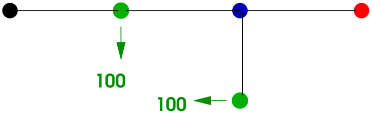
Controlling variability in power systems

Daniel Bienstock

Columbia University

APAM Nov 17 2017

A simple example:



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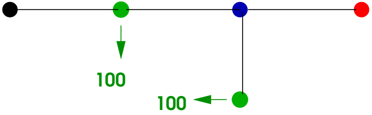
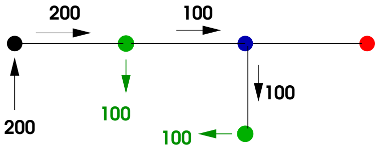


Only one solution:



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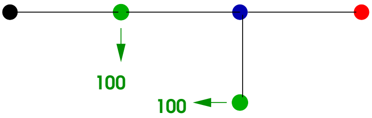

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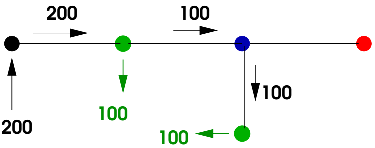
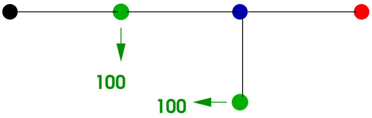


But what if the red node suddenly injects power?



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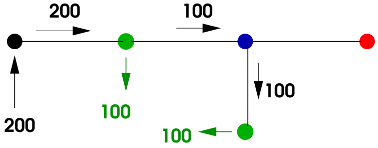

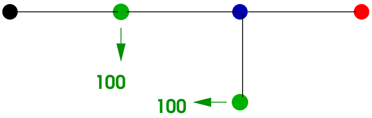

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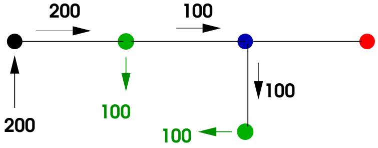
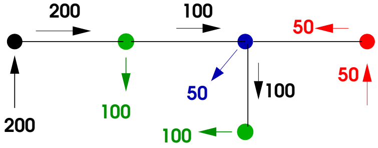
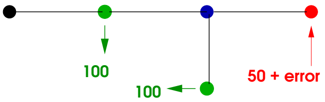


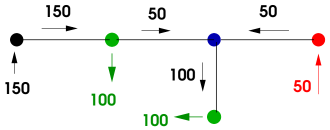
If red node suddenly injects power, offset using blue node:



Better example: red node is cheap but unreliable source



Base case solution:



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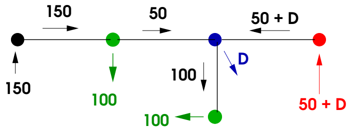
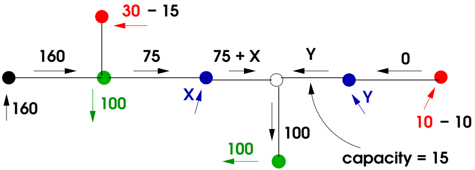
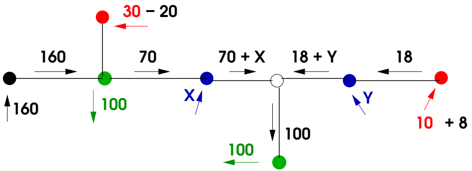


Examples:

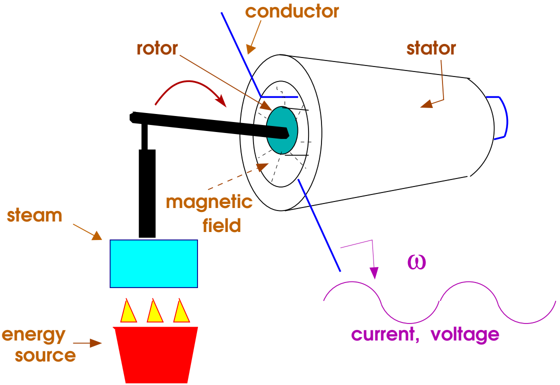


Any combination with $X + Y = 25$ "works" so long as $Y \leq 15$.



Any combination with $X + Y = 12$ "works" so long as $Y \leq -3$.

Power engineering for non-engineers



Power engineering for non-power engineers

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AC Power Flows

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Real-time:

AC Power Flows

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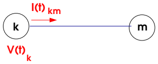
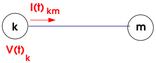


- Voltage at bus k: $v_k(t) \ = \ V_k^{max} \cos(\omega t + \theta_k^V)$
- Current injected at k into km: $i_{km}(t) \ = \ I_{km}^{max} \cos(\omega t + \theta_{km}^I)$.

AC Power Flows

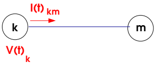
Real-time:



- Voltage at bus k: $v_k(t) \; = \; V_k^{max} \cos(\omega t + \theta_k^V)$
- Current injected at k into km: $i_{km}(t) \; = \; I_{km}^{max} \cos(\omega t + \theta_{km}^I)$.
- Power injected at k into km: $p_{km}(t) \; = \; v_k(t) i_{km}(t)$.

Averaged over period T:

- $p_{km} \; \doteq \; \frac{1}{T} \int_0^T p(t) \; = \; \frac{1}{2} V_k^{max} I_{km}^{max} \cos(\theta_k^V - \theta_{km}^I)$.



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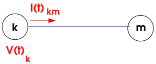

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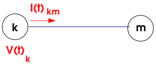

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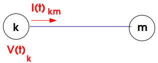
$$V_k \doteq \frac{V_k^{max}}{\sqrt{2}} e^{j\theta_k^V}, \quad I_{km} \doteq \frac{I_{km}^{max}}{\sqrt{2}} e^{j\theta_{mk}^I}$$



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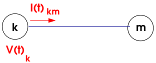

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- $p_{km} = |V_k||I_{km}| \cos(\theta_k^V - \theta_{km}^I) = \mathcal{R}\mathrm{e}(V_k I_{km}^*)$
- $q_{km} \doteq Im(V_{km} I_{km}^*)$ and $S_{km} \doteq p_{km} + jq_{km}$

- $V_k \doteq \frac{V_k^{max}}{\sqrt{2}} e^{j\theta_k^V}, \quad I_{km} \doteq \frac{I_{km}^{max}}{\sqrt{2}} e^{j\theta_{mk}^I}$ (voltage, current)

$$p_{km} \;=\; \mathcal{R}\mathrm{e}(V_k I_{km}^*), \quad q_{km} \;=\; Im(V_{km} I_{km}^*) \tag{1}$$

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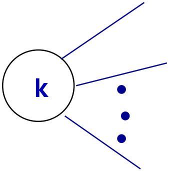
Network Equations



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Network Equations

$$\sum_{km \in \delta(k)} p_{km} \; = \; \hat{P}_k, \quad \sum_{km \in \delta(k)} q_{km} \; = \; \hat{Q}_k \quad \forall\, k \tag{5}$$

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Network Equations

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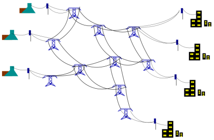
Generator: $\hat{P}_k, |V_k|$ given. Other buses: $\hat{P}_k, \hat{Q}_k$ given.

Optimal power flow (economic dispatch, tertiary control)



- Used periodically to handle the next time window (e.g. 15 minutes, one hour)
- Choose generator outputs
- Minimize cost (quadratic)
- Satisfy demands, meet generator and network constraints
- **Constant load (demand) estimates for the time window**

DC-OPF:

$$\min \; c(p) \quad \text{(convex piecewise-linear or quadratic)}$$

s.t.

$$B\theta = p - d \tag{6}$$

$$|y_{ij}(\theta_i - \theta_j)| \;\leq\; u_{ij} \quad \text{for each line } ij \tag{7}$$

$$P_g^{min} \;\leq\; p_g \;\leq\; P_g^{max} \quad \text{for each bus } g \tag{8}$$

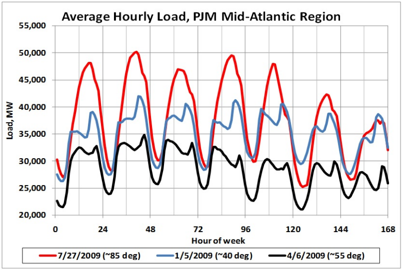
Notation:

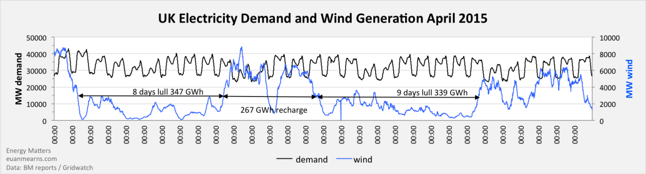
$p \;=\;$ vector of generations $\in \mathcal{R}^n$, $\quad d \;=\;$ vector of loads $\in \mathcal{R}^n$

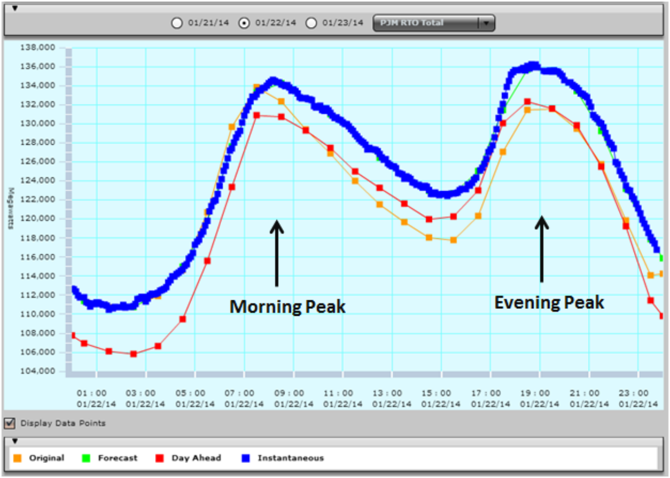
$B \;\in \mathcal{R}^{n \times n}$, $\quad$ (bus susceptance matrix)

$$\forall i,j: \quad B_{ij} = \begin{cases} -y_{ij}, & ij \;\in\; \mathcal{E} \text{ (set of lines)} \\ \sum_{k;\{k,j\} \in \mathcal{E}} y_{kj}, & i = j \\ 0, & \text{otherwise} \end{cases}$$

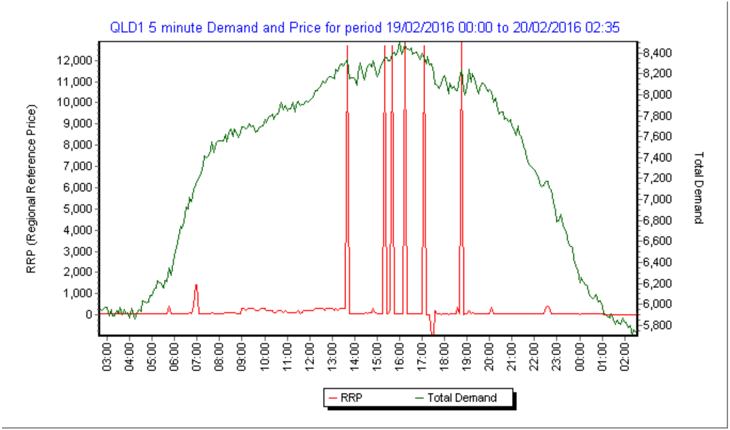
Managing changing demands



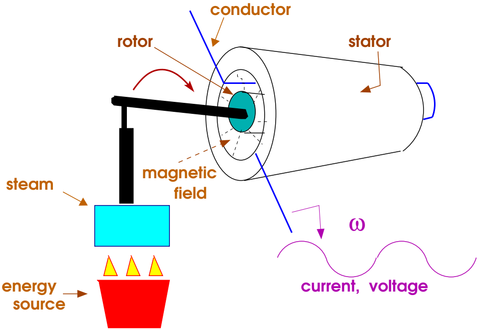




01/21/14
01/22/14
01/23/14
PJM RTO Total
Megawatts
138,000
136,000
134,000
132,000
130,000
128,000
126,000
124,000
122,000
120,000
118,000
116,000
114,000
112,000
110,000
108,000
106,000
104,000
01 : 00 01/22/14
03 : 00 01/22/14
05 : 00 01/22/14
07 : 00 01/22/14
09 : 00 01/22/14
11 : 00 01/22/14
13 : 00 01/22/14
15 : 00 01/22/14
17 : 00 01/22/14
19 : 00 01/22/14
21 : 00 01/22/14
23 : 00 01/22/14
Morning Peak
Evening Peak
Display Data Points
Original
Forecast
Day Ahead
Instantaneous


QLD1 5 minute Demand and Price for period 19/02/2016 00:00 to 20/02/2016 02:35
RRP (Regional Reference Price)
Total Demand
12,000
11,000
10,000
9,000
8,000
7,000
6,000
5,000
4,000
3,000
2,000
1,000
0
8,400
8,200
8,000
7,800
7,600
7,400
7,200
7,000
6,800
6,600
6,400
6,200
6,000
5,800
03:00
04:00
05:00
06:00
07:00
08:00
09:00
10:00
11:00
12:00
13:00
14:00
15:00
16:00
17:00
18:00
19:00
20:00
21:00
22:00
23:00
00:00
01:00
02:00
RRP
Total Demand

What happens when there is a generation/load mismatch



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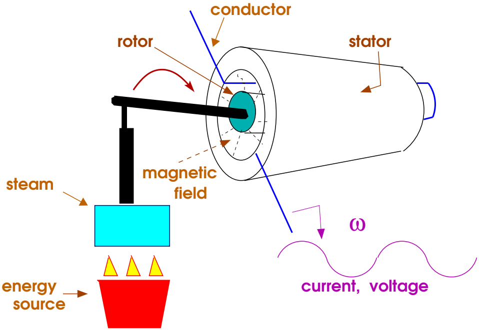
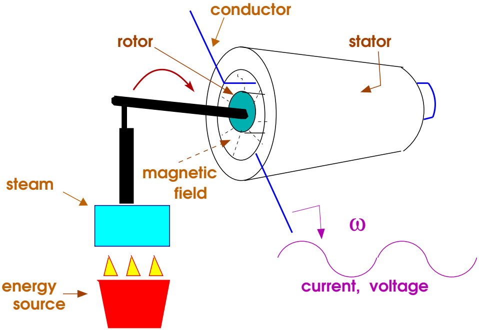


Frequency response:

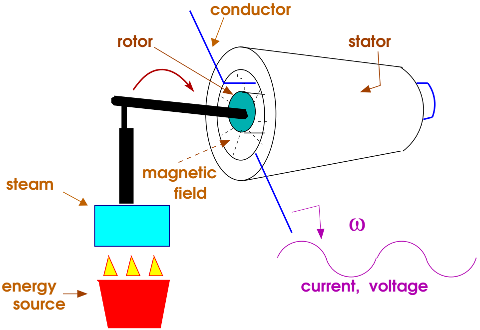
What happens when there is a generation/load mismatch



Frequency response:

mismatch ΔP

What happens when there is a generation/load mismatch



Frequency response:

mismatch $\boldsymbol{\Delta P}$ $\Rightarrow$ **frequency change** $\boldsymbol{\Delta\omega} \approx -c\,\Delta P$

The swing equation

$$H\dot{\omega} \; = \; p_m(t) - p_e(t) - D\omega$$

- $\omega \;\; = \omega(t)$ = frequency
- $p_m(t)$ = mechanical power supplied by motor
- $p_e(t)$ = electrical power supplied by motor
- $D > 0$ (tamping)

Managing changing demands

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Managing changing demands

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THE ENERGY CHALLENGE

Wind Energy Bumps Into Power Grid's Limits

Mike Groll/Associated Press

The Maple Ridge Wind farm near Lowville, N.Y. It has been forced to shut down when regional electric lines become congested.

By MATTHEW L. WALD
Published: August 26, 2008

When the builders of the Maple Ridge Wind farm spent $320 million to put nearly 200 wind turbines in upstate New York, the idea was to get paid for producing electricity. But at times, regional electric lines have been so congested that Maple Ridge has been forced to shut down even with a brisk wind blowing.

CIGRE -International Conference on Large High Voltage Electric Systems '09

- Large unexpected fluctuations in wind power can cause additional flows through the transmission system (grid)
- Large power deviations in renewables must be balanced by other sources, which may be far away
- Flow reversals may be observed – control difficult
- A solution – expand transmission capacity! Difficult (expensive), takes a long time
- Problems **already observed** when renewable penetration high

CIGRE -International Conference on Large High Voltage Electric Systems '09

- "Fluctuations" – 15-minute timespan
- Due to turbulence ("storm cut-off")
- Variation of the same order of magnitude as mean
- Most problematic when renewable penetration starts to exceed $20 - 30\%$
- Many countries are getting into this regime

Control model

1. $\bar{w}_i + w_i$ = output of renewable at bus i.
   $\bar{w}_i$ = forecast, w_i = error (uncertain).
2. δ_j = response at bus j.

Generic linear control:

$$\delta_j = -\sum_i \alpha_{ji} w_i$$

- $\mathcal{A}$: matrix of all values α_{ji}; $(\mathcal{A}, \ldots, \mathcal{A}) \in \mathcal{K}$

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- e.g. $\sum_j (1 - \alpha_{ji}) = 0 \quad \forall i$ for "full-dimensional" uncertainty set.

Chance-constrained problems (one period)

Optimization Problem

$$\min_{P^g, \mathcal{A}} \quad \sum_k c_k(P^g_k)$$

s.t. the following system is feasible:

$\rightarrow$ **Flow balance:**

$$B\ \boldsymbol{\theta} \;=\; P^g + \overbrace{\bar{w} + \ \boldsymbol{w}}^{\textbf{renewables}} \;-\; \overbrace{\mathcal{A}\ \boldsymbol{w}}^{\textbf{linear control}} \;-\; P^d$$

$\rightarrow$ **Line limits:**

$$\mathbf{P}(|f_{km}| > f^{\max}_{km}) \;<\; \boldsymbol{\epsilon_{km}} \quad \forall \quad km$$

(ignoring e.g. generator constraints)

Safety-constrained problems (one period)

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- **Caution!** SOCP, but not easy in larger cases.
  - Should use sparse formulation.
  - Should use first-order or outer-envelope method.

Previous work on chance-constrained OPF (review)

- Bienstock, Chertkov, Harnett
- Roald, Andersson, several coauthors

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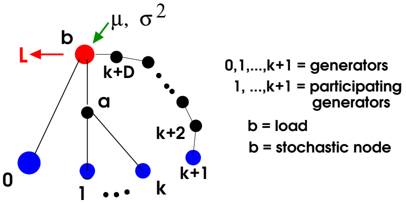
   can be implemented as convex optimization problem under suitable assumptions

2. However such convex problems (SOCPs) beyond solvers
   But first-order methods fast and accurate

An extreme example of variability



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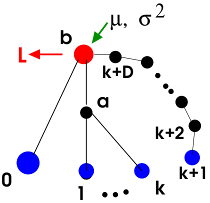

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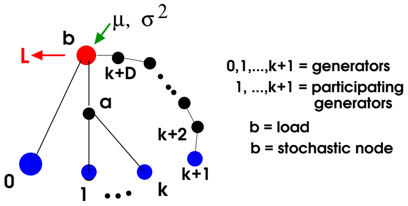
- Stochastic injection at bus $\boldsymbol{b} = \boldsymbol{\omega}$, $\mathbf{E}(\omega) = \mu$, $\mathbf{Var}(\omega) = \sigma^2$. $\boldsymbol{2\sigma > \mu}$.
- Linear generation cost function at i $(0 \le i \le k+1)$: $c_i p_i$. $c_0 < c_1 = c_2 \ldots = c_k < c_{k+1}$.
- Safety parameters equal to 3.
- (specify line limits later)



Unique optimal safety-constrained solution:

- $P_0^g \;=\; L - \mu - 3\sigma$.
- For $1 \leq i \leq k$: $\alpha_i = 1/k$ and $P_i^g = 3\sigma/k$.
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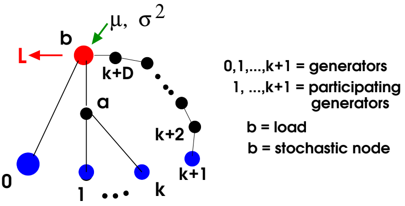

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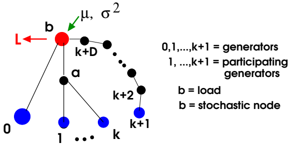

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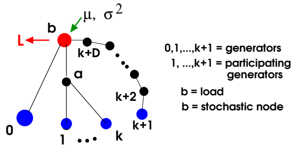

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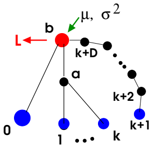

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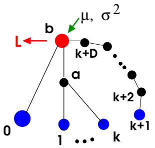

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  $\boldsymbol{D = 10,}$ $\rightarrow$ sum of line flow variances $\boldsymbol{\approx 1.35\,\sigma^2}$.

Variance-aware SCOPF

$$\min_{P^g, \mathcal{A}} \quad \sum_k c_k(P_k^g) \quad + \quad \boldsymbol{\Delta(v^2)}$$

$$\text{s.t.} \sum_j (1 - \alpha_{ji}) \; = \; 0 \quad \forall i; \quad \sum_i (P_i^g + \bar{w}_i - P_i^d) \; = \; 0$$

$$\pi_{km}^T y_{km} |\mathbf{E}(f_{km})| \; + \; \boldsymbol{\nu_{km}} \, \mathbf{Std}(f_{km}) \;\; \leq \;\; f_{km}^{\max} \quad \forall \; km \qquad (9)$$

- $\boldsymbol{v^2} \;\doteq\;$ vector with entries $\mathbf{Var(f_{ij})}$.
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4. Logarithmic barrier function.

$$\Delta_{ij} \;=\; -\rho \log(f_{km}^{\max} \;-\; |\mathbf{E}(f_{km})| \;-\; \boldsymbol{\nu_{km}}\, \mathbf{Std}(f_{km}))$$

where $\rho > 0$

Variance-aware SCOPF

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Correction vs Formal Variance-Aware Optimization

Several groups of authors:

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Correction vs Formal Variance-Aware Optimization

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1. Outer-approximation algorithm for chance-constrained DC-OPF converges in **few** iterations.
2. Only a few lines are "at risk" in realistic cases.
3. **Low-hanging fruit:** instead of variance-aware optimization, first solve SC-OPF problem, and then **correct** or **adjust** to reduce variability without increasing cost (by much).

GENERIC CORRECTION TEMPLATE

Input: an instance of the safety-constrained problem and a variance metric.

Step I. Solve safety-constrained OPF problem, with solution $(\bar{P}^g, \bar{\mathcal{A}})$.

Step II. Perform a small number of adjustment iterations which shift $(\bar{P}^g, \bar{\mathcal{A}})$ to a new feasible solution that attains an improved value of the variance metric, while at the same time increasing generation cost in a moderate manner.

Reroute($\tau, \hat{\mathcal{A}}$)

- $\hat{\mathcal{A}}$ **:** a given control matrix

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- So $\textbf{Var}(f_{km}) \; = \; y_{km}^2 \pi_{km}^T (I - \hat{\mathcal{A}}) \Omega (I - \hat{\mathcal{A}}^T) \pi_{km} \; \doteq \; \hat{V}_{km}$ is **fixed** for every km
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- $\mathbf{0} < \boldsymbol{\tau} < \mathbf{1}$.

$$\min_{P^g} \quad \sum_k c_k(P_k^g)$$

$$\text{s.t.} \quad \sum_i (P_i^g + \bar{w}_i - P_i^d) \ = \ 0$$

$$\pi_{km}^T y_{km} |\mathbf{E}(f_{km})| + \nu_{km} \sqrt{\hat{\boldsymbol{V}}_{\boldsymbol{km}}} \ \leq \ (1 - \boldsymbol{\tau}) f_{km}^{\max} \quad \forall \ km$$

Here, $\boldsymbol{f_{km}}$ is an implicit function of $\boldsymbol{P^g}$ and $\hat{\boldsymbol{A}}$

VShift($\bar{P}^g$)

- $\bar{P}^g$ **:** generation vector. $\bar{f}$ **:** corresponding flow vector.

$$\min_{\mathcal{A}} \quad \sum_{km \in \mathcal{F}(\bar{f})} \Delta_{km} \left(y_{km}^2 \pi_{km}^T (I - \mathcal{A}) \Omega (I - \mathcal{A}^T) \pi_{km} \right)$$

$$\text{constraints on } \mathcal{A}: \quad \sum_j (1 - \alpha_{ji}) = 0 \quad \forall i$$

$\rightarrow$ Keep power flows fixed, improve on control

Correction procedure

Input: Feasible solution $(P^{g,0}, \mathcal{A}_0)$, $0 < \tau < 1$.

For $k = 1, 2, \ldots, K$ **perform iteration k:**

1. Run $\textit{Reroute}(\mathcal{A}_{k-1}, \tau)$. **If** infeasible **STOP.** **Else** let $P^{g,k}$ be optimal.
2. Solve $\textit{VShift}(P^{g,k})$, with solution $\hat{\mathcal{A}}_k$.
3. Set $\mathcal{A}_k \leftarrow (1-\lambda)\mathcal{A}_{k-1} + \lambda\hat{\mathcal{A}}_k$. $0 < \lambda < 1$ chosen so that $P^{g,k}, \mathcal{A}_k$ feasible
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Thm.: In convex case if we stop in Step **4**, then Δ minimum.

Example on case2746wp

- 2746 buses, 3514 branches, 520 generators
- 22 stochastic injection sites, sum of mean injections 4611.57 (penetration 18.5% penetration)
- average ratio of standard deviation to mean $\approx$ 30%.

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- Variance-shifting SOCP has approximately 1.4×10^{05} variables and constraints and 1×10^{06} nonzeros
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