Practice mid term solutions

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1 (a) Optimal rule (WSPT)
   (b) NP Hard
   (c) Preemptive EDD
   (d) NP Hard
   (e) NP Hard
   (f) NP hard
   (g) SRPT-FM

2 There are many lower bounds, am listing only 1 here. (a) 60, 1 machine has to process the job with $p_j = 60$
   (b) Find LB on page 128 of the text.
   (c) 71, 60 can be processed only after 10 which in turn can be processed only after 1.

3 (a) The sense of intuition should help you solve the problem. The jobs that are given to be not completed by their deadlines can be scheduled last immaterial of the order. The objective value would only depend on these jobs. The deal is to arrange the jobs that meet the deadlines in a feasible way. All these jobs have $U_j = 0$. These set of jobs have to be scheduled according to EDD in order to make sure that $L_m ax$ is negative.

   (b) Use part (a). Run the algorithm in (a), for the indicated (k-1) jobs + 1 job from the remaining $n - k + 1$ jobs. In total, $n - k + 1$ times. The optimal schedule is the one with minimum objective value among all these schedules.

4 SPT rule is optimal. Prove it based on exchange argument ideas. Suppose SPT is not optimal, there exists a pair of adjacent jobs in the claimed optimal schedule ($S1$) such that $p_j < p_k$ and j is processed after job k. Keeping all other jobs the same, switch the positions of jobs j and k to obtain a new schedule ($S2$). Obj($S2$) - Obj($S1$) $= 2^{t+p_j} - 2^{t+p_k}$ where t is the starting time of processing job k in the original schedule. Clearly, Obj($S2$) - Obj($S1$) le 0. resulting in the contradiction $S1$ is optimal. Hence, SPT is an optimal schedule.

5 Suppose you have the solution to the 3 partition problem, to wit, you have a way to partition $3t$ numbers into t sets of 3 numbers each such that each sum upto b. Take the above $3t$ numbers and append t more ones. ($1's$) and pose the $4t$ partition question on the above set with $b' = b + 1$. The solution to the $4t$ partition problem then is to take the solution of the $3t$ partition problem and append the element 1 to each group. Hence, we have reduced 3

partition to an instance of 4 partition in polynomial time. Since, 3P is NP hard, hardness of 4P also follows.