Sorting

Heapsort $O(n\lg n)$ time

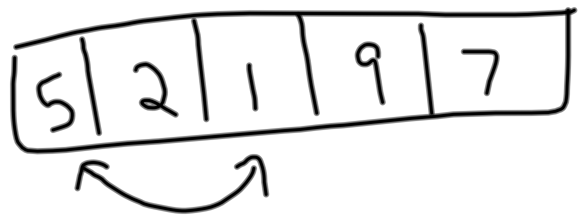
Several $O(n\lg n)$ time sorts

No $o(n\lg n)$ time sorts known.

We'd like to say, no sort runs in $o(n\lg n)$ time.

$\equiv$ There is no alg. that on all inputs runs in $o(n\lg n)$ time.

Sorts do comparisons & swaps



Restrict attention to comparisons.

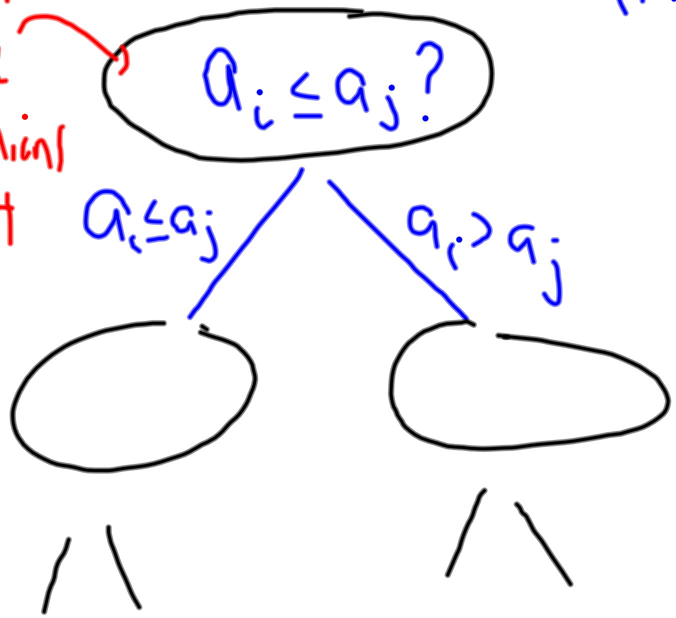
Any sorting alg. that accesses the data using only comparisons & swaps must do $\Omega(n \lg n)$ comparisons.

Assume: elements are distinct $\leq$

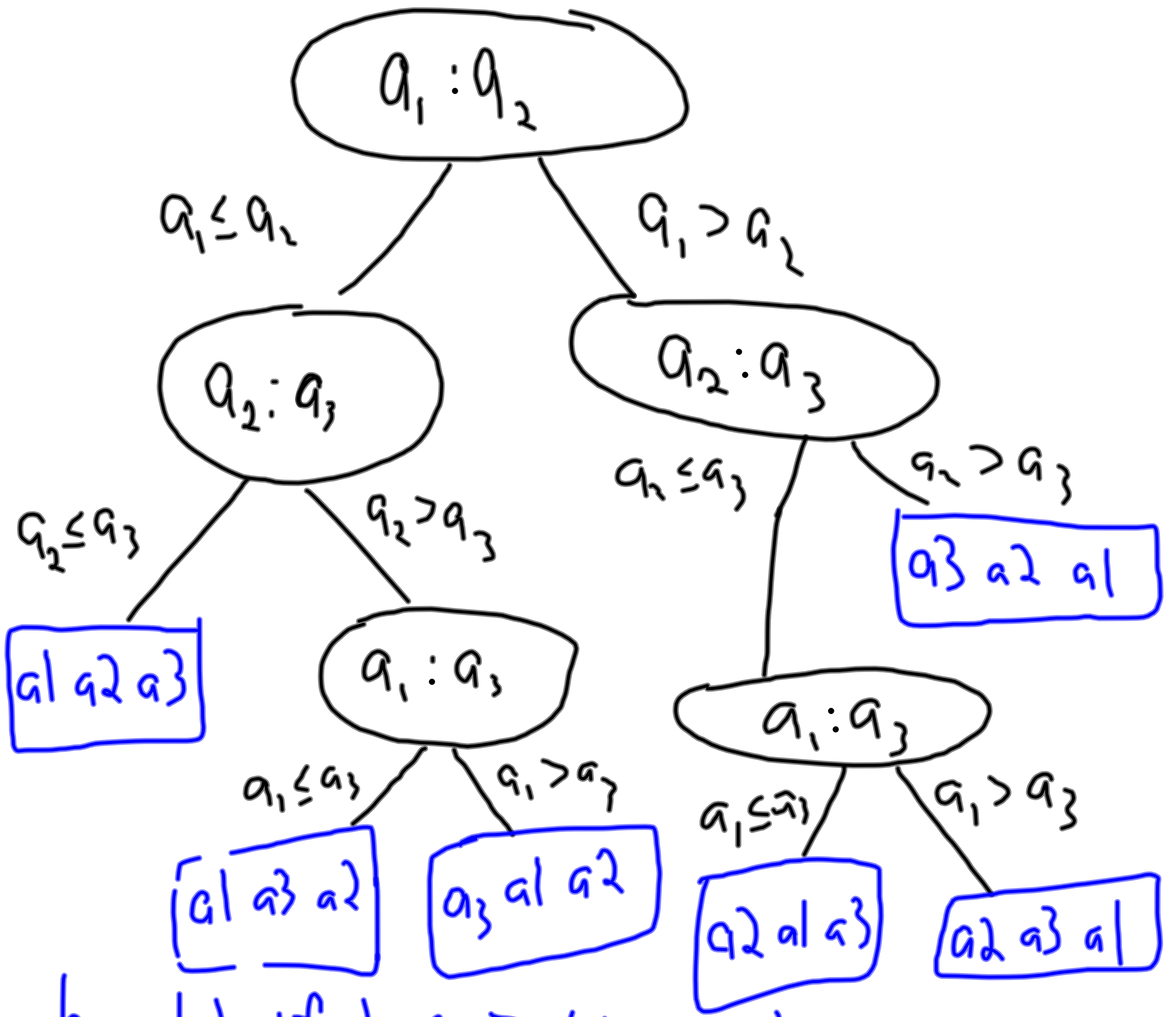
Decision Tree

$a_1 \dots a_n$

set of possible permutations of input



3 items $a_1\ a_2\ a_3$



height of tree = running time

Every permutation has to be at a leaf.

$$\#\text{leaves} \geq n!$$

$$\#\text{leaves} \leq 2^h$$

$$2^h \geq n!$$

$$\Rightarrow \quad h \geq \lg(n!)$$

$$h = \Omega(n \lg n).$$

$\Rightarrow$ any alg. must do at least $\Omega(n \lg n)$ comparisons.

Stirling's Approx.

$$n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n\left(1+\Theta\left(\frac{1}{n}\right)\right)$$

$$\sim \left(\frac{n}{e}\right)^n$$

$$\ln\left(\frac{n}{e}\right)^n = n\ln n - n\ln e$$

$$= n(\ln n - 1)$$

$$\sim n\lg n$$

C.S. $O(n+k)$

$1 \ldots k$

R.S. $O(d(n+b))$

$k = b^d$

d digits

b base

Col. dir.

$O(n + 27^{20})$

$O(20(n+27))$

$n = 20{,}000$

$\lg n = 15$

$O(n \lg n)$

integers ⇒ fixed precision floating point

⇒ rationals