe f, walks clauses by clause

k ≡ # literals in a clause

If $k = 1$

$$x_1 \xrightarrow{f} (x_1 \vee x_1 \vee x_1)$$

If $k = 2$

$$(x_1 \vee x_2) \xrightarrow{f} (x_1 \vee x_2 \vee x_2)$$

If $k = 3$

$$(x_1 \vee x_2 \vee x_3) \xrightarrow{f} (x_1 \vee x_2 \vee x_3)$$

If $k = 4$

$C = (x_1 \vee x_2 \vee x_3 \vee x_4) \xrightarrow{f} C' = (x_1 \vee x_2 \vee y_1)$

$\wedge (\bar{y}_1 \vee x_3 \vee x_4)$

For any setting of the x_i's.

C is true iff C' can be true.

$\Rightarrow$ If C is true at least one literal x_1, x_2, x_3, x_4 is true. If x_1 or x_2 is true set $y_1 = F$ then $\bar{y}_1$ is true & both clauses of C' are true.

If x_3 or x_4 is true, set $y_1 = T$, both clauses of C' are true.

$\Leftarrow$ If C is false, $x_1 = x_2 = x_3 = x_4 = F$. Then C' reduces to $y_1 \wedge \bar{y}_1$ which must be false

$k = 5$

$$(x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5) \xrightarrow{F}$$

$$(\overset{F}{x_1} \vee \overset{F}{x_2} \vee \overset{T}{y_1}) \wedge (\overset{F}{\bar{y}_1} \vee \overset{F}{x_3} \vee \overset{T}{y_2})$$

$$\wedge\ (\overset{F}{\bar{y}_2} \vee \overset{F}{x_4} \vee \overset{F}{x_5})$$

$$(x_1 \vee x_2 \ \ldots \ x_k) \to (x_1 \vee x_2 \vee y_1)$$

$$\wedge\ (\bar{y}_1 \vee x_3 \vee y_2)$$

$$\wedge\ (\bar{y}_2 \vee x_4 \vee y_3)$$

$$\vdots$$

$$\wedge\ (\bar{y}_{k-3} \vee x_{k-1} \vee x_k)$$

$k \le n$

Each clause C $\rightarrow$ atmost n clauses in C'

adds at most n variables

SAT
n vars.
m clauses
$\longrightarrow$
3SAT
vars $\le n + mn$
clauses $\le mn$

f is poly time

2SAT $\leq$ 3SAT

To show 2-SAT is NP-comp.

Show SAT $\leq$ 2-SAT

$(x_1 \vee x_2 \vee x_3)$ $(x_1 \vee y_1)$

$(\bar{y}_1 \vee x_2)$

(

SAT

$\underbrace{\qquad}_{\text{poly time}}$

3SAT( )

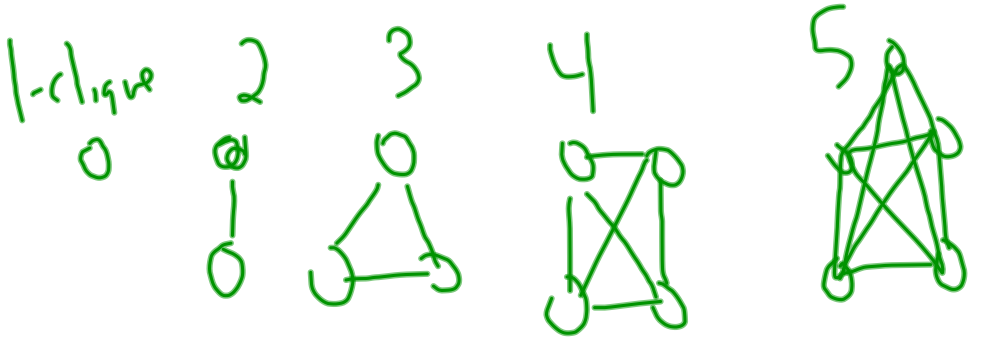
$\underbrace{\qquad}_{\text{poly time}}$

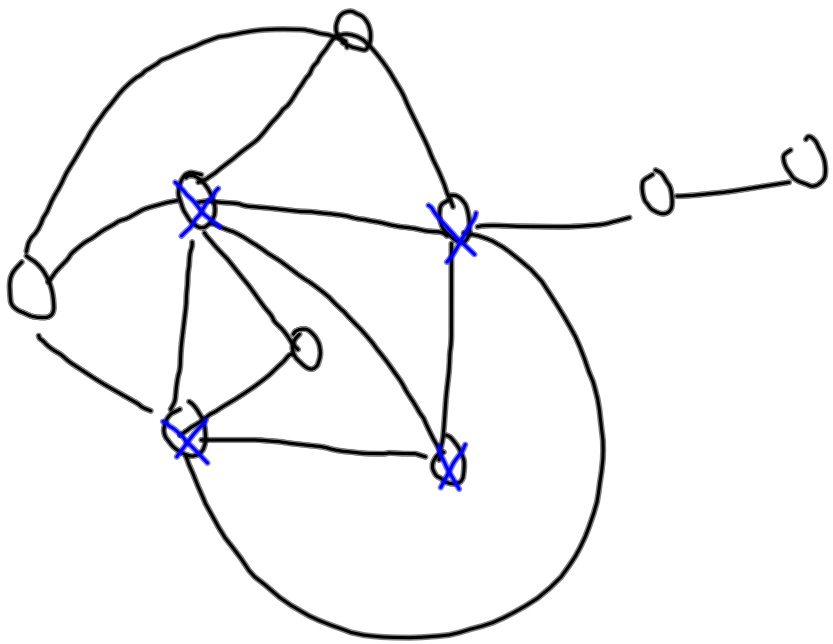
$\forall X \in NP \; X \leq SAT$

$SAT \leq 3SAT$

$\therefore \; \forall X \in NP \quad X \leq 3\text{-}SAT$

Clique Given a graph $G=(V,E)$ & an int. k, does G has a k-clique, that is, a subset $V' \subseteq V$ w/ $|V'|=k$ s.t. $\forall\, x,y \in V'$ $x \neq y$, edge (x,y) exists





Clique is NP-c.

1) Clique ∈ NP.

2) 3-SAT

$$\overset{C_1}{(x_1 \vee \bar{x}_2 \vee x_3)} \wedge \overset{C_2}{(\bar{x}_1 \vee x_2 \vee \bar{x}_3)} \wedge \overset{C_3}{(x_1 \vee x_2 \vee x_3)}$$

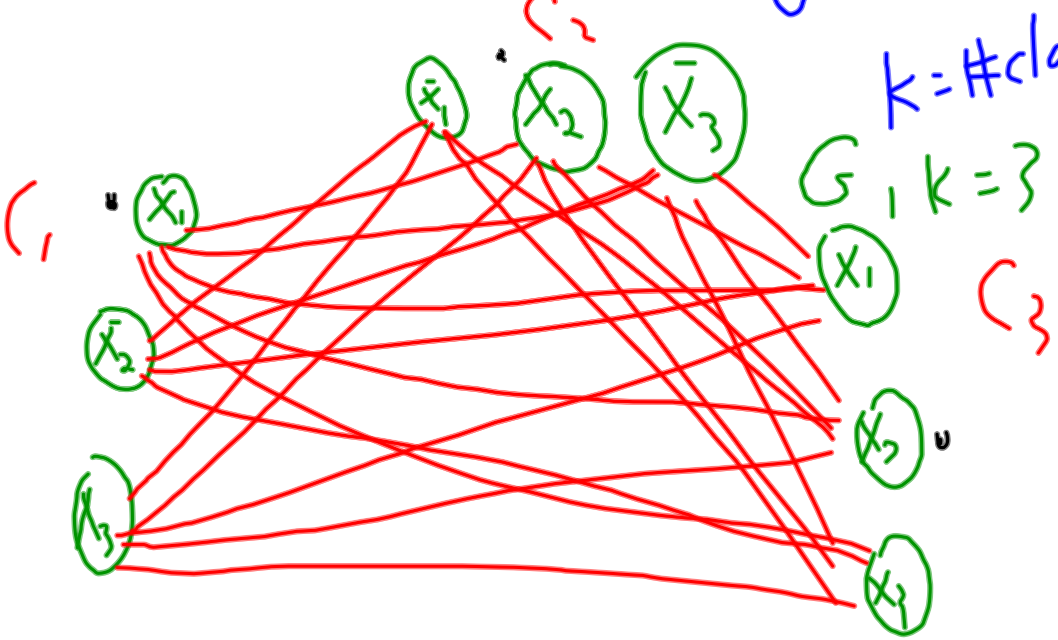
T F F   F T T   T T F

node for each appearance of var

edge between pairs of nodes in diff. clauses that can simultaneously be true

$k = \#\text{clauses}$

$G, k = 3$



ϕ is sat. iff G has a $\overset{(k)}{3}$-clique

$\Rightarrow$ ϕ is sat., given a sat. assignment, pick one literal per clause, by constuction all the edges are there

$\Leftarrow$ Suppose G has a k-clique, it must contain exactly 1 literal/clause, and by construction, all can be set to true simultaneously.