VC, TSP 2-approx.

Set Cover $O(\lg n)$-approx.

Subset Sum $(1+\varepsilon)$-approx, for any $\varepsilon > 0$.

running time will depend on ε

FPTAS poly in $n, \frac{1}{\varepsilon}$

PTAS poly in n for fixed ε

PTAS $O(n^{1/\varepsilon})$ FPTAS $O(n^2 (\frac{1}{\varepsilon})^3)$

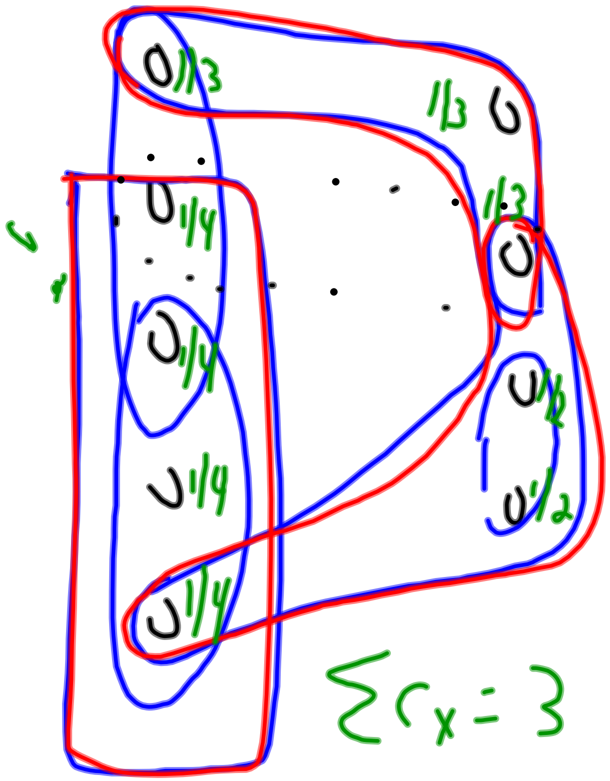
Set Cover

X = bunch of elements

$\mathcal{F}$ = family of subsets of X

A cover $C \subseteq \mathcal{F}$ s.t. $\bigcup_{S_i \in C} S_i = X$.

min $|C|$

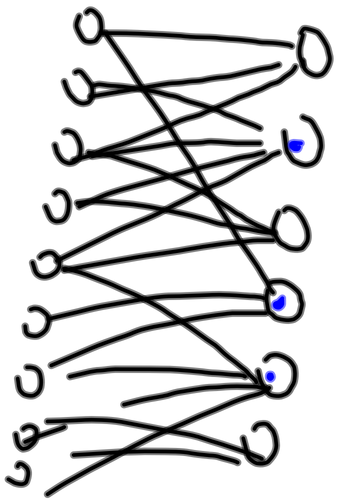


elements = skills

sets = people

Choose small set of people to cover all skills

elts.



sets

$\sum c_x = 3$

$|C| = 3$

look at an optimal sol'n and show that for any S in the opt. sol'n

$$\sum_{x \in S} c_x = O(\lg n)$$

Greedy

Repeat

pick the set w/ the max # of uncovered elements

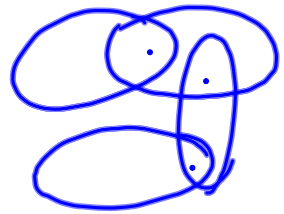
Alg chooses $S_1 \ldots S_k$

If x is covered for the first time in S_i

then

C - alg cover $\qquad c_x = \dfrac{1}{|S_i - (S_1 \cup \cdots \cup S_{i-1})|}$

C^* - opt. cover



$$|C| = \sum_{x \in X} c_x$$

$$\cancel{|C^*|} \le \sum_{S \in C^*} \underbrace{\sum_{x \in S} c_x}$$

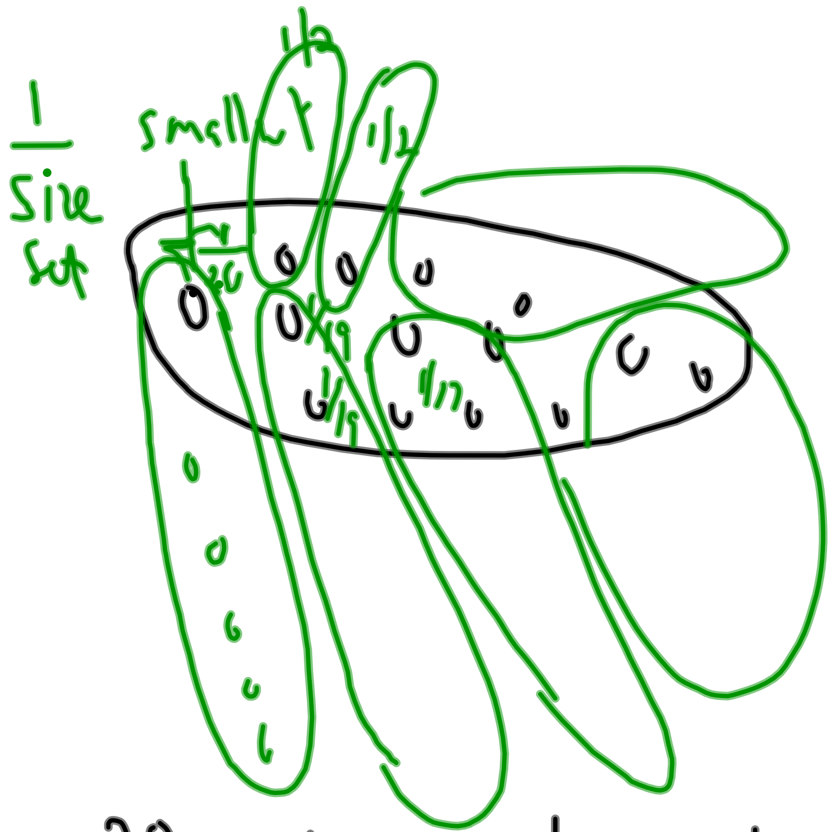
$\sum_{x \in X} c_x$

n = # elts.

will show

$$\sum_{x \in S} c_x = O(\lg n)$$

$$|C| = \sum_{x \in X} c_x \le \sum_{S \in C^*} \sum_{x \in S} c_x = \sum_{S \in C^*} O(\lg n) = |C^*| \cdot O(\lg n)$$



20
19
17
17
17
14
14
13

$$1 \cdot \frac{1}{20} + 2 \cdot \frac{1}{19} + 0 \cdot \frac{1}{17} + 3 \cdot \frac{1}{17} + \cdots$$

For $S \in C^*$ $\sum_{x \in S} c_x = O(\lg n)$

let u_i = # of uncovered elts in S after greedy chooses $S_1, \dots, S_i$

$u_0 = |S|$

$\vdots$

$u_k = 0$

In i^{th} iteration of greedy

$u_i - u_{i-1}$ elts. of S are covered, and assigned cost $\frac{1}{u_{i-1}}$

$$\sum_{x \in S} c_x = \sum_{i=1}^{k} (u_{i-1} - u_i) \cdot \frac{1}{u_{i-1}}$$

$$= \sum_{i=1}^{k} \sum_{i=u_i+1}^{u_{i-1}} \frac{1}{u_{i-1}}$$

$$H(x) = \sum_{j=1}^{x} \frac{1}{x}$$

$$\leq \sum_{i=1}^{k} \sum_{j=u_i+1}^{u_{i-1}} \frac{1}{j}$$

$$\leq \sum_{i=1}^{k} \left(\sum_{j=1}^{u_{i-1}} \frac{1}{j} - \sum_{j=1}^{u_i} \frac{1}{j} \right)$$

$H_0 - H_1$

$H_1 - H_2$

$H_2 - H_3$

$$\leq \sum_{i=1}^{k} \left(H(u_{i-1}) - H(u_i) \right) = H(u_0) - H(u_k)$$

$$\leq H(u_0) = H(|S|) \leq H(n) \leq O(\lg n)$$

$S = \{1, 4, 5\}$ $t = 8$

Find subset of S whose sum is $\leq t$, and is large as possible.

Careful Rounding - represent an exponential sized set approx by a polynomial sized set.

$$0,\ 1,\ \overset{4}{\underline{4,\ \cancel{5}}},\ 6,\ \overset{9}{\underline{9,\ \cancel{10}}}$$

{1,4,5} 7

0 0

1 0 1

4 0 1 4 5

5 0 1 4 5 ~~5~~ 6 9 10

0 1 4 5 6 9 10 7 8 11 12

13 16 17

Fix δ.

Trim$(L) \to L'$ remove some elts. w/ prop.

If $y \in L - L'$

then $\exists\, z \in L'$ s.t. $\frac{y}{1+\delta} \le z \le y$

$L = \langle 10, \cancel{11}, 12, 15, 20, \cancel{21}, \cancel{22}, 23, \cancel{24}, 29 \rangle$

$\delta = .1$ # elts. in list $\le \lg_{1.1} 29$.

$\longrightarrow \boxed{\log_{1+\delta} t}$ polynomial in n, $\frac{1}{\varepsilon}$, $\log t$

$\longrightarrow$ output is close to the best possible answer

$\ge (1-\varepsilon) t^*$

ε - desired accuracy

- add elt. to list
- Trim w/ $\delta = \frac{\varepsilon}{2n}$ $\quad n = |S|$

$$\log_{1+\delta} t = \frac{\ln t}{\ln 1+\delta} = \frac{\ln t}{\ln\left(1+\frac{\varepsilon}{2n}\right)}$$

$$\frac{x}{1+x} \le \ln(1+x) \le x$$

$$\le \frac{\ln t \left(1+\frac{\varepsilon}{2n}\right)}{\varepsilon/2n}$$

$$= \frac{2n \ln t}{\varepsilon} \cdot \left(1+\frac{\varepsilon}{2n}\right)$$

$$\le \frac{4n \ln t}{\varepsilon}$$

$$\left(1+\frac{\varepsilon}{2n}\right)^n \leq 1+O(\varepsilon)$$