- all comparisons are with pivot.

$$T(n) = T(x) + T(n-x-1) + O(n)$$

when the split puts x elts. on left side.

if $x = \frac{n}{2}$ $\quad T(n) = 2T(\frac{n}{2}) + O(n) \Rightarrow O(n \lg n)$

$x = 1$ $\quad T(n) = T(1) + T(n-2) + O(n)$

$= T(n-2) + O(n)$

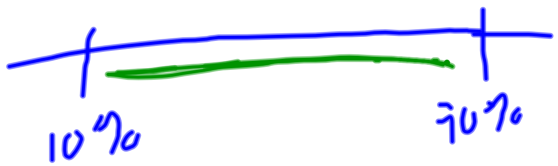
$= n + (n-2) + (n-4) + \cdots + 1 = O(n^2)$

What happens in avg. case?

If $\alpha = \frac{n}{10}$ $\quad T(n) = T\left(\frac{n}{10}\right) + T\left(\frac{9n}{10}\right) + O(n)$

$$= O(n \lg n).$$ (need proof)

How often is the split $\frac{1}{10}, \frac{9}{10}$ or better?



80% of time

Formally

$T(n)$ = expected running time of q.s.

$$T(n) = \sum_{\substack{i\text{th smallest} \\ \text{elt. is pivot}}} \Pr(i\text{th smallest is pivot}) \cdot \left(\text{Exp. running time when } i\text{th smallest is pivot}\right)$$

$$\boxed{T(n) = \sum_{i=1}^{n} \frac{1}{n}\left(T(i-1) + T(n-i) + O(n)\right)}$$

one can show by induction $T(n) = O(n \lg n)$

Different analysis:

Count executions of line 6

$\equiv$ counting comparisons

- all comps. w/ pivot
- each pair of elts. is compared at most once

rename data $z_1 \dots z_n$ in sorted order.

$$Z_{ij} = \{z_i, z_{i+1}, \dots z_j\}$$

$X_{ij} = I\{z_i \text{ is compared w/ } z_j\}$

$X = \text{total \# comps}$

$$X = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} X_{ij}$$

$$E[X] = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E[X_{ij}]$$

$$= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \Pr(z_i \text{ is compared w/ } z_j)$$

z_i is compared to z_j iff

either z_i or z_j is chosen as pivot

before any other element in Z_{ij}.

$$\Pr(z_i \text{ comp. to } z_j) = \frac{2}{j-i+1}$$

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{2}{j-i+1} \quad (k = j-i+1)$$

$j=i+1$
$k=i+1-i+1=2$
$j=n$
$k=n-i+1$

$$= \sum_{i=1}^{n-1} \sum_{k=2}^{n-i+1} \frac{2}{k} \le \sum_{i=1}^{n-1} \underbrace{\sum_{k=1}^{n} \frac{2}{k}} \le \sum_{i=1}^{n-1} 2(\ln n + 1)$$

$$= O(n \lg n).$$