# Identifying and Controlling Risky Contingencies of Transmission Systems

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## N - K criterion revisited

**Previous work**: Salmeron and Wood, Donde et al, Turitsyin, Hines

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## Example: August 14 2003

U.S. - Canada report on blackout:

"Because it had been hot for several days in the Cleveland-Akron area, more air conditioners were running to overcome the persistent heat, and consuming relatively high levels of reactive power – further straining the area's limited reactive generation capabilities."

$\longrightarrow$ A **system-wide** condition that impedes the system

$\longrightarrow$ Not a cause, but a contributor

$\longrightarrow$ Look for combined events ?

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  - Perhaps **N - K** does not necessarily capture all interesting events?
- How can we deal with both types of problems?

## A continuous interdiction model

- A fictitious adversary is trying to interdict the transmission system.
- This adversary negatively alters the physical parameters of equipment, e.g. transmission lines, so as to impede transmission.
- The adversary has a budget available (both system-wide and per-line).
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  - $\sum_{km} \boldsymbol{\lambda_{km}} \leq \boldsymbol{\Lambda}$ (global limit)

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- The adversary has a budget available (both system-wide and per-line).
- Adversary maximizes the impact (e.g. voltage loss) over the available budget.
- A continuous, non-convex optimization problem with **simple** constraints. **No emumeration!**

## A blast from the past: Bienstock and Verma, 2007

- **DC approximation** to power flows.
- Adversary **increases reactances** of lines.
- **Limit** on total percentage-increase of reactances, and on per-line increase.
- Adversary maximizes the maximum **line overload**:

$$\max_{\boldsymbol{x},\boldsymbol{\theta}} \quad \max_{km} \left\{ \frac{|\theta_k - \theta_m|}{u_{km} \ \boldsymbol{x_{km}}} \right\}$$

$$\text{s.t.} \quad \boldsymbol{B_x}\theta = d$$

$$\boldsymbol{x} \text{ within budget}$$

  - Variables: reactances $\boldsymbol{x}$, phase angles $\boldsymbol{\pi}$
  - $\boldsymbol{x_{km}}$ = reactance of $\boldsymbol{km}$, $\boldsymbol{u_{km}}$ = limit of $\boldsymbol{km}$, $\boldsymbol{B_x}$ = bus susceptance matrix, $\boldsymbol{d}$ = net injections (given)

- Continuous, but non-smooth problem.

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- Continuous, smooth, **nonconvex**.

# Technical point

$$
\begin{array}{rl}
\max\limits_{\boldsymbol{x},\boldsymbol{\theta},\boldsymbol{\alpha}} & \sum\limits_{km} (\alpha^+_{km} - \alpha^-_{km}) \dfrac{(\theta_k - \theta_m)}{u_{km} \ \boldsymbol{x_{km}}} \\
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Function to maximize: $\quad \boldsymbol{F(x, \alpha)} \doteq \sum\limits_{km} (\alpha^+_{km} - \alpha^-_{km}) \frac{(\theta_k - \theta_m)}{u_{km} \, x_{km}}$

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Function to maximize: $\boldsymbol{F(x,\alpha)} \doteq \sum_{km} (\alpha^+_{km} - \alpha^-_{km}) \frac{(\theta_k - \theta_m)}{u_{km} \, x_{km}}$

- **Fact:** The gradient and the Hessian of $F(x, \alpha)$ can be efficiently computed
- Optimization problem solved using **LOQO** ( **IPOPT** an option)

## And what happens?

- Algorithm scales well (2007): CPU times of $\sim 1$ hour for studying systems with thousands of lines.

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- Algorithm scales well (2007): CPU times of $\sim 1$ hour for studying systems with thousands of lines.
- Optimal * attack concentrated on a handful of lines
- Significant part of the budget expended on many lines, with visible impact

Table 6: **Attack patterns**

| single = 20 total = 60 | | single = 10 total = 30 | | single = 10 total = 40 | |
|---|---|---|---|---|---|
| **Range** | **Count** | **Range** | **Count** | **Range** | **Count** |
| $[1, 1]$ | 8 | $[1, 1]$ | 1 | $[1, 1]$ | 14 |
| $(1, 2]$ | 72 | $(1, 2]$ | 405 | $(1, 2]$ | 970 |
| $(2, 3]$ | 4 | $(2, 9]$ | 0 | $(2, 5]$ | 3 |
| $(5, 6]$ | 1 | $(9, 10]$ | **3** | $(5, 6]$ | 0 |
| $(6, 7]$ | 1 | | | $(6, 7]$ | 1 |
| $(7, 8]$ | **4** | | | $(7, 9]$ | 0 |
| $(8, 20]$ | 0 | | | $(9, 10]$ | **2** |

**"single"** = max multiplicative increase of a line's reactance

**"total"** = max total multiplicative increase of line reactances

## Today: the AC power flows setting

As before, adversary increases impedances, subject to budgets

Adversary wants to **maximize:**

- Phase angle differences across ends of a lines
- Voltage deviations (loss)

**Alternative** version:

- There is a **recourse** action: shed load so as to maintain feasibility of all power flow constraints (limits)
- Adversary wants to maximize the amount of lost load

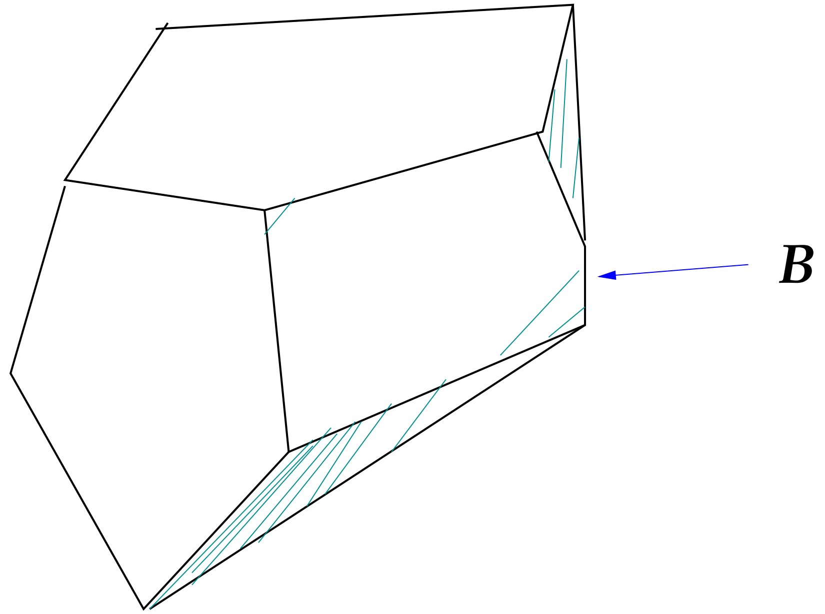
Generically:

$$\begin{aligned} \mathbf{max} \quad & \boldsymbol{\mathcal{F}(x)} \\ \text{s.t.} \quad & \boldsymbol{x} \in \mathcal{B} \end{aligned}$$

- $\boldsymbol{x}$ = impedances, $\boldsymbol{\mathcal{B}}$ = budget constraints
- $\boldsymbol{\mathcal{F}(x)}$= meausure of phase angle differences, voltage loss, load loss
- Challenge 1: $\boldsymbol{\mathcal{F}(x)}$ is implicitly defined

# Basic methodology: **Frank-Wolfe**

$$\begin{array}{ll} \textbf{max} & \color{red}{\mathcal{F}(\boldsymbol{x})} \\ \text{s.t.} & \color{red}{\boldsymbol{x}} \in \mathcal{B} \quad \text{(within budget)} \end{array}$$



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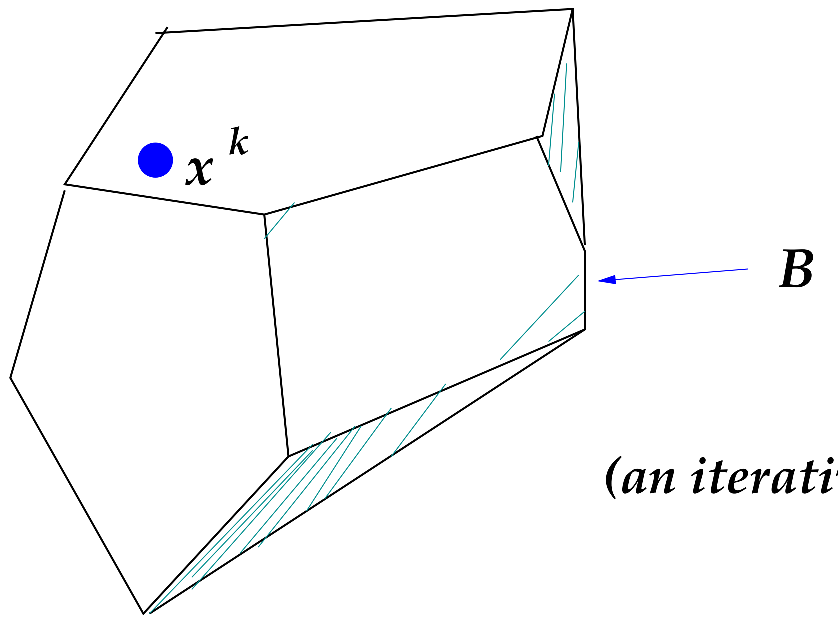


***(an iterative algorithm)***

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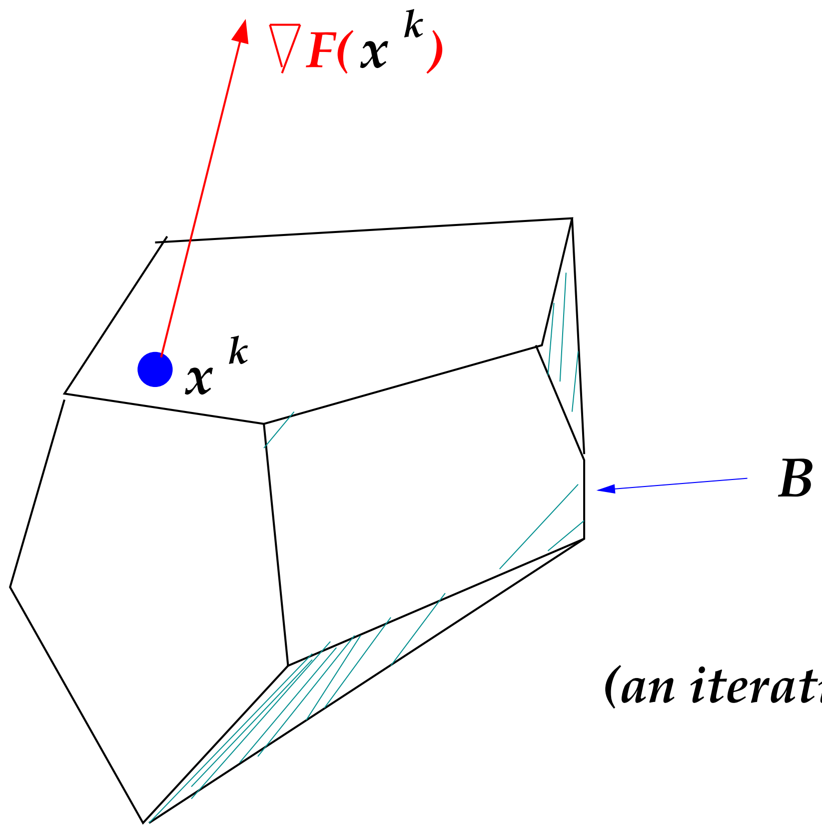


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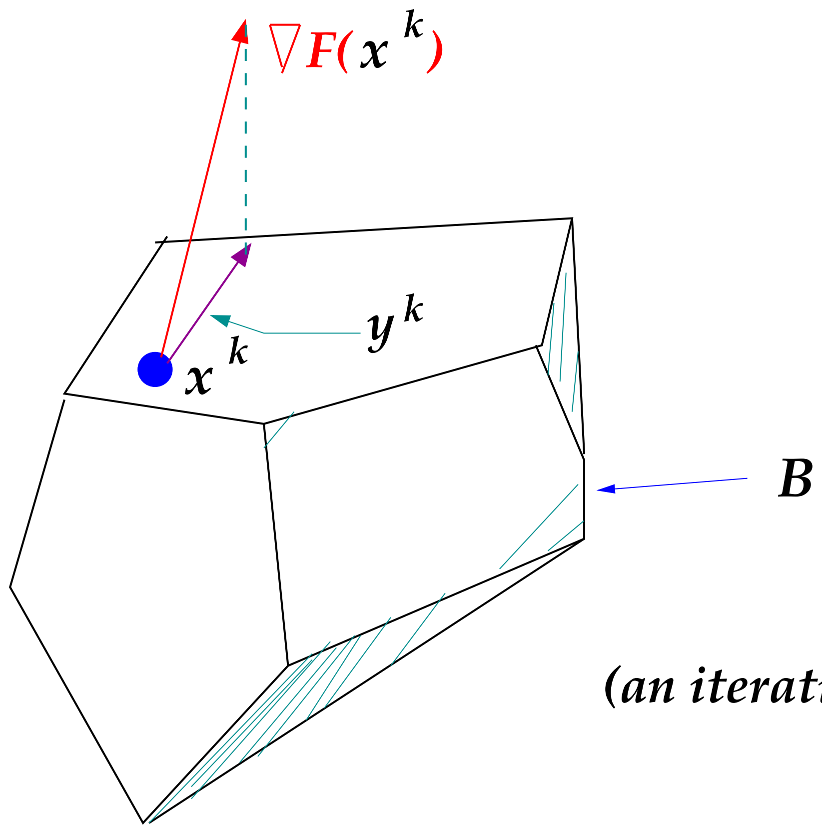


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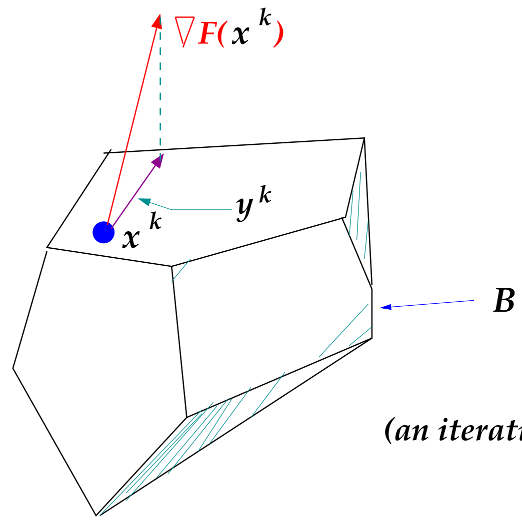


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$$\boldsymbol{y^k} \text{ solves} \quad \begin{array}{ll} \textbf{max} & [\nabla \mathcal{F}(x^k)]^T y \\ \text{s.t.} & x^k + y \in \mathcal{B} \quad \text{(within budget)} \end{array}$$

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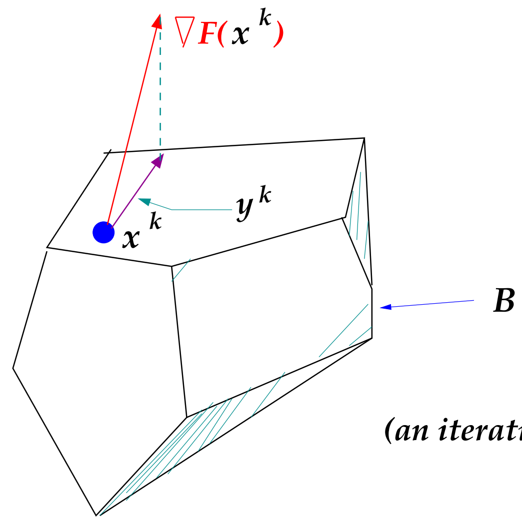


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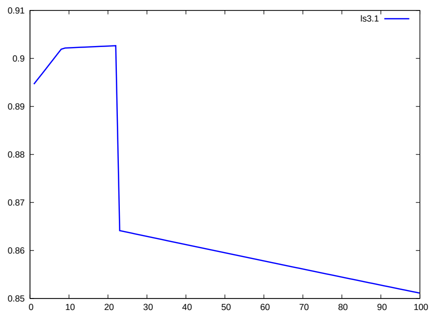
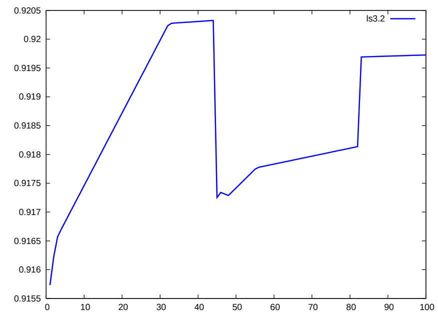
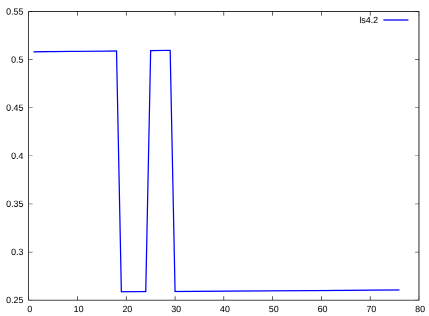
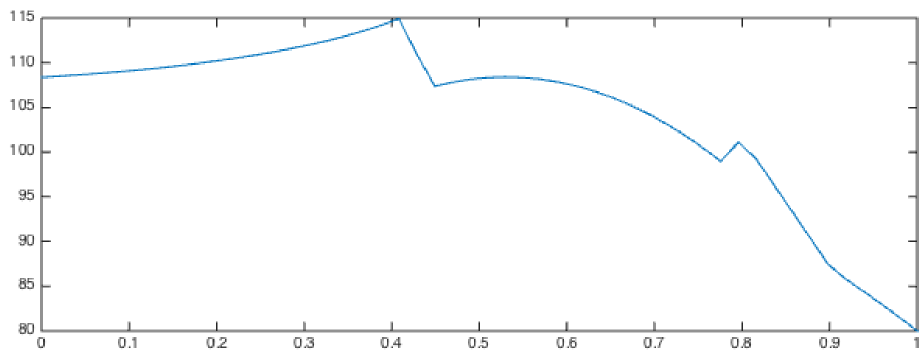
**Final step** is a **line search**: $x^{k+1} = x^k + \alpha y^k$, where $0 \leq \alpha \leq 1$ is the stepsize.

# Line searches



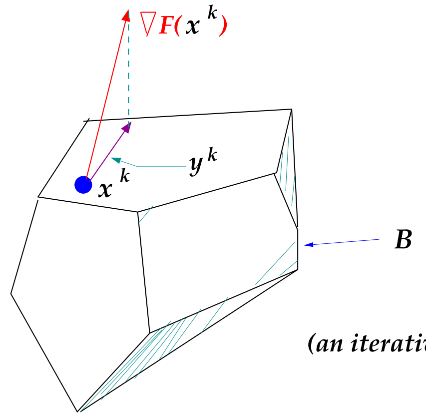



# Challenge 2

$$\begin{array}{ll} \textbf{max} & \mathcal{F}(x) \\ \text{s.t.} & x \in \mathcal{B} \quad \text{(within budget)} \end{array}$$




*(an iterative algorithm)*

- Recall: $\mathcal{F}(x)$ measures e.g. the largest phase angle difference using reactances $x$
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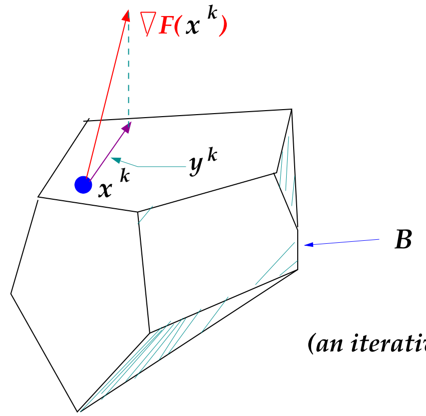


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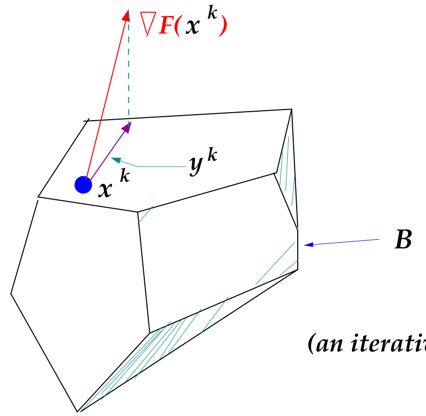


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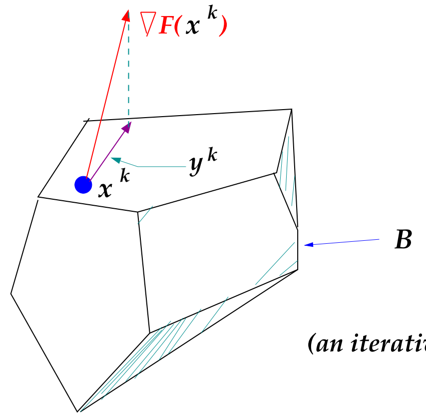


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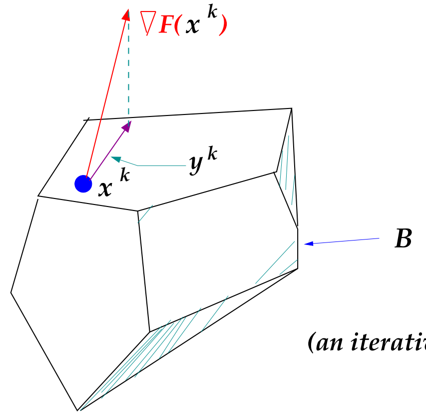


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- "**Solution**": Estimate $\nabla\mathcal{F}(x)$ in parallel over several cores
- **Alternative:** only estimate some of the components of $\nabla\mathcal{F}(x)$:
  - **Random** subset of small size
  - **Most promising** subset

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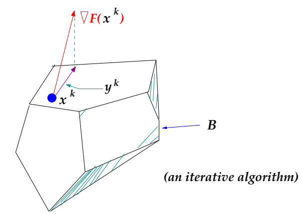


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- And we estimate $\nabla\mathcal{F}(\boldsymbol{x})$ using finite differences
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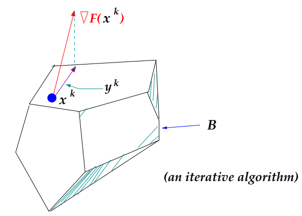


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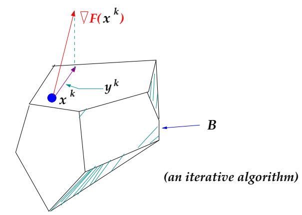


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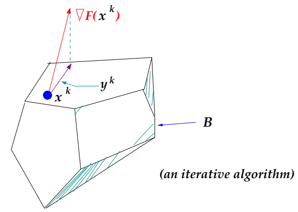


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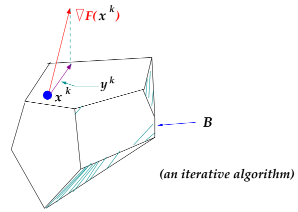
- **Challenge!** PF **often does not converge** for interesting $x$
- **solution:** solution OPF-like problem:
  minimize sum of square of all violations (load mismatch, line limits, etc)

# Challenge 3

$$\begin{array}{lll} \textbf{max} & \mathcal{F}(x) & \\ \text{s.t.} & x \in \mathcal{B} & \text{(within budget)} \end{array}$$




- $\mathcal{F}(x)$ measures e.g. the sum of voltage losses with reactances $x$
- And we estimate $\nabla\mathcal{F}(x)$ using finite differences
- **Q:** How do we compute $\mathcal{F}(x)$, for given reactances $x$?
- **A:** Ideally, a **PF** (load flow) calculation
- **Challenge!** PF **often does not converge** for interesting $x$
- **solution:** solution OPF-like problem:
  minimize sum of square of all violations (load mismatch, line limits, etc)
- **solution?** violations still observed
- **solution?** Add to definition of $\mathcal{F}(x)$ sum of weighted square violations

$\longrightarrow$ Currently using **IPOPT** within Matpower (fastest for **our** purposes)

$\longrightarrow$ Infeasible cases verified using SDP relaxation

# Example: phase angle attack on Polish grid (from Matpower)

**1** obj=2620.72 step=1.00 [ **263** 8.00; **300** 8.00; **728** 8.00; ]

**2** obj=2641.52 step=1.00 [ **305** 8.00; **306** 8.00; **309** 8.00; ]

**3** obj=2649.34 step=1.00 [ **168** 8.00; **263** 8.00; **321** 8.00; ]

**5** obj=2765.47 step=0.50 [ **51** 4.00; **261** 4.00; **263** 4.00; **300** 4.00; **321** 4.00; **322** 4.00; ]

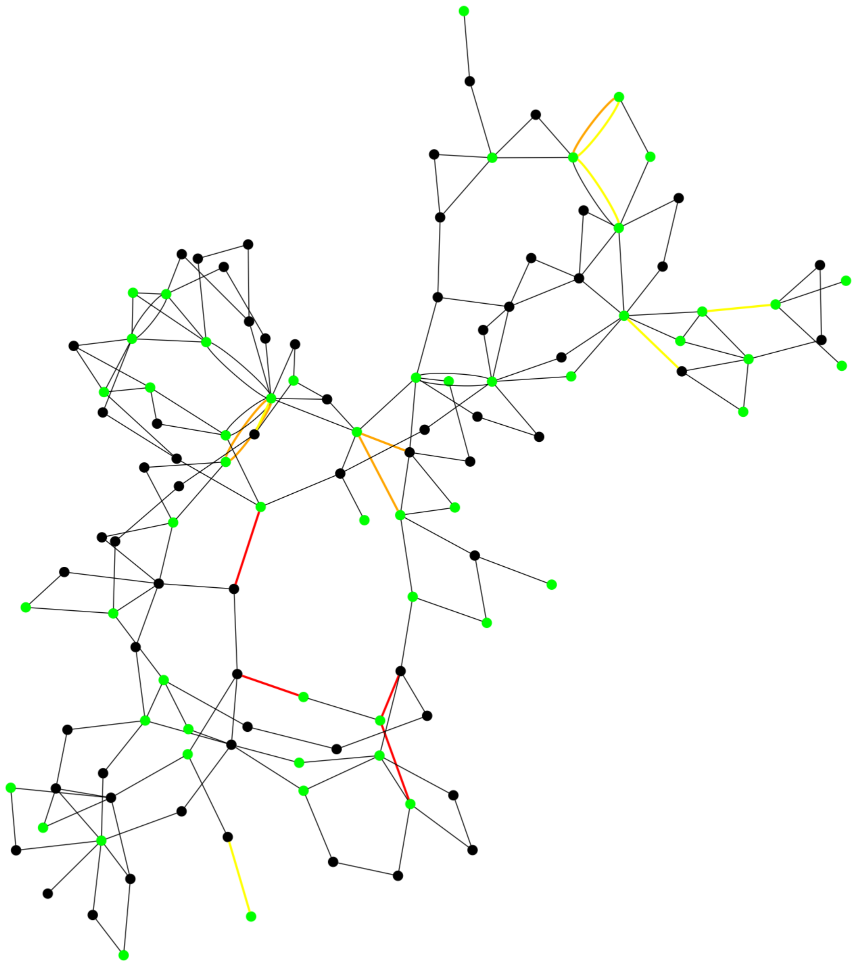
**13** obj=2944.01 step=0.12 [ **305** 2.60; **168** 2.32; **322** 2.17; **169** 1.90; **321** 1.85; **263** 1.57; **309** 1.50; **32** 1.15; **51** 1.08; **261** 1.08; **170** 1.00; **171** 1.00; **306** 0.85; **39** 0.75; **281** 0.75; **166** 0.57; **310** 0.57; **8** 0.43; **264** 0.43; **300** 0.42; ]

**20** obj=2950.54 step=0.03 [ **169** 2.53; **305** 2.38; **168** 1.88; **322** 1.77; **321** 1.76; **309** 1.74; **166** 1.44; **170** 1.28; **263** 1.28; **261** 1.14; **32** 0.93; **51** 0.88; **171** 0.81; **306** 0.69; **39** 0.61; **281** 0.61; **264** 0.59; **260** 0.51; **310** 0.46; **8** 0.35; **300** 0.34; ]

**27** obj=2958.08 **step=0.00** [ **169** 2.80; **305** 2.53; **321** 2.00; **309** 1.97; **168** 1.63; **263** 1.58; **322** 1.53; **166** 1.38; **261** 1.11; **170** 1.11; **32** 0.81; **51** 0.76; **264** 0.76; **281** 0.75; **171** 0.71; **306** 0.60; **39** 0.53; **260** 0.44; **310** 0.40; **8** 0.30; **300** 0.30; ]

# Example: phase angle attack on 118-bus

Three top-attacked lines in red:



## Fact: phase angle attack cannot be isolated to a few lines

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Experiment on 118-bus case:

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(3) What happens? Phase angle difference $\rightarrow \boldsymbol{\pi/2}$ ?

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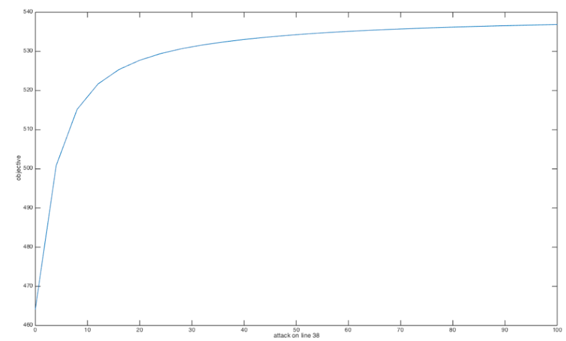
From $\approx \mathbf{10}$ to $\approx \mathbf{40}$.



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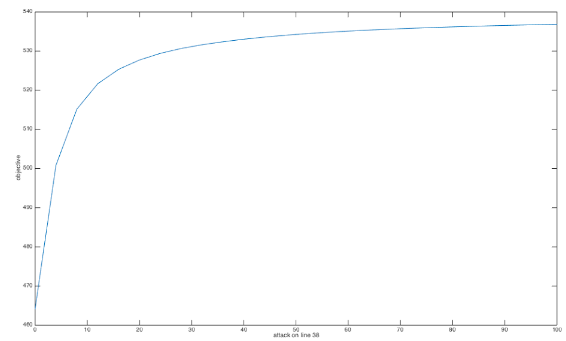
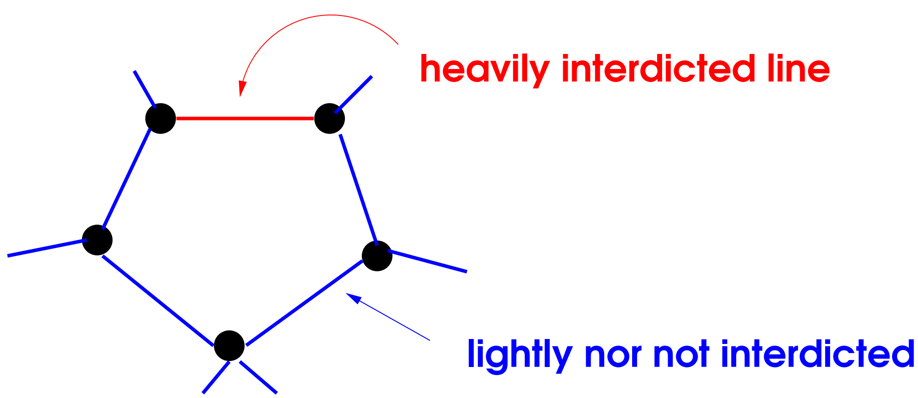
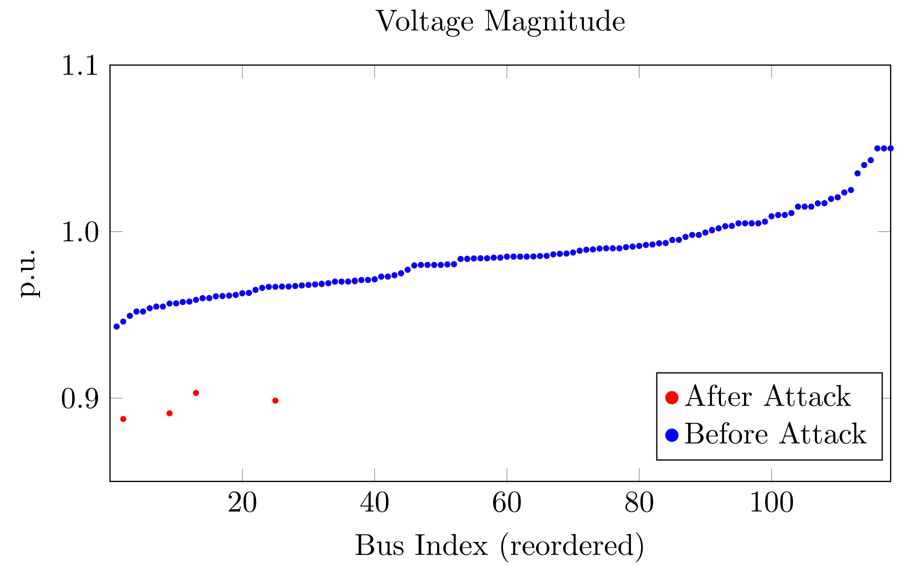
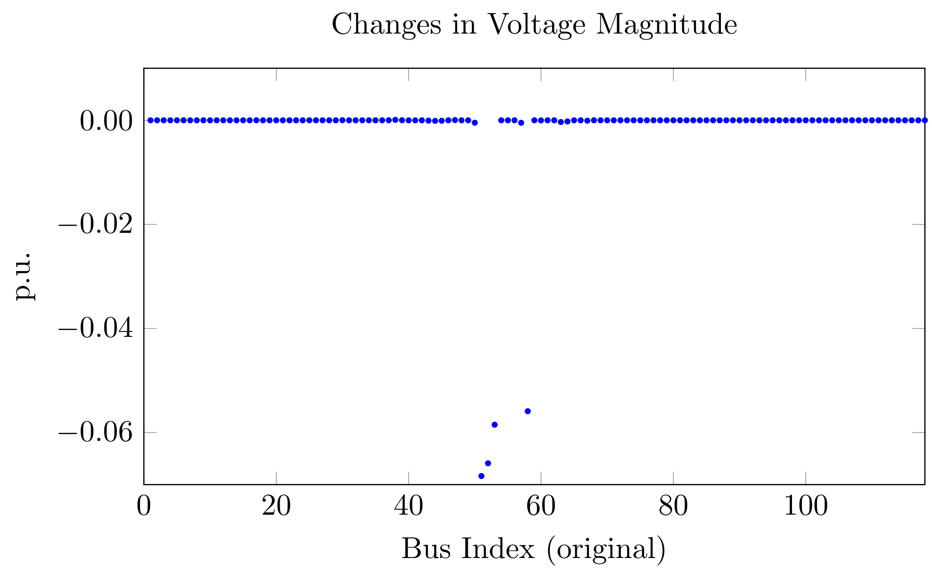


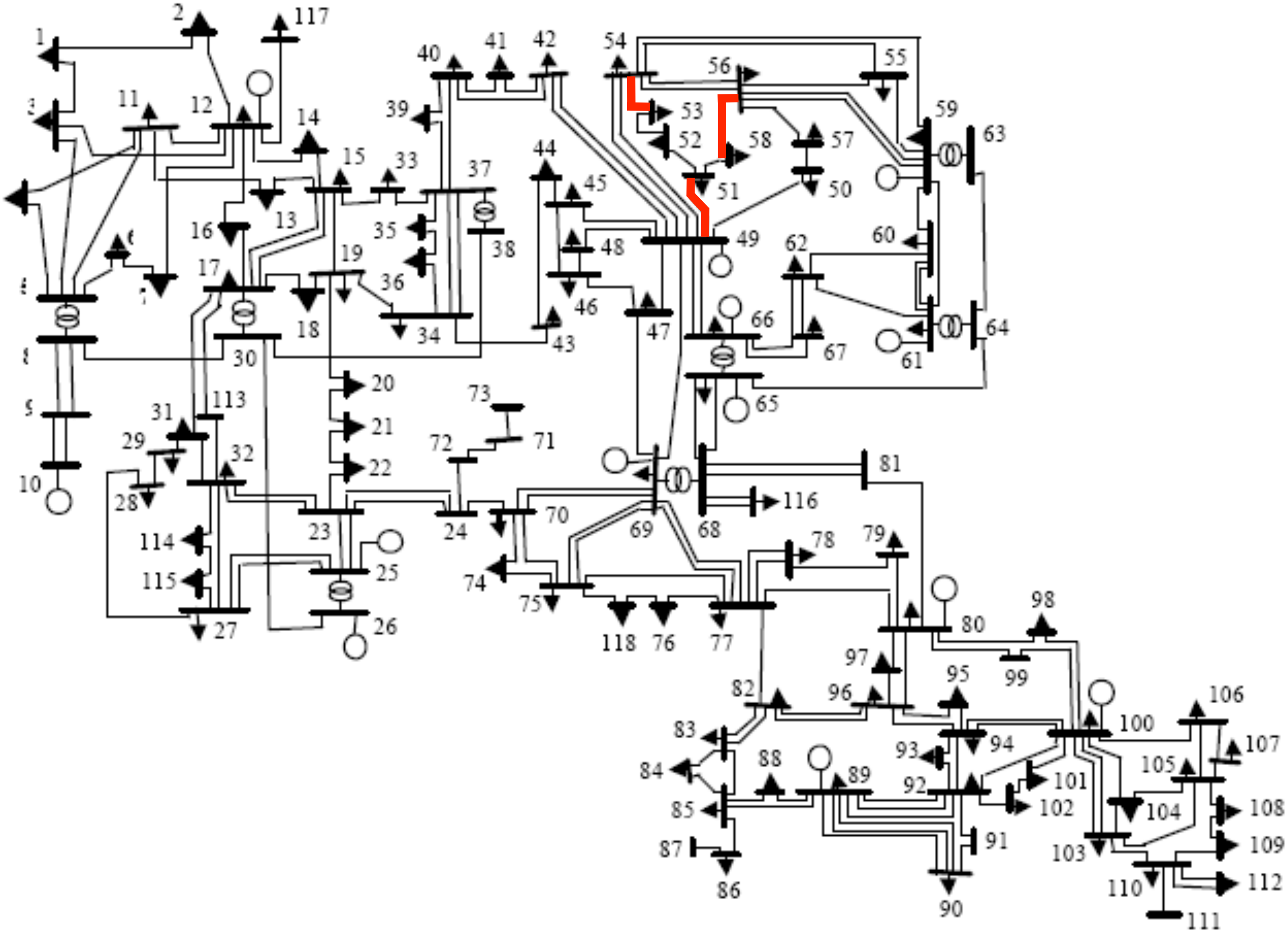



# Voltage attack on 118-bus

"Triple the reactance of at most three lines"





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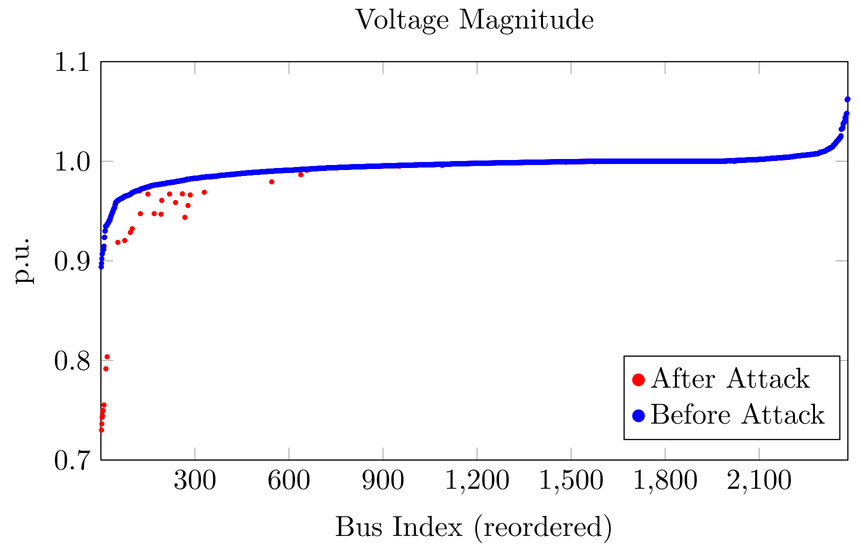
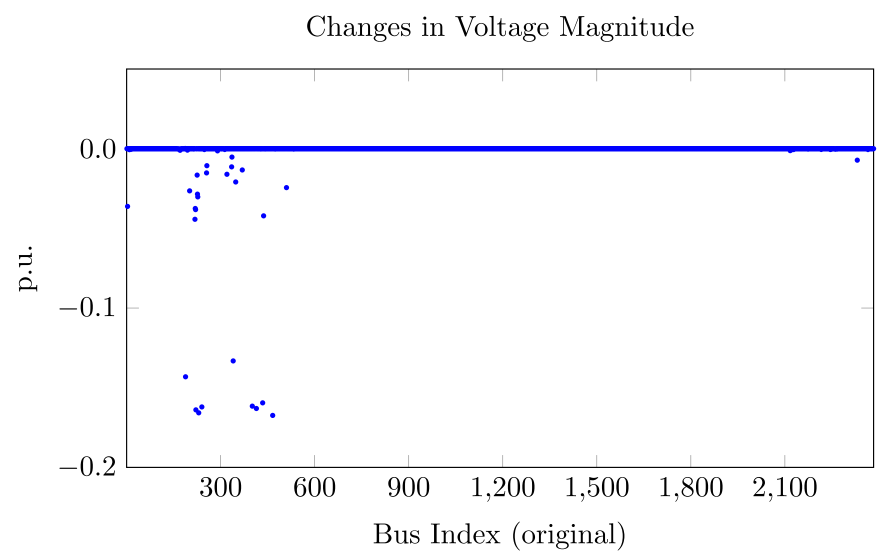
# Voltage attack on 2383-bus Polish

"Double the reactance of at most three lines"




→ Primarily 4 lines interdicted