

# Shifting Bottleneck Heuristic

## Algorithm

### 1. Initialization

- $M_0 = \emptyset$  (scheduled machines)
- $G$  = only conjunctive arcs
- $C_{\max}$  = critical path in  $G$ .

### 2. (Choice of machine.) For each $M_i \in M - M_0$ ,

- generate the  $1|r_j|L_{\max}$  schedule
- compute  $L_{|max}(i)$ .

### 3. Scheduling the bottleneck machine

- Let  $k$  be the machine that maximizes  $L_{\max}(i)$
- Schedule  $k$  by the  $1|r_j|L_{\max}$  solution
- Update  $G$
- $M_0 = M_0 \cup \{k\}$ .

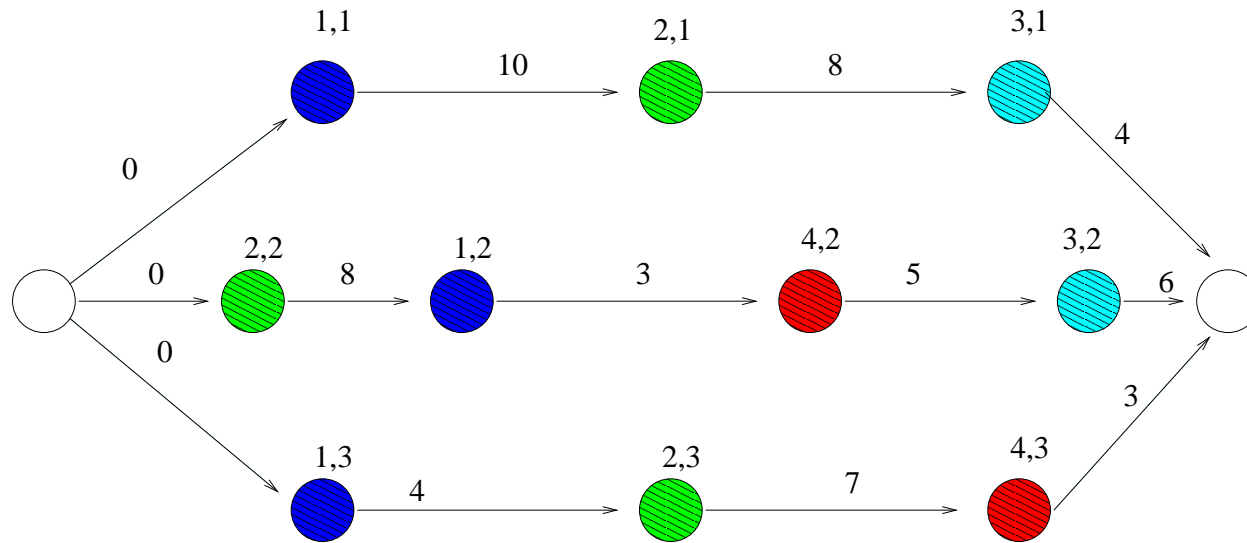
### 4. (Resequence already scheduled machines.) For each $M_i \in M_0 - \{k\}$

- Delete disjunctive arcs for  $M_i$  from  $G$
- Form the  $1|r_j|L_{\max}$
- Reschedule  $M_i$  according to this schedule

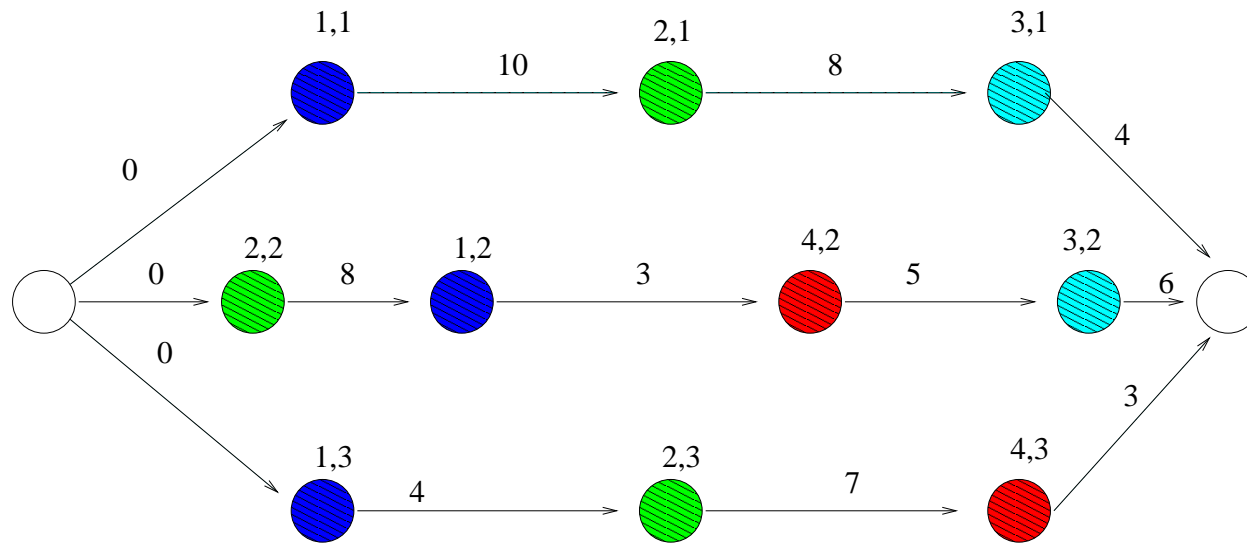
### 5. If $M = M_0$ stop, else go to 2

# Example

jobs	machine sequence	processing times
1	1,2,3	$p_{11} = 10, p_{21} = 8, p_{31} = 4$
2	2,1,4,3	$p_{22} = 8, p_{12} = 3, p_{42} = 5, p_{32} = 6$
3	1,2,4	$p_{13} = 4, p_{23} = 7, p_{43} = 3$



# Iteration 1



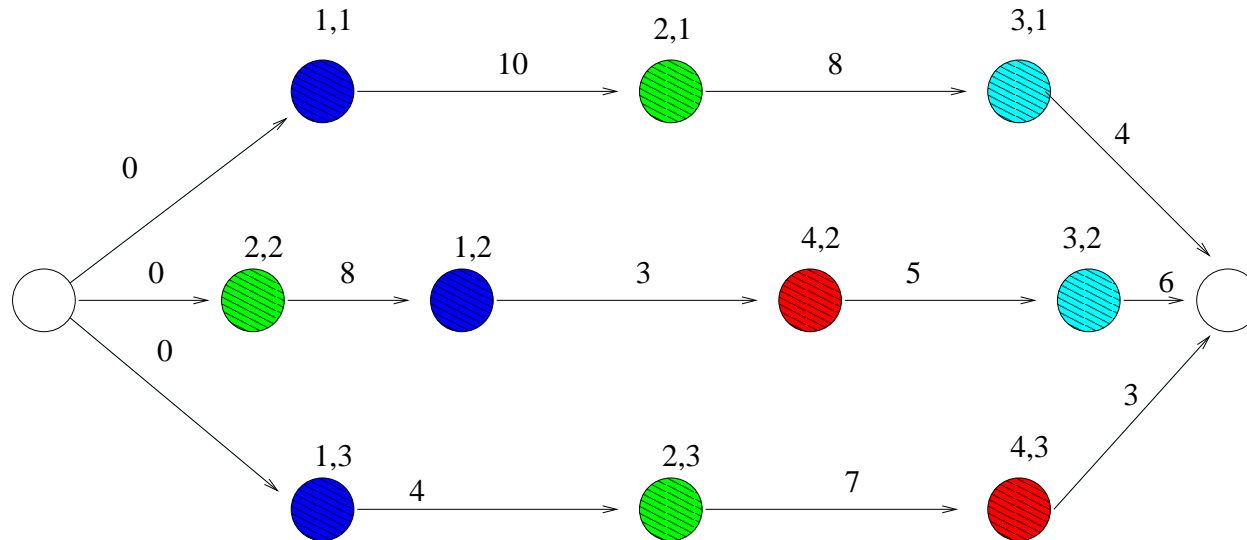
Form the  $1|r_j|l_{\max}$  problems (Recall  $d_j = LB - (CP - p_j)$ )

## Machine 1

job	1	2	3
$r_j$	0	8	0
$p_j$	10	3	4
$d_j$	10	11	12

Optimal schedule 1,2,3,  $L_{\max}(1) = 5$

## Iteration 1 (cont)



Form the  $1|r_j|l_{\max}$  problems

### Machine 2

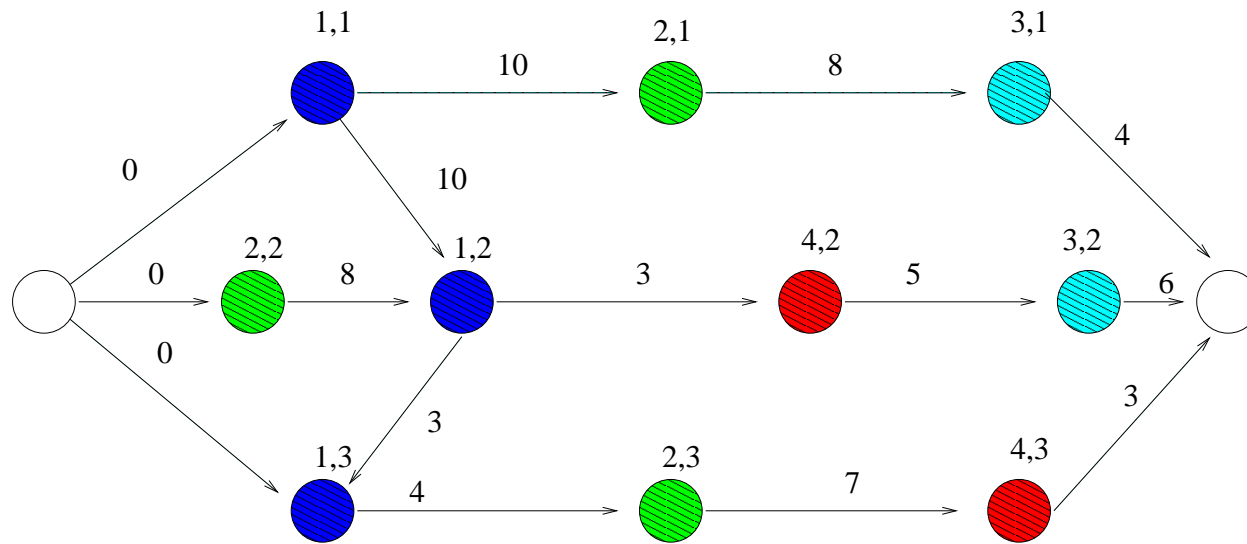
job	1	2	3
$r_j$	10	0	4
$p_j$	8	8	7
$d_j$	18	8	19

**Optimal schedule** 2,3,1  $L_{\max}(2) = 5$

**Similarly**  $L_{\max}(3) = 4$ .  $L_{\max}(4) = 0$ .

**Schedule**  $M_1$  in the order 1,2,3.

## Iteration 2



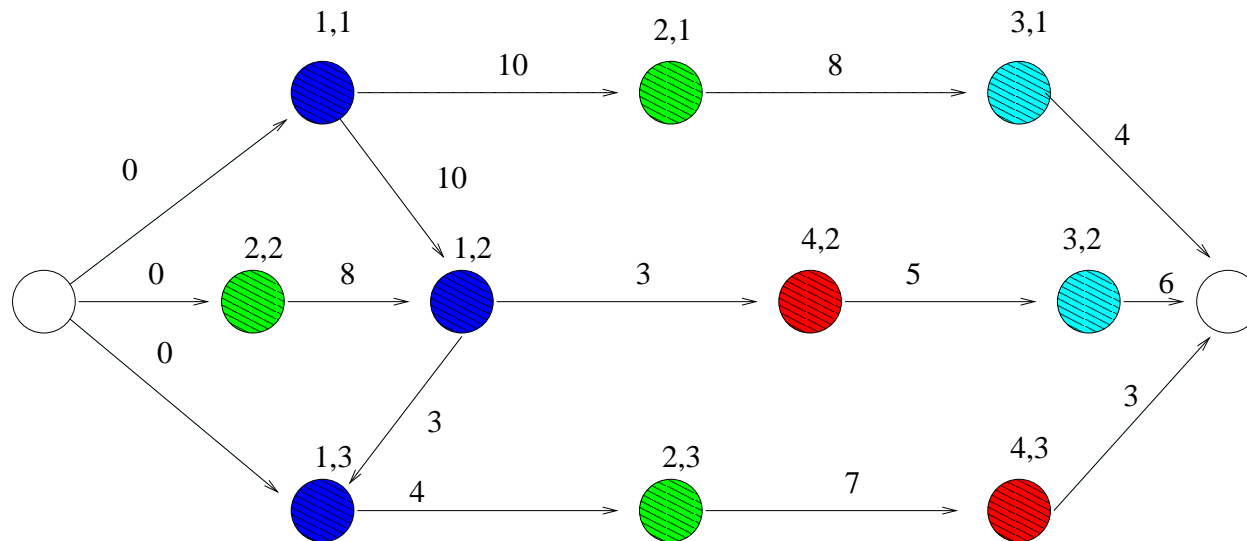
Form the  $1|r_j|l_{\max}$  problems

### Machine 2

job	1	2	3
$r_j$	10	0	17
$p_j$	8	8	7
$d_j$	23	10	14

Optimal schedule  $2,1,3$ ,  $L_{\max}(2) = 1$

## Iteration 2 (cont)



Form the  $1|r_j|L_{\max}$  problems

### Machine 3

job	1	2
$r_j$	18	18
$p_j$	4	6
$d_j$	27	27

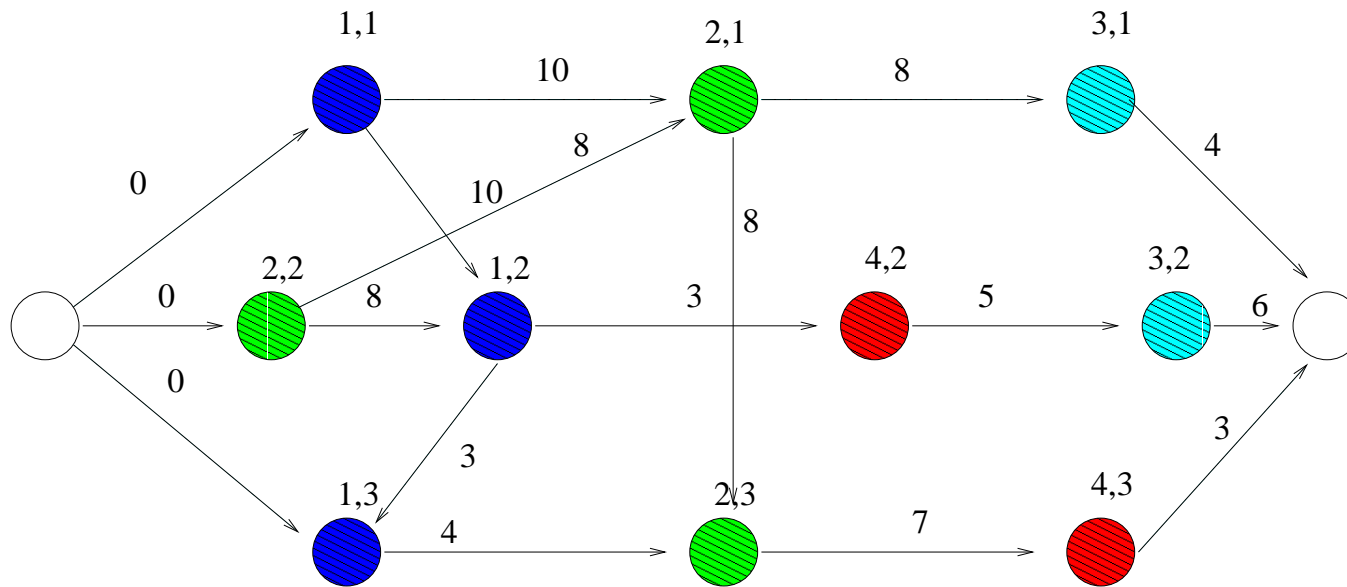
**Optimal schedule** (either),  $L_{\max}(3) = 1$

**Similarly**  $L_{\max}(4) = 0$ .

**Schedule**  $M_2$  in the order 2,1,3.

**Resequence** Trying to resequence machine 1 does not help.

## Iteration 3

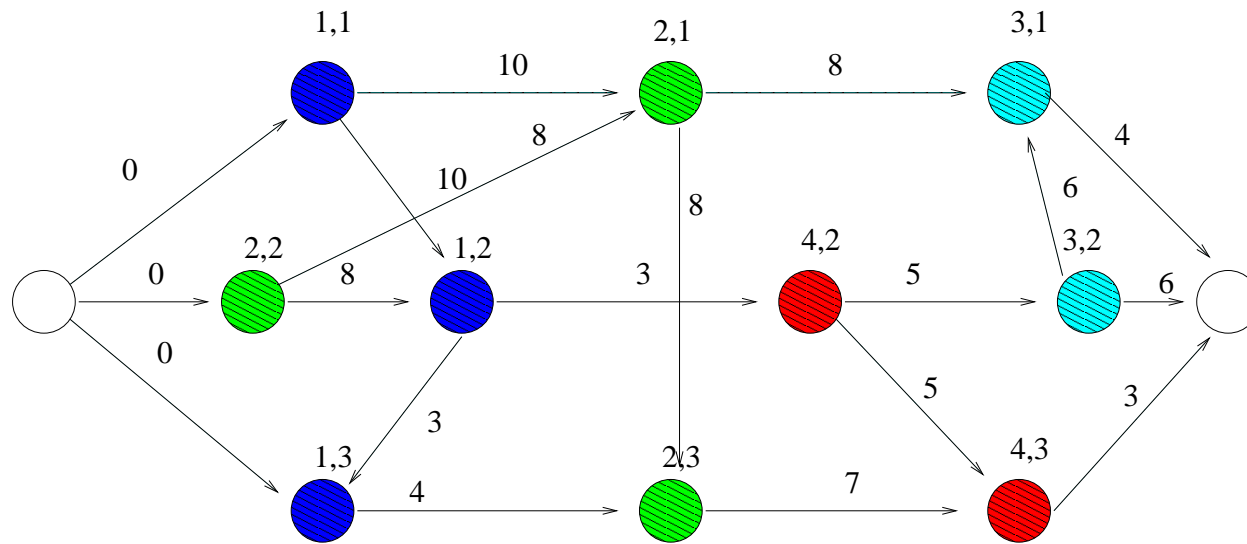


Form the  $1|r_j|L_{\max}$  problems

Machine 3 and Machine 4 both have  $L_{\max}() = 0$ .

Resequencing does not help.

# Final schedule



Critical path length is 28. (Colors between pictures do not correspond)

