Reductions

Reduction: **Problem A reduces to Problem B if, given a "black box" (subroutine) for B, one can solve A using a (polynomial) number of calls to the subroutine.**

Trivial Example:

- **B is addition –** $B(x, y) = x + y$
- **A multiplication by 3.**
- **A reduces to B because we can multiply by 3 :** $A(z) = B(z, B(z, z))$ **.**

More Reduction Examples

- **A is max flow, B is linear programming**
- **A is** $1||\sum C_j$ **, B is** $1||\sum w_j C_j$
- **A is** $P||C_{\max}$ **, B is** $P|prec|\sum w_j C_j$

Reductions for NP-completeness

- **For technical reasons, We will only consider decision versions of problems.**
- **e.g. $P||C_{\max}$; Given m machines, n jobs and a number B, does the optimal schedule have makespan less than B.**
- **e.g. Shortest Paths: Given a graph G with weights on the edges, two distinguished vertices s and t and a number B, is the shortest path from s to t of length less than B .**
- **The decision version and the optimization version of a problem are "equivalent," that is they each reduce to each other.**

Reduction Example

Vertex Cover A **vertex cover** of a graph **G=(V,E)** is a set of vertices **V'**, such that for every edge (x,y), at least one of x and y is in **V'**. The vertex cover problem is given a graph **G** and a number **k** and asks whether **G** has a vertex of size at most **k**.

Clique A **clique** is a set of vertices such that each pair of vertices has an edge between them. The **clique problem** is given a graph and a number ℓ and asks when a graph has a clique of size at least ℓ .

Question: Show that vertex cover reduces to clique.