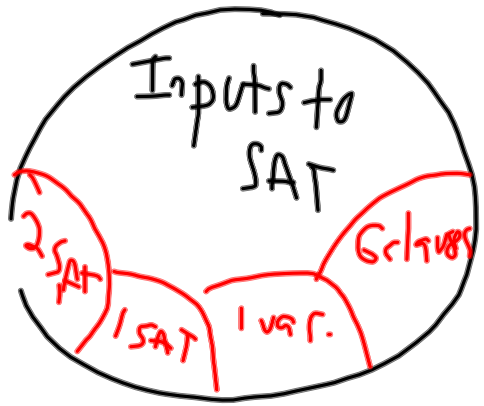
SAT

3-SAT

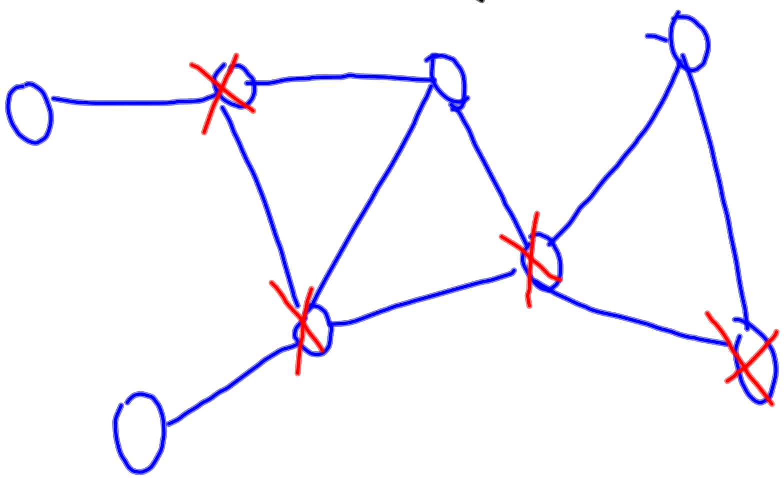
Clique

3-SAT $\leq$ Clique

no assumptions on the ~~3-~~SAT instance



Vertex Cover Given a graph $G=(V,E)$ & int k. A vertex cover is a subset $V' \subseteq V$ of verts. s.t. $\forall (x,y) \in E$, either $x \in V'$, $y \in V'$ (or both).
Is there a V.C. of size $= k$.



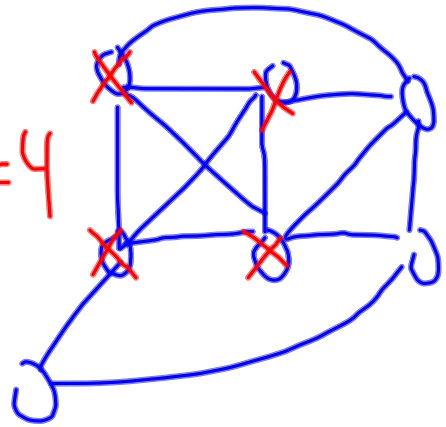
V.C. is NP-c.

1) V.C $\in$ NP.

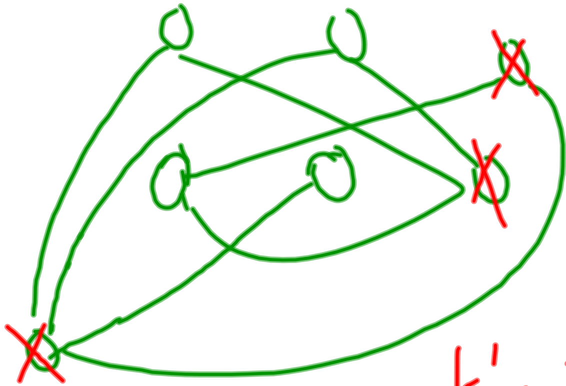
2) Clique $\leq$ V.C.

G

$k=4$

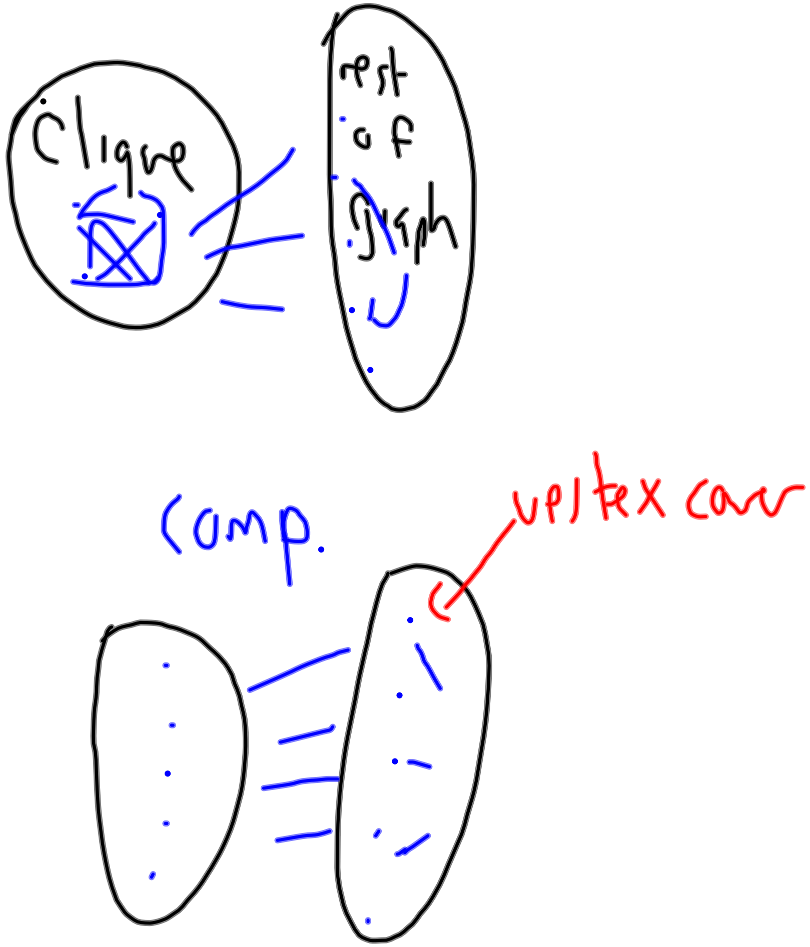


G^c



$k'=3$

V' is a clique of size k in G $\Longleftrightarrow$ $V-V'$ is a V.C. of size $|V|-k=k'$ in G^c


clique
rest of graph
comp.
vertex cover

Subset Sum

Given a set of integers S, target t.

Is there a $S' \subseteq S$ s.t. $\sum_{s_i \in S'} s_i = t$.

e.g.

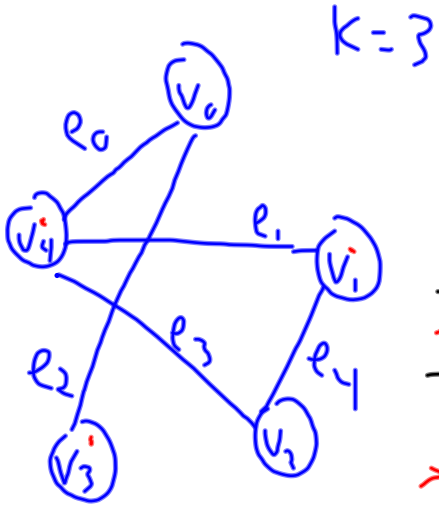
$\{1, 4, 16, 64, 256, 1040, 1041, 1093, 1284, 1344\}$

$t = 3754$

$S' = \{1, 16, 64, 256, 1040, 1093, 1284\}$

$SS \in NP$ ✓

VC ≤ S.S.



	e_4	e_3	e_2	e_1	e_0
v_0	0	0	1	0	1
→ v_1	1	0	0	1	0
→ v_2	1	1	0	0	0
→ v_3	0	0	1	0	0
→ v_4	0	1	0	1	1
	2	1	0	1	0
	1	1	1	2	1

$t = \{1\ 1\ 1\ 1\ 1\}$

choose a set of rows s.t. at least one "1" in each col. then I have a V.C.

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problems:

1) carries (base 4)

2) no unique target sum

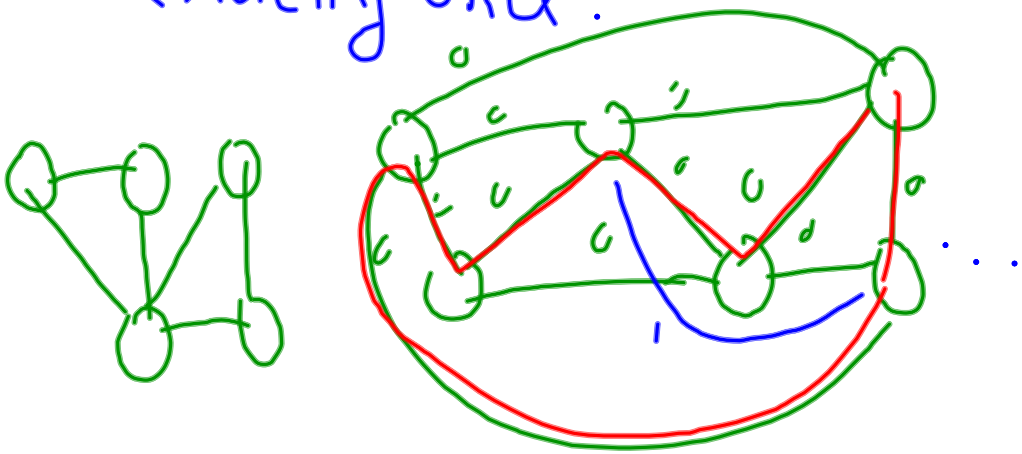
	velt	e_4	e_3	e_2	e_1	e_0	base 4
X_0	1	0	0	1	0	1	1041
→ X_1	1	1	0	0	1	0	1284
X_2	1	1	1	0	0	0	1344
→ X_3	1	0	0	1	0	0	1044
→ X_4	1	0	1	0	1	1	1093
→ Y_0	0	0	0	0	0	1	1
Y_1	0	0	0	0	1	0	4
→ Y_2	0	0	0	1	0	0	16
→ Y_3	0	0	1	0	0	0	64
→ Y_4	0	1	0	0	0	0	256
							3754
t	(3)	2	2	2	2	2	

- If V.C. of size k,
  then S.S. has a solution
- If S.S. has a sol, then
  G has a V.C. of size k

If VC is hard ⇒ SS in hard
If SS is easy ⇒ VC is easy

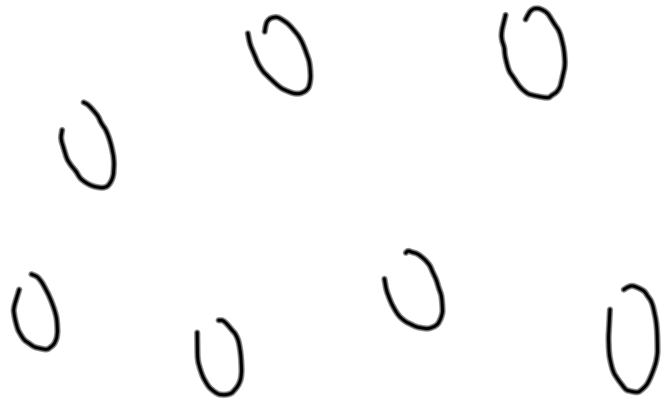
Hamiltonian Cycle

Given a graph $G=(V,E)$ is there a cycle visiting each vertex exactly once.



Travelling Salesman Problem

Given a graph $G=(V,E)$ and $d(u,v)$ between every pair of vertices, find the H.C. of minimum total distance.



HC $\leq$ TSP.

Red

Given G input to H.C.

make G', input to TSP

by setting $d(u,v) = 1$ if $(u,v) \notin E$

$d(u,v) = 0$ if $(u,v) \in E$.

HC is G $\iff$ TSP is G' has a tour of length 0.