Dealing w/ NP-complete Problems

- solve small instances
- solve instances w/ special structure
- Heuristics (find not necc. opt. solutions) (max, min problem)
  - greedy
  - simulated annealing, genetic algs., tabu search, GRASP, ...

no guarantee on quality of solution

Approximation Algs

Problem X, instance I, want to minimize

Let OPT(I) be the best possible solution (min)

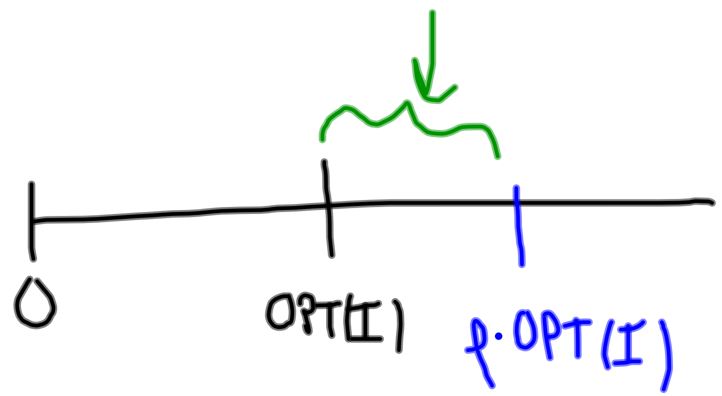
A ρ-approx. alg. for X, called A, on any input I,

1) runs in polynomial time

2) return a solution of value A(I) where

$$A(I) \le \rho \, OPT(I).$$

($\rho \ge 1$, small ρ are better)



If $\rho = 1.1$, 10% relative error

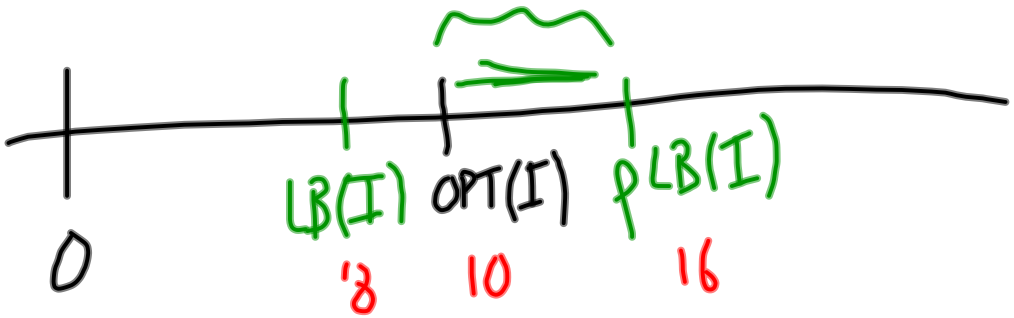
Need to accomplish this, w/o knowing OPT(I).

1) Compute a lower bound $LB(I)$ on $OPT(I)$, some poly time computable value (object) s.t. $LB(I) \leq OPT(I)$

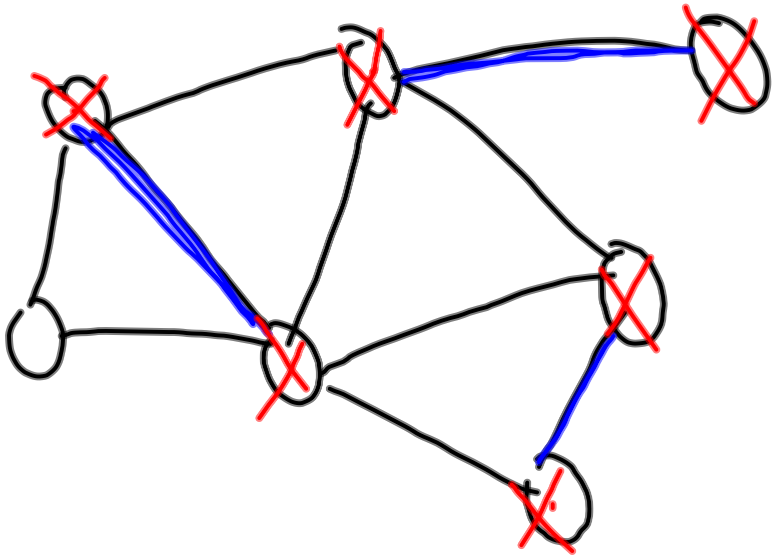
2) Use $LB(I)$ to find a solution $A(I)$ w/ $A(I) \leq \rho \cdot LB(I)$.

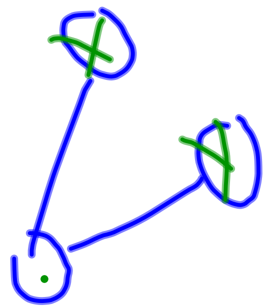
Conclude $A(I) \leq \rho\, LB(I) \leq \rho\, OPT(I)$

Really



VC Give a 2-approx alg ($\rho = 2$)





matching A set of edges that share no endpoints (Find in polynomial time)

maximum matching is matching w/ most edges

This is our lower bound

$MM(I)$ = size of MM

$OPT(I)$ = size of opt. vertex cover

$$MM(I) \leq OPT(I).$$

Alg.

1) Compute a max. matching

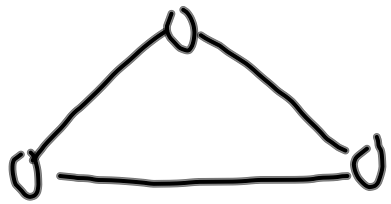
2) For each edge in the matching, include both endpoints in the vertex cover. (VC)

The set of vertices we choose is a vertex cover, because if some edge (u,v) has neither u nor v in the cover, then (u,v) could be added to the matching, contradicting the fact that we have a maximum matching.

$$VC(I) = 2\,MM(I) \leq 2\,OPT(I)$$

$$\therefore \quad VC(I) \leq 2\,OPT(I).$$

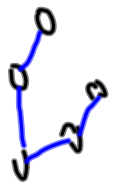
2-approx for symmetric TSP w/ Δ-inequality

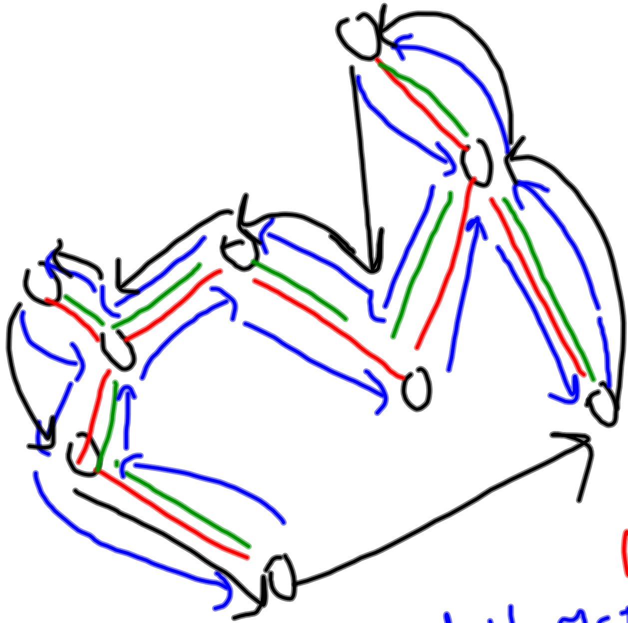


- $w(a,b)$ - dist betw. a & b
- $w(a,b) \geq 0$
- $w(b,a) = w(a,b)$

For any a,b,c · $w(a,b) \leq w(a,c) + w(c,b)$

w.o. Δ-ineq., you cannot approx. TSP (unless P=NP)





dist = eucl. dist.

LB = MST

OPT - opt. TSP tour

$MST(I) \leq OPT(I)$

double $MST(I) = 2\ MST(I)$

black tour $\leq$ double MST

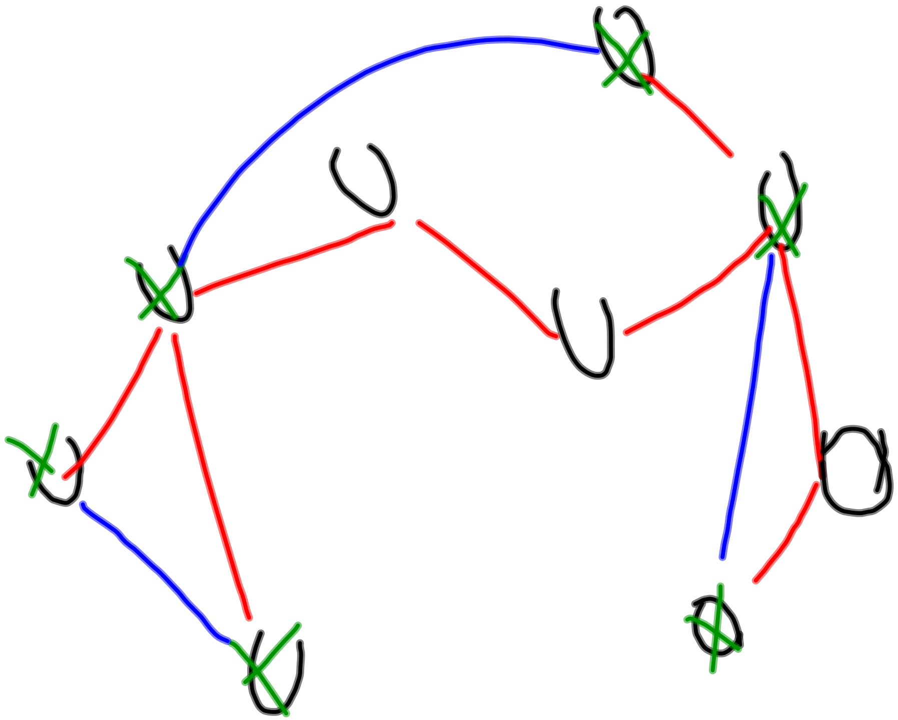
tour - connect all the vertices w/ edges

- edge vertex has degree 2

black tour $\leq$ double MST $= 2\ MST(I) \leq 2\ OPT(I)$.

2-approximation

1. Find MST
2. Double each edge
3. Find an Euler tour in doubled edges
4. Shortcut to Euler tour to a TSP tour



$$\text{Matching} \le \frac{1}{2}\,OPT(I)$$

$$|S| \le OPT(I)$$