# Selection

Given a set of $n$ numbers, want to find the $i^{th}$ smallest.

Cases

$i = 1$ minimum $\Theta(n)$

$i = n$ maximum

$i = n/2$ median

sort, return $A[n/2]$.

Median is "no harder than" sorting

Is median "easier than" sorting

$$T(n) = M + T\left(\frac{n}{2}\right) + O(n)$$

If $M = O(n)$

$$T(n) = T\left(\frac{n}{2}\right) + O(n)$$

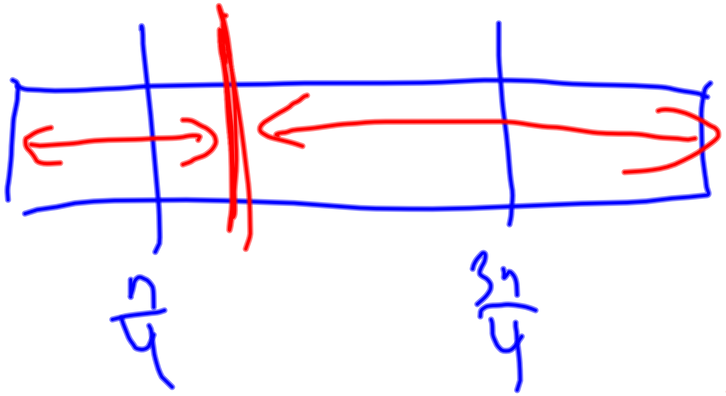
$n^{\lg_2 1}$ $\quad$ $n$

$1$ $\quad$ $n$

$O(n)$.

Find something "close" to median, and not increase assymptotic running time

Suppose we found an element in the middle half (between $\frac{n}{4}$ $\frac{3n}{4}$ smallest)




Always recurse on a piece of size $\leq \frac{3}{4}n$.

$$T(n) = T\left(\tfrac{3}{4}n\right) + O(n) = O(n)$$

$$n^{\log_{4/3} 1} \qquad n$$

$$T(n) \le T\left(\frac{n}{5}\right) + T\left(\frac{3n}{4}\right) + O(n)$$

Claim $T(n) = O(n)$

$$T(n) \le T\left(\frac{n}{5}\right) + T\left(\frac{3n}{4}\right) + kn$$

Prove $T(n) \le cn$ for some $c$

Pf

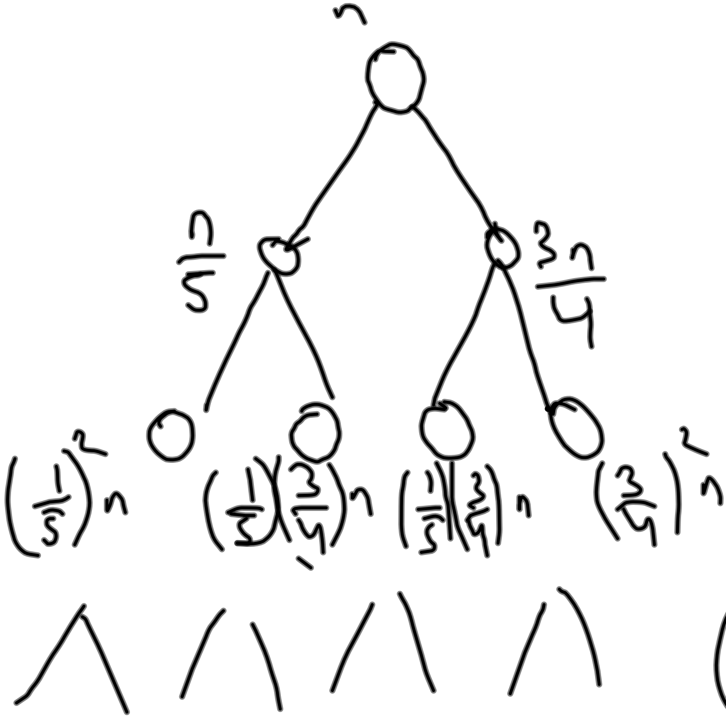
$$\begin{aligned} T(n) &\le T\left(\frac{n}{5}\right) + T\left(\frac{3n}{4}\right) + kn \\ &\le \frac{cn}{5} + \frac{3cn}{4} + kn \\ &= \frac{19}{20}cn + kn \\ &= cn - \underbrace{\frac{1}{20}cn + kn} \le cn \end{aligned}$$

if $-\frac{1}{20}cn + kn \le 0$

if $\frac{1}{20}cn \ge kn$

$c \ge 20k.$

Choose $c = 20k$. ⊗




$n$

$$\left(\frac{1}{5}+\frac{3}{4}\right)n = \frac{19}{20}n$$

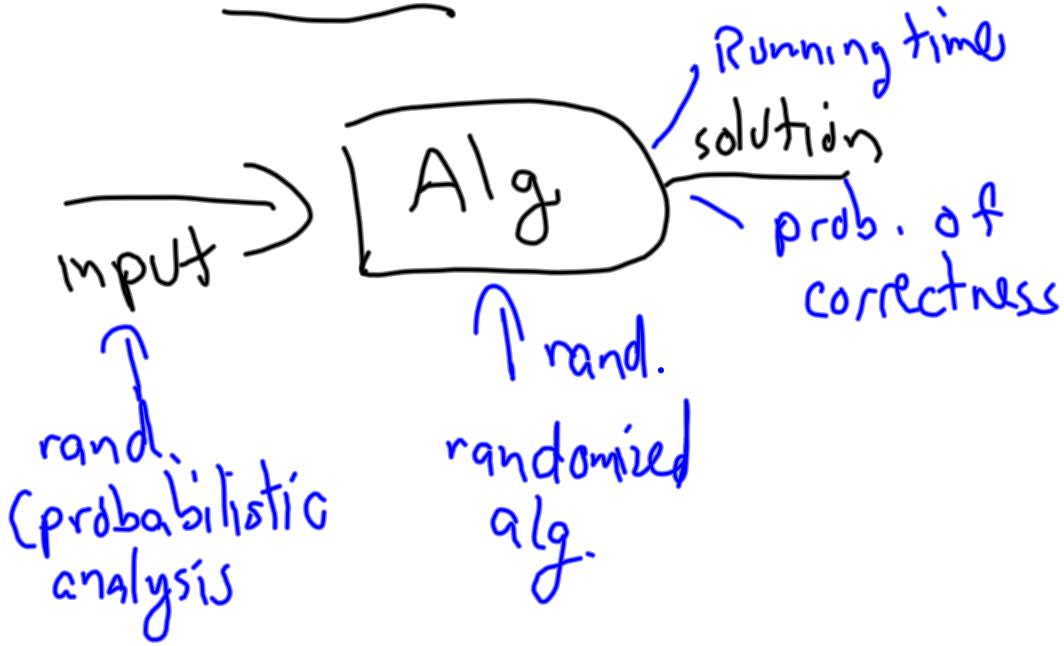
$$\left(\frac{1}{5}+\frac{3}{4}\right)^2 n = \left(\frac{19}{20}\right)^2 n$$

$$\left(\frac{1}{5}+\frac{3}{4}\right)^3 n = \left(\frac{19}{20}\right)^3 n$$

$$= O(n)$$

# Randomized Algs.

Randomization is a tool or resource.




Las Vegas
correct, running time is random

correctness random, running time is det.
Monte Carlo

# Hiring Problem

best = 0
for i = 1 to n
    interview candidate i
    if i is better than best
        hire i
        best = i

count # hirings.
$O(n)$ hirings
· $1 \le$ hirings $\le n$.

avg. is not $\frac{n}{2}$.

Assume all orderings of candidates are equally likely.

expected value $(X) = \sum_{\text{outcomes of } X} Pr(X \text{ occurs}) \cdot \text{Value}(X)$.

$h = \#$ hirings

$h(\pi_i) = \#$ hirings when the order of the candidates is permutation $\pi_i$

$$E[h] = \sum_{\text{perm. } \pi_i} \frac{1}{n!} h(\pi_i).$$