Tanker Scheduling

Ships have:

- capacity
- draught (minimum depth to float)
- range of speeds and fuel consumption
- location and available time

Ports have:

- weight limits
- draught
- other physical restrications
- government restrictions

Tanker Scheduling (cont)

Cargo has

- type
- load port
- destination prot
- time constraints
- load and unload times

A company will own ships and may rent ships. It is more expensive to rent ships.

Objective: minimize cost

- operarting costs for company shipos
- charter rates
- fuel costs
- port charges

Formulation

Notation:

Parameters

- n - number of cargoes
- T - number of company owned tankers
- p - number of ports

plus data for all of the above.

Compute

- S_i - the set of possible schedules for ship i. $a_{ij}^l = 1$ if under schedule l ship i transports cargo j.
- c_j^* is amount paid to transport cargo j on a ship that is not company owned.
- c_i^l - incremental cost of operating a company-owned ship i under schedule l versus keeping ship i idle.
- Compute the profit for operationg ship i according to schedule l $- \pi_i^l = \Sigma_{j=1}^n a_{ij}^l c_j^* - c_i^l$.

Formulation

Decision variable: x_i^l **if ship** i **follows schedule** l**.**

Formulation

$$\begin{aligned} \textbf{maximize} \quad & \sum_{i=1}^{T} \sum_{l \in S_i} \pi_i^l x_i^l \\ \textbf{subject to} \quad & \\ \sum_{i=1}^{T} \sum_{l \in S_i} a_{ij}^l x_i^l & \leq 1 \quad j = 1, \ldots, n \\ \sum_{l \in S_i} x_i^l & \leq 1 \quad i = 1, \ldots, T \\ x_i^l & \in \{0,1\} \quad l \in S_i, i = 1, \ldots, T \end{aligned}$$

Solution **Set packing. Use branch and bound.**

Example

- 3 ships
- 12 cargoes

Analysis of the data show that each of the ships has five feasible schedules:

Schedules	a^1_{1j}	a^2_{1j}	a^3_{1j}	a^4_{1j}	a^5_{1j}	a^1_{2j}	a^2_{2j}	a^3_{2j}	a^4_{2j}	a^5_{2j}	a^1_{3j}	a^2_{3j}	a^3_{3j}	a^4_{3j}	a^5_{3j}
cargo 1	1	0	0	1	1	0	1	0	0	0	0	0	0	1	0
cargo 2	1	0	0	0	0	1	0	0	0	0	0	1	0	1	1
cargo 3	0	0	1	0	1	0	0	0	1	1	0	0	0	0	0
cargo 4	0	1	1	1	0	1	0	1	0	0	0	0	0	0	0
cargo 5	1	1	0	0	0	0	0	0	1	0	0	0	1	0	1
cargo 6	0	0	0	1	1	0	1	0	0	1	1	0	0	0	0
cargo 7	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1
cargo 8	0	1	0	0	0	0	1	0	1	1	1	0	0	0	0
cargo 9	0	0	1	0	0	0	1	0	0	1	1	1	1	0	0
cargo 10	0	1	0	0	0	1	0	0	0	0	1	1	0	0	0
cargo 11	0	0	0	0	0	0	1	1	0	0	0	1	1	1	0
cargo 12	0	0	0	1	0	0	0	0	0	0	1	0	1	1	1

Costs

Charter cost (CC) for transporting a particular cargo by charter:

Cargo	1	2	3	4	5	6	7	8	9	10	11	12
CC	1429	1323	1208	512	2173	2217	1775	1885	2468	1928	1634	741

Operating costs of the tankers under each one of the schedules is also given:

Schedule l	1	2	3	4	5
cost of tanker 1 (c_1^l)	5608	5033	2722	3505	3996
cost of tanker 2 (c_2^l)	4019	6914	4693	7910	6866
cost of tanker 3 (c_3^l)	5829	5588	82824	3338	4715

We can compute the profit for each schedule

Schedule l	1	2	3	4	5
profit of tanker 1 (π_1^l)	-683	1465	1466	1394	858
profit of tanker 2 (π_2^l)	1629	834	1113	-869	910
profit of tanker 3 (π_3^l)	1525	1765	-1268	1789	1297

IP

Now we can give an IP

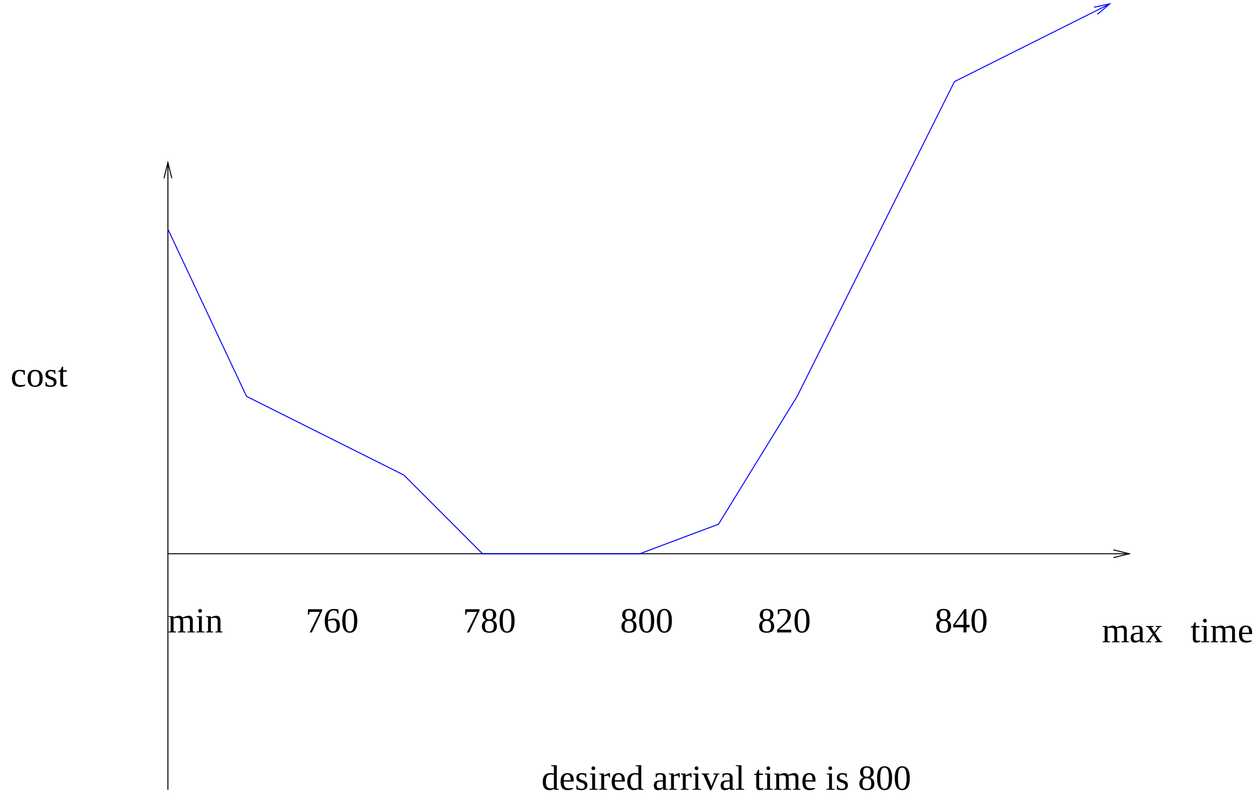
$$
\begin{aligned}
\textbf{maximize} & -733x_1^1 + 1465x_1^2 + 1466x_1^3 + 1394x_1^4 + 858x_1^5 \\
& +1629x_2^1 + 834x_2^2 + 1113x_2^3 + -869x_2^4 + 910x_2^5 \\
& +1525x_3^1 + 1765x_3^2 + -1268x_3^3 + 1789x_3^4 + 1297x_3^5 \\
& \textbf{subject to} \\
x_1^1 + x_1^4 + x_1^5 + x_2^2 + x_3^4 &\leq 1 \\
x_1^1 + x_2^2 + x_3^2 + x_3^4 + x_3^5 &\leq 1 \\
x_1^3 + x_1^5 + x_2^4 + x_2^5 &\leq 1 \\
x_1^2 + x_1^3 + x_1^4 + x_2^1 + x_2^3 &\leq 1 \\
x_1^1 + x_1^2 + x_2^4 + x_3^3 + x_3^5 &\leq 1 \\
x_1^4 + x_1^5 + x_2^2 + x_2^5 + x_3^1 &\leq 1 \\
x_2^3 + x_2^4 + x_3^5 &\leq 1 \\
x_1^2 + x_2^1 + x_2^3 + x_2^4 + x_2^5 &\leq 1 \\
x_1^3 + x_2^2 + x_2^5 + x_3^1 + x_3^2 + x_3^3 &\leq 1 \\
x_1^2 + x_2^1 + x_3^1 + x_3^2 &\leq 1 \\
x_2^2 + x_2^3 + x_3^2 + x_3^3 + x_3^4 &\leq 1 \\
x_1^4 + x_3^1 + x_3^3 + x_3^4 + x_3^5 &\leq 1 \\
\\
x_1^1 + x_1^2 + x_1^3 + x_1^4 + x_1^5 &\leq 1 \\
x_2^1 + x_2^2 + x_2^3 + x_2^4 + x_1^5 &\leq 1
\end{aligned}
$$

$$x_3^1 + x_3^2 + x_3^3 + x_3^4 + x_3^5 \leq 1$$
$$x_i^l \in \{0, 1\}$$

Optimal solution Schedule 3 for ship 1, schedule 4 for ship 3. Ship 2 remains idle. Cargoes 5,6,7,8,10 are transported by charters. Value = 3255.

Train timetabling

- **One track with many stations (think 1/9 subway line or commuter rail).**
- **Trains can pass at statations but not between stations.**
- **Stations are numbered 0 to L.**
- **Tracks are numbered 1 to $L+1$.**
- **Track i connectes station $j-1$ with j.**
- **Time is measured in minutes (1 to 1440).**
- **You are given preferred arrival, departure, travel, and holdover times for each train at each station.**
- **There is a piecewise linear function measuring the cost (lost revenue) of deviating from the desired time.**


cost
min
760
780
800
820
840
max
time
desired arrival time is 800

IP

Variables

- y_{ij} = **time train** i **enters link** j **(leaves station** $j-1$**)**
- z_{ij} = **time train** i **exits line** j **(arrives at station** j**)**

We compute

- $\tau_{ij} = z_{ij} - y_{ij}$ **(travel time of train** i **in link** j**)**
- $\delta_{ij} = y_{i,j+1} - z_{ij}$ **(dwelling time of train** i **in station** j**)**

We are given costs for each of these quantities:

- $c^a_{ij}(z_{ij})$ **- costs for train** i **arriving at station** j
- $c^d_{ij}(y_{ij})$ **- costs for train** i **departing from station** j
- $c^\tau_{ij}(\tau_{ij})$ **- costs for travel time of train** i **in link** j
- $c^\delta_{ij}(\delta ij)$ **- costs for travel time of train** i **dwelling in station** j**.**

Each of these costs is piecewise linear, and we are given min and max values. Also, minimum and maximum headway values H

T **is the set of possible trains.**

Variable: $x_{hij} = 1$ **is train** h **immediately precedes train** i **on link** j**.**

IP

$$\textbf{minimize} \sum_{i\in T}\sum_{j=1}^{L}\Big(c^a_{ij}(z_{ij}) + c^d_{i,j-1}(y_{ij}) + c^\tau_{ij}(\tau_{ij})\Big) + \sum_{i\in T}\sum_{j=1}^{L-1}(c^\delta_{ij}(\delta ij))$$

subject to

$$\begin{array}{rcl}
y_{ij} & \geq y^{\min}_{ij} & i\in T, j=1,\ldots,L \\
y_{ij} & \leq y^{\max}_{ij} & i\in T, j=1,\ldots,L \\
z_{ij} & \geq z^{\min}_{ij} & i\in T, j=1,\ldots,L \\
z_{ij} & \leq z^{\max}_{ij} & i\in T, j=1,\ldots,L \\
\tau_{ij} & = z_{ij} - y_{ij} & i\in T, j=1,\ldots,L \\
\tau_{ij} & \geq \tau^{\min}_{ij} & i\in T, j=1,\ldots,L \\
\tau_{ij} & \leq \tau^{\max}_{ij} & i\in T, j=1,\ldots,L \\
\delta_{ij} & = y_{i,j+1} - z_{ij} & i\in T, j=1,\ldots,L \\
\delta_{ij} & \geq \delta^{\min}_{ij} & i\in T, j=1,\ldots,L-1 \\
\delta_{ij} & \leq \delta^{\max}_{ij} & i\in T, j=1,\ldots,L-1 \\
y_{i,j+1} - y_{h,j+1} + (1-x_{hij})M & \geq H^d_{hij} & i\in T, j=1,\ldots,L \\
z_{ij} - z_{hj} + (1-x_{hik})M \geq H^a_{hij} & i\in T, j=1,\ldots,L & \\
\sum_{h\in\{T-i\}} x_{hij} & = 1 & i\in T, j=1,\ldots,L \\
x_{hij} & \in \{0,1\} &
\end{array}$$

Solution

Can solve using heuristic similar to shifting bottleneck heuristic. Set one train (by importance) and resolve LP relaxation.