Sample average approximation and cascading failures of power grids

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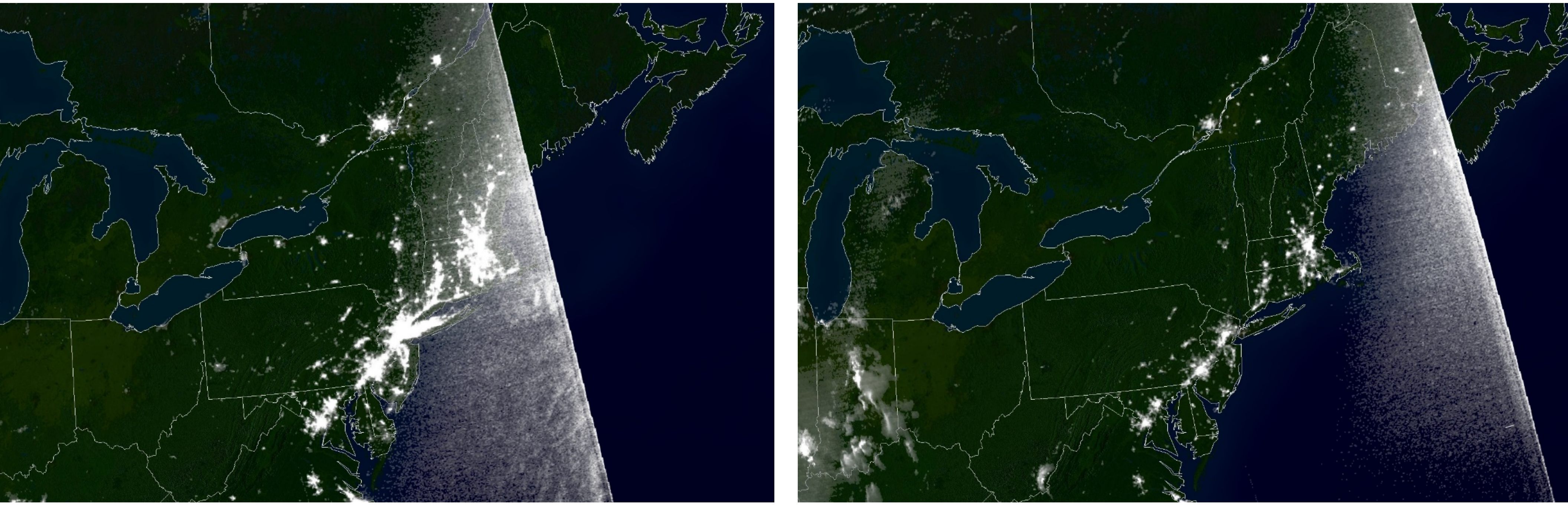
GraphEx '15

August 14 2003



Approximately 50 million people affected

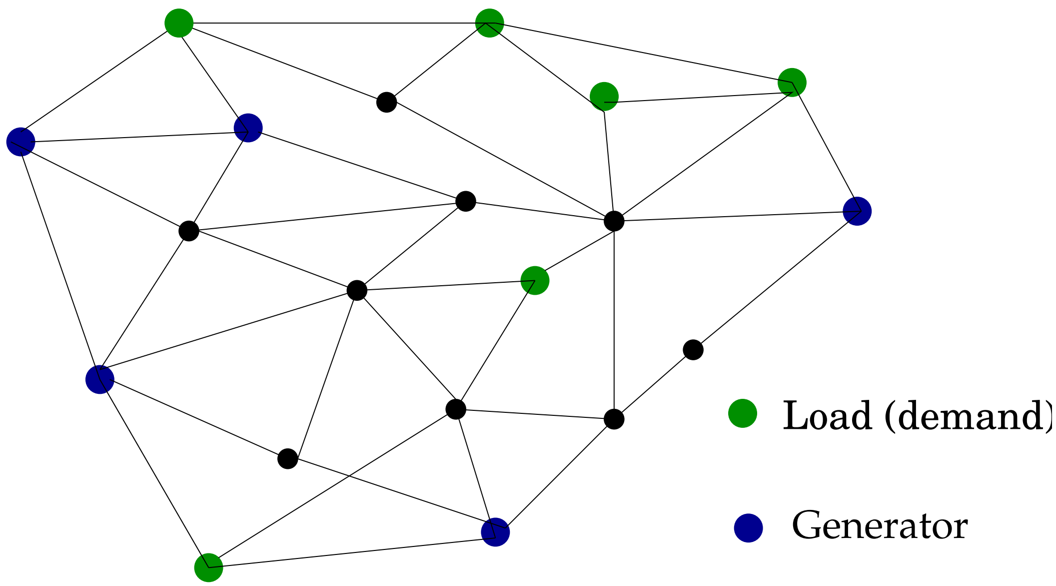
Other large-scale cascading failures

- Italy, 2003
- San Diego, 2011
- India, 2012

At fault:

unexpected event, cascading mechanism, noise and human error

Cascade cartoon



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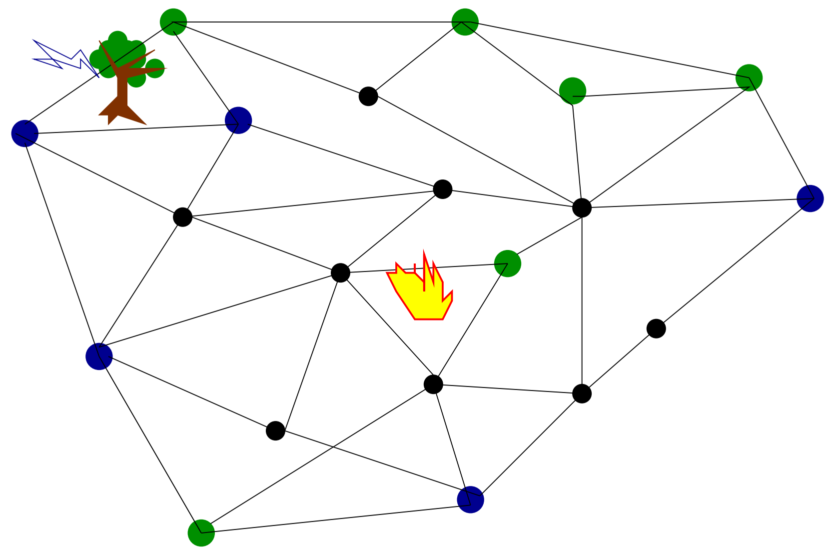

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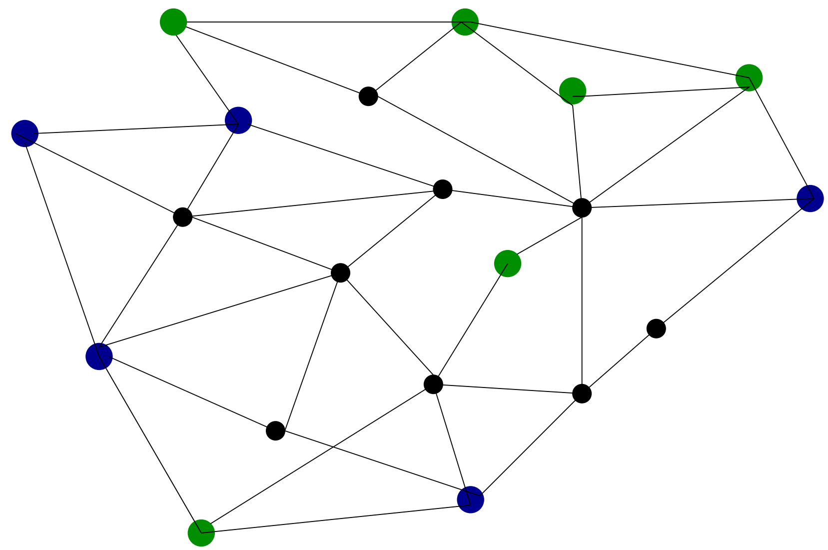

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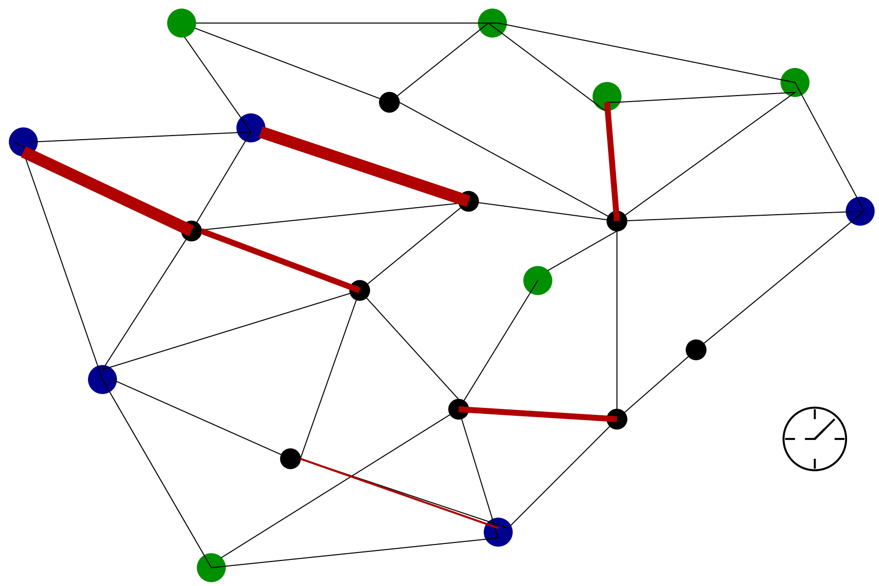

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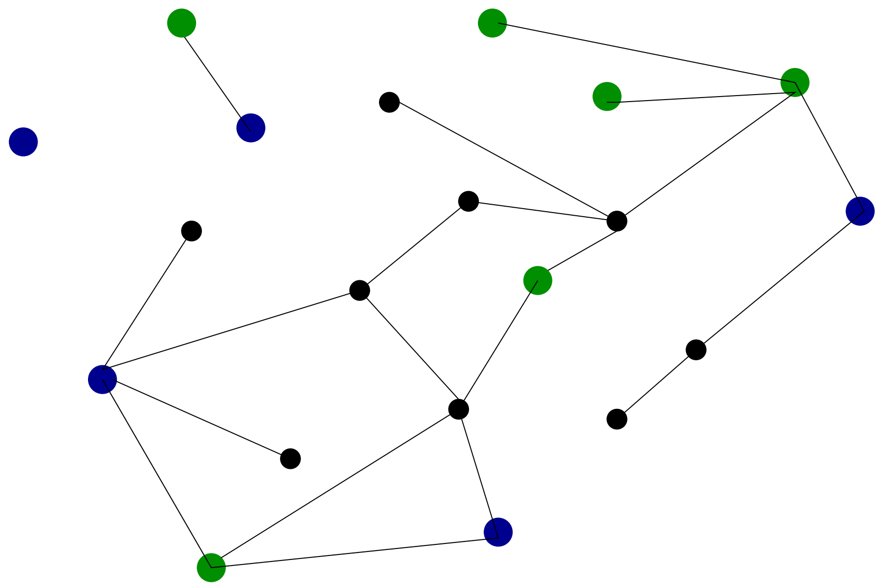

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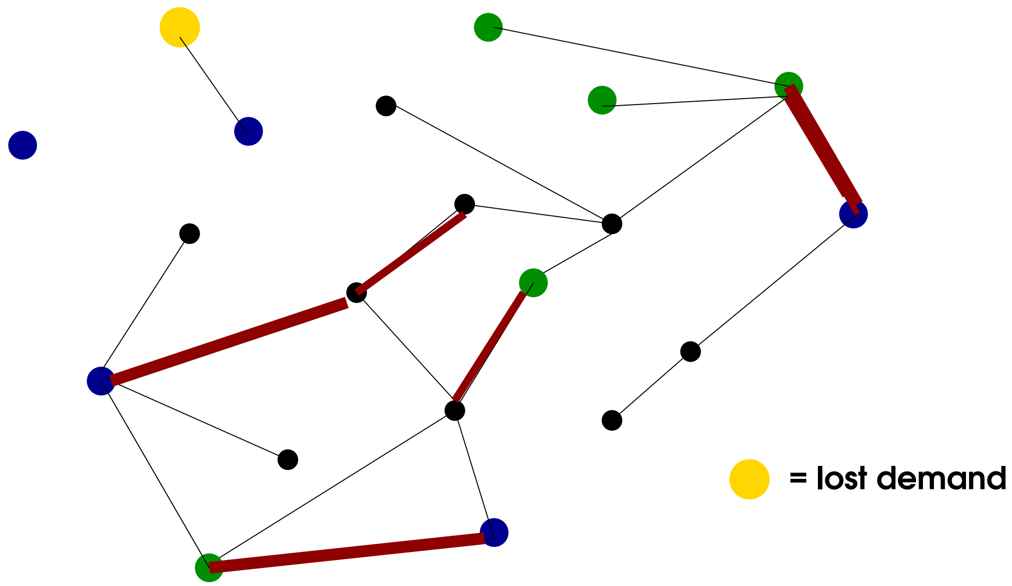

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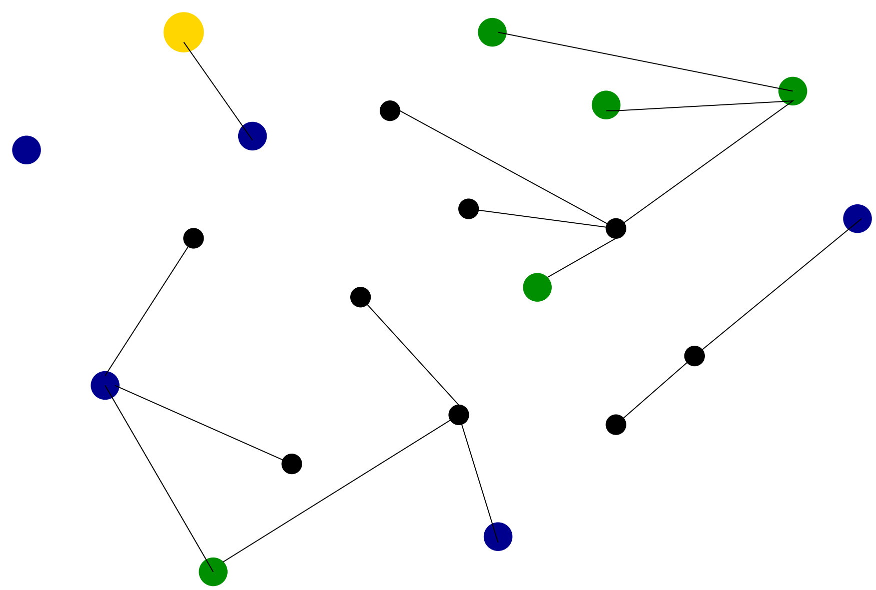

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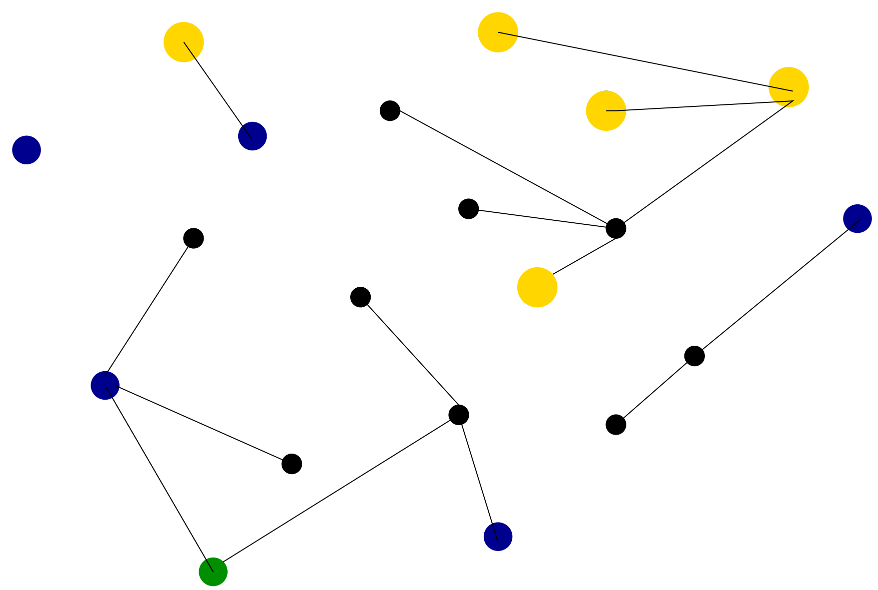

A quote from:

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Cause **1** of the blackout was **"inadequate system understanding"**

(stated at least twenty times)

Cause **2** of the blackout was **"inadequate situational awareness"**

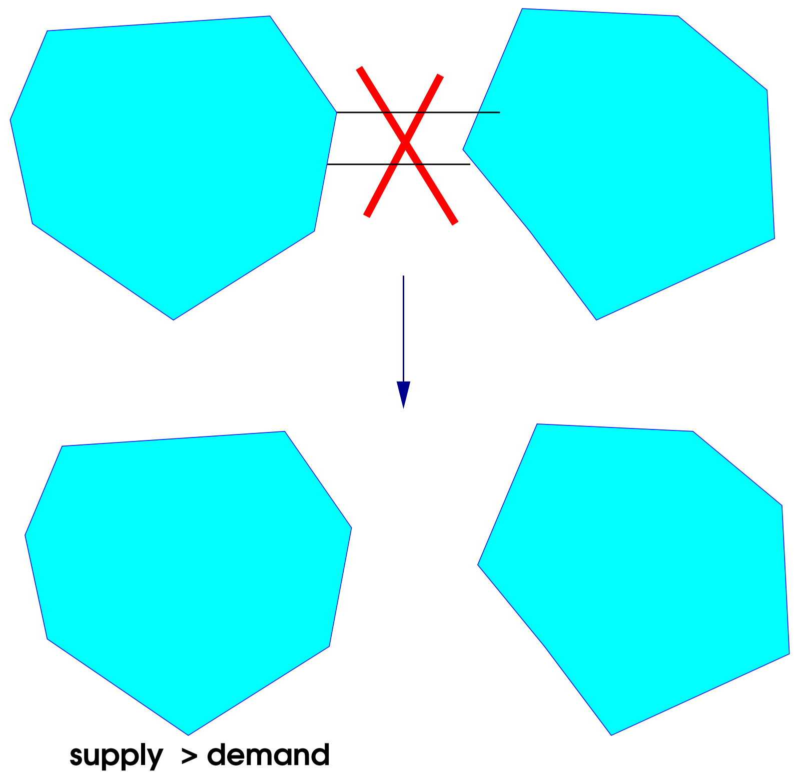
Very approximate cascade model

$\rightarrow$ Initial fault event takes place (an “act of God”).

For $\boldsymbol{t = 1, 2, \ldots,}$

1. Reconfigure demands and generator output levels (if islanding occurs).

Islanding



The "swing" equation

- A second-order differential equation used to explain swings in a motor's frequency in response to a change of loads.

- To properly analyize islanding, we need to consider systems of swing equations, plus physics of power flows.

- Which is a very difficult computational problem.

- Primary, secondary frequency response.

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Power flow problem in rectangular coordinates, simplest form

Variables: Complex voltages $\boldsymbol{e_k + j f_k}$, power flows $\boldsymbol{P_{km}, Q_{km}}$

Notation: For a bus $\boldsymbol{k}$, $\boldsymbol{\delta(k)}$ = set of lines incident with $\boldsymbol{k}$;
$\boldsymbol{V}$ = set of buses

$$\forall km: \quad P_{km} = \boldsymbol{g_{km}}(e_k^2 + f_k^2) - \boldsymbol{g_{km}}(e_k e_m + f_k f_m) + \boldsymbol{b_{km}}(e_k f_m - f_k e_m) \tag{1a}$$

$$\forall km: \quad Q_{km} = -\boldsymbol{b_{km}}(e_k^2 + f_k^2) + \boldsymbol{b_{km}}(e_k e_m + f_k f_m) + \boldsymbol{g_{km}}(e_k f_m - f_k e_m) \tag{1b}$$

$$\forall km: \quad |P_{km}|^2 + |Q_{km}|^2 \leq \boldsymbol{U_{km}} \tag{1c}$$

$$\forall k: \quad \boldsymbol{P_k^{\min}} \leq \sum_{km \in \delta(k)} P_{km} \leq \boldsymbol{P_k^{\max}} \tag{1d}$$

$$\forall k: \quad \boldsymbol{Q_k^{\min}} \leq \sum_{km \in \delta(k)} Q_{km} \leq \boldsymbol{Q_k^{\max}} \tag{1e}$$

$$\forall k: \quad \boldsymbol{V_k^{\min}} \leq e_k^2 + f_k^2 \leq \boldsymbol{V_k^{\max}}. \tag{1f}$$

Solving AC power flow problems

- When considering a grid in stable operation, Newton-Raphson or similar works very well. Convergence in seconds

- But no theoretical foundation for this behavior exists.

- When studying a grid under distress, Newton-Raphson (or similar) does **not** work well. Non-convergence.

- Recently, renewed interest in semidefinite relaxations, and techniques from real algebraic geometry.

- These methodologies are much more accurate but also much slower.

- The mathematics is the same as that for systems of *polynomial equations*: Hilbert's and Smale's 17th problems.

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2. New power flows are instantiated.

3. The next set of faults takes place.

Line tripping

1. If a power line carries too much power, it will overheat.

2. At a critical temperature, the line will **fail**.

3. Before that point, the line will **sag**. A physical contact would lead to immeadite **tripping**.

4. If a line is overloaded for too long, it will be protectively **tripped**.

5. Simplification: there is a **line limit** beyond which a line is considered overloaded.

5. IEEE Standard 738. An adaptation of the heat equation so as to take into account ...

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5. IEEE Standard 738. An adaptation of the heat equation so as to take into account ... the state of the universe, pretty much

(current work: appropriate stochastic variants of the heat equation)

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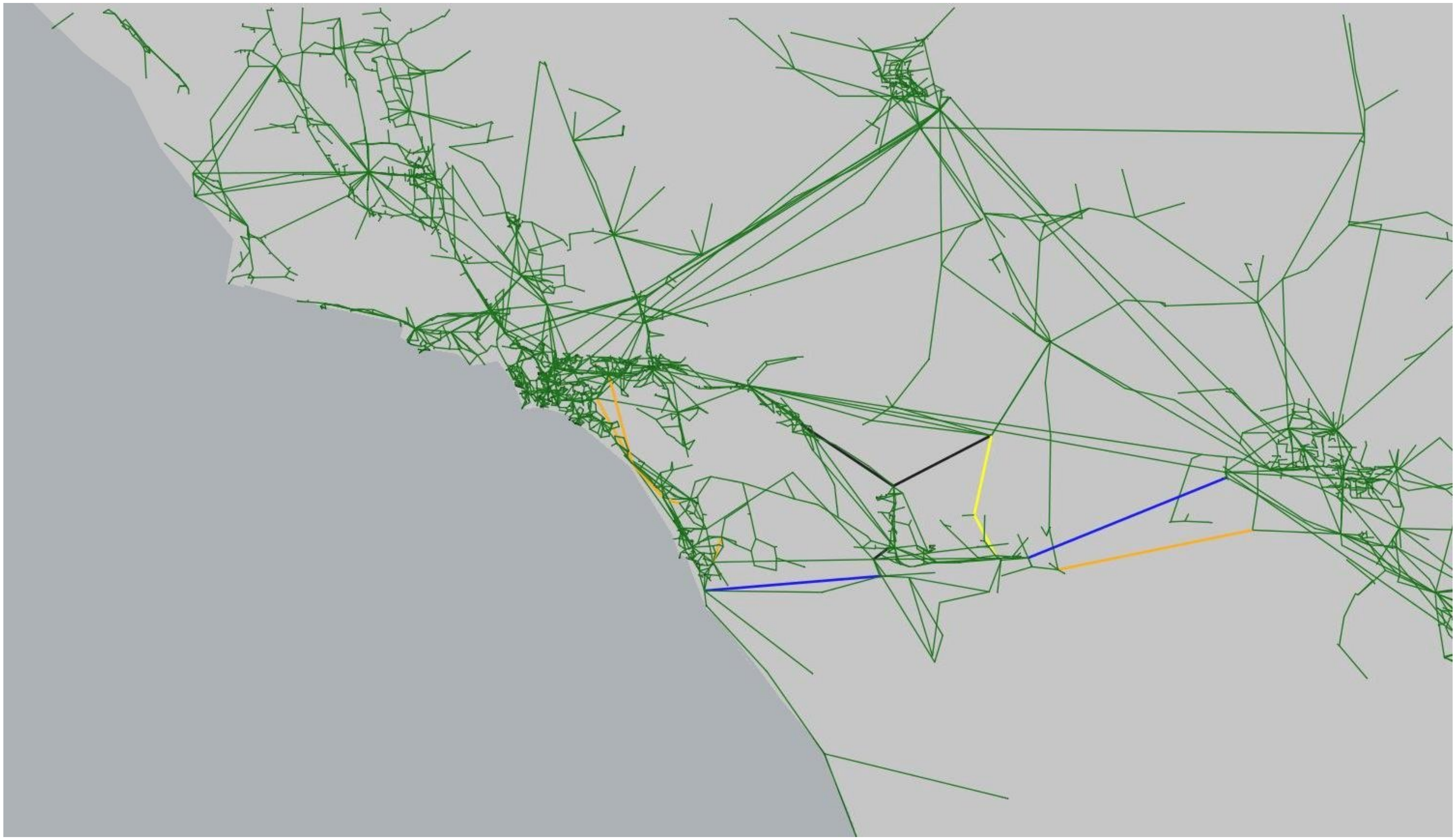
4. STOP if no more faults

Simulation of 2011 San Diego Event

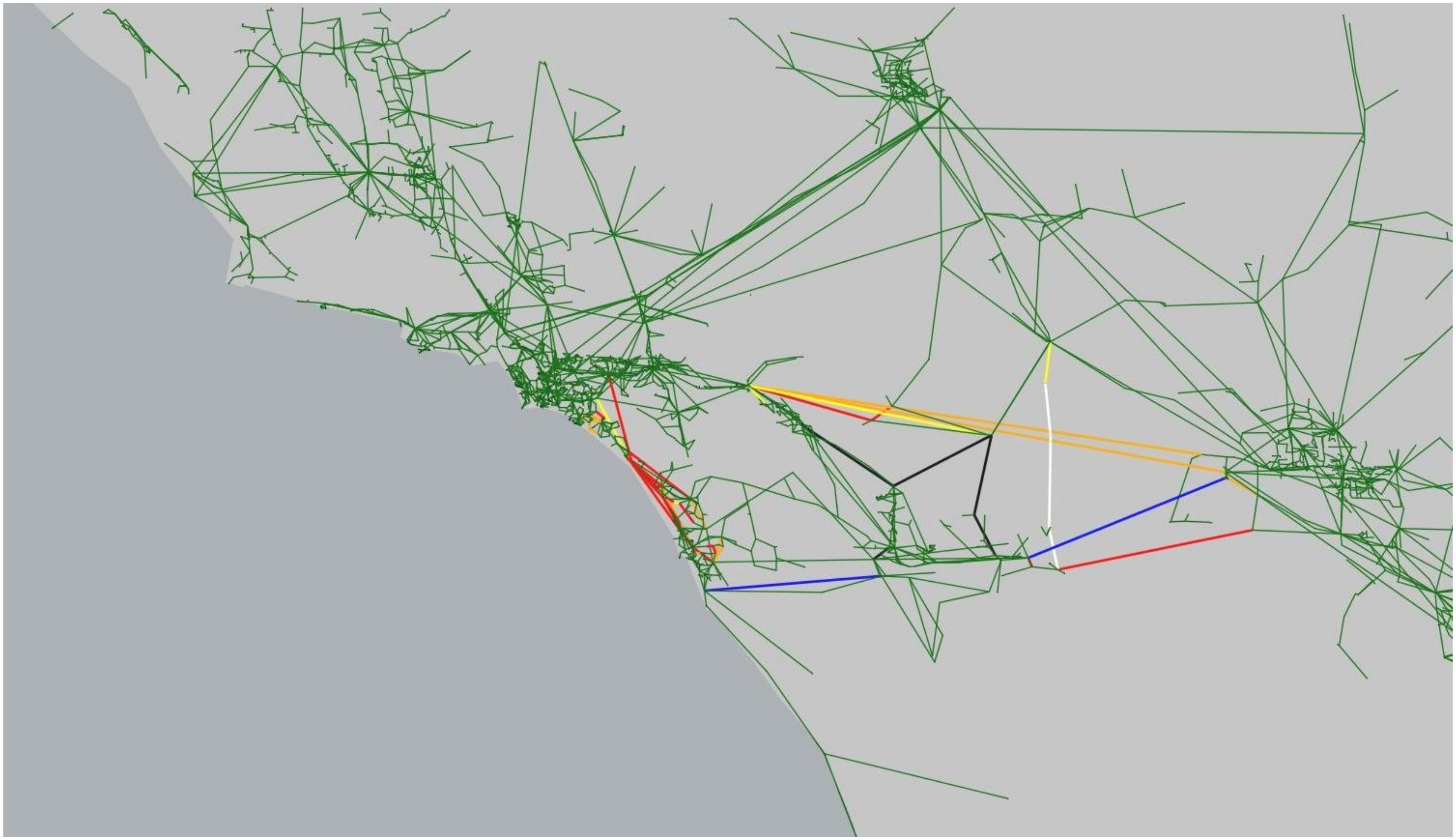
Joint work with A. Bernstein, D. Hay, G. Zussman, M. Uzunoglu (EE Dept. Columbia)

- Initiating event: human error
- **We do not have complete or exact data**
- Nevertheless, in our simulations a cascade does take place, with similar characteristics at the initial stages
- “inadequate system understanding”

Beginning (partial picture: 13K buses total)



More



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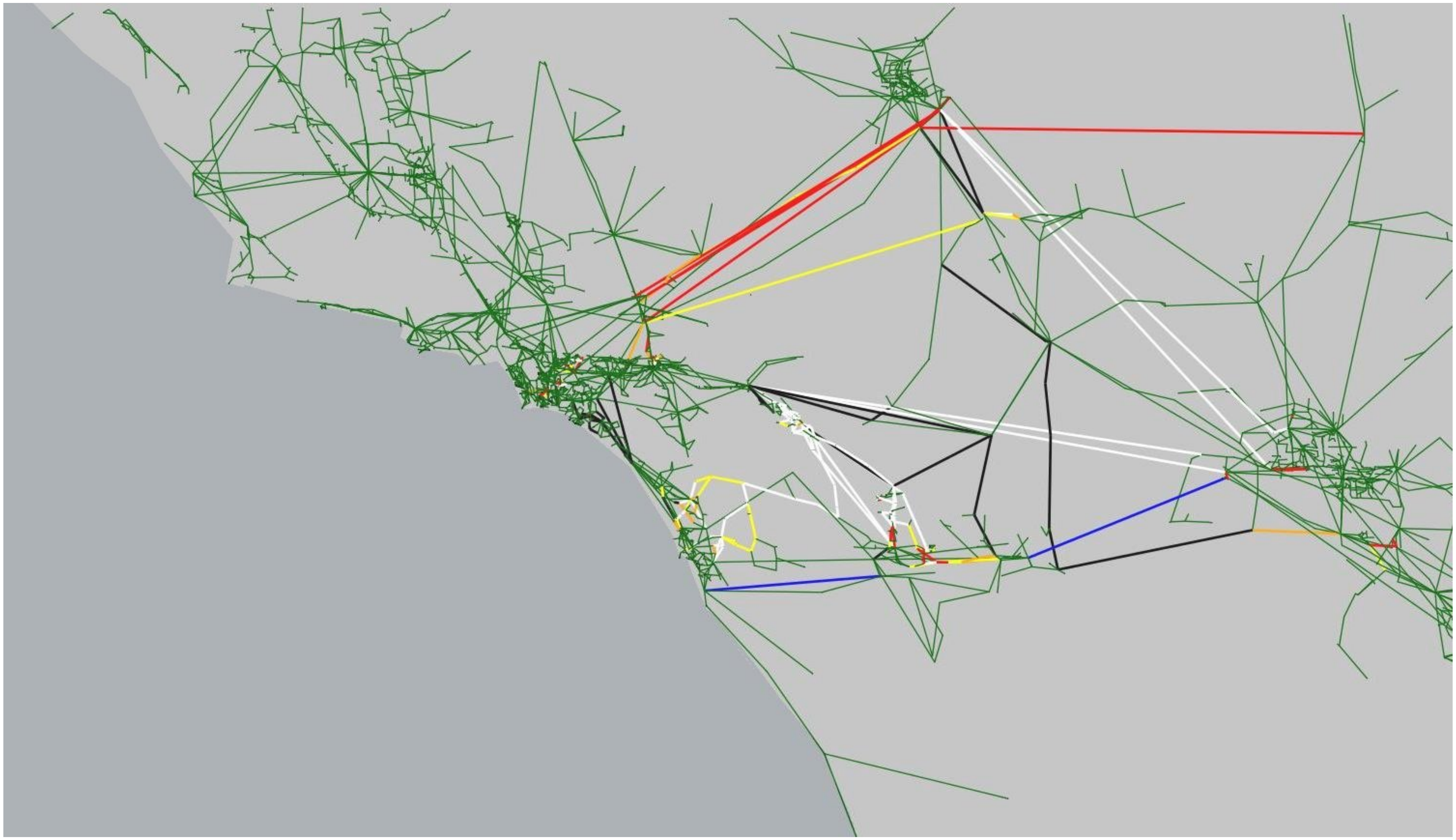

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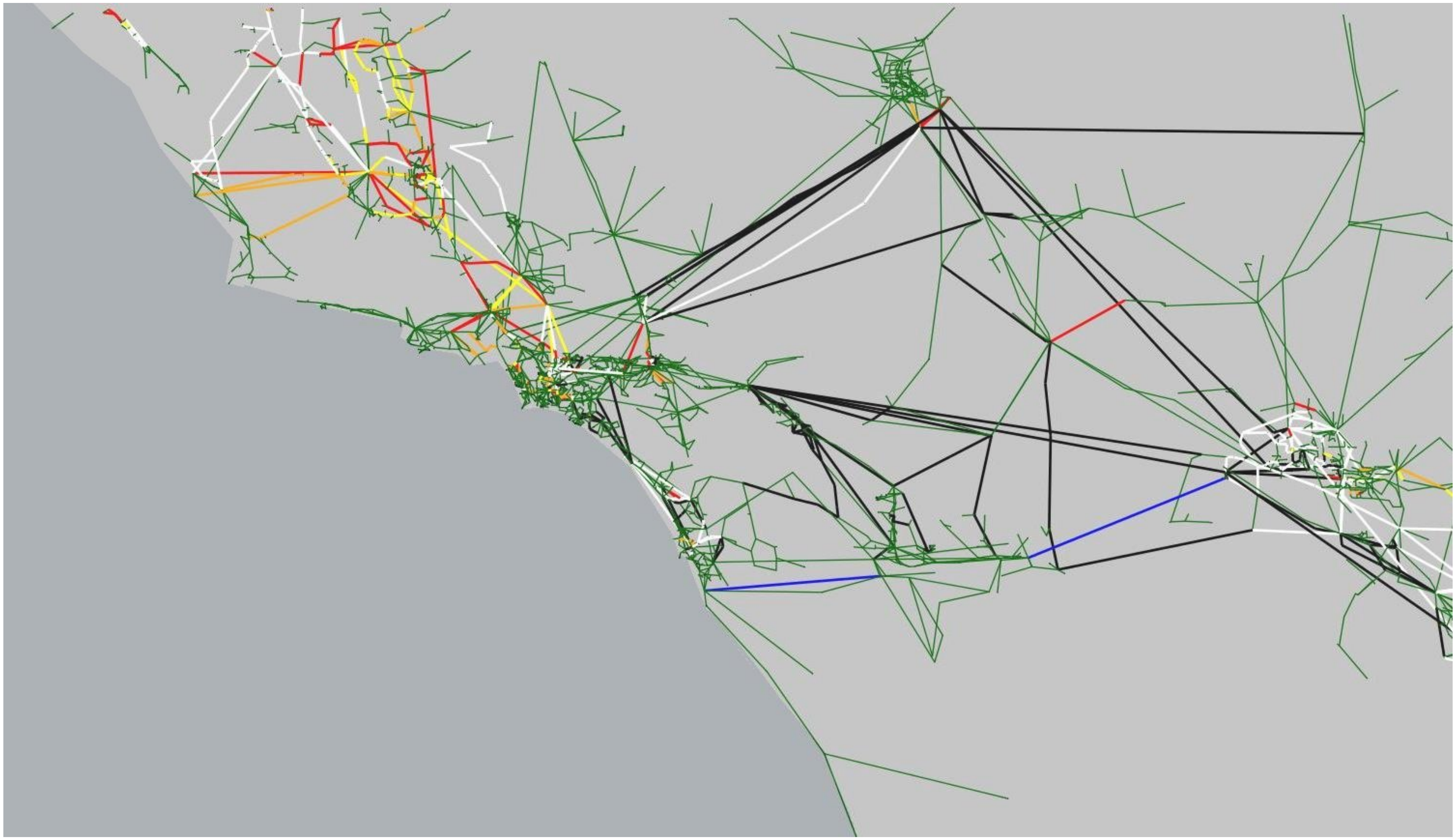

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0. Compute control on the basis of initial data.

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3*a*. Obtain measurements.

3*b*. Apply control.

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4. If $\boldsymbol{t = T}$ proportionally shed enough load to stop cascade.

A basic form of affine control

- Control consists of nonnegative parameters $\boldsymbol{u^1, u^2, \ldots, u^T}$.

- At time $\boldsymbol{t}$, on an island $\boldsymbol{C}$ with max line overload $\boldsymbol{\kappa^C}$, scale demands by

$$\mathbf{1} + \boldsymbol{u^t}\,(\mathbf{1} - \boldsymbol{\kappa^C})$$

- $\boldsymbol{\kappa^C} > \mathbf{1}$ implies loss of demand

- Easy to apply?

Clairvoyant control

- For any island $\boldsymbol{C}$ that exists at time $\boldsymbol{t}$, a parameter $\boldsymbol{u}^{t,C}$.
- At time $\boldsymbol{t}$, on island $\boldsymbol{C}$ with max line overload $\boldsymbol{\kappa}^{C}$, scale demands by

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- $\boldsymbol{\kappa}^{C} > \mathbf{1}$ implies loss of demand
- Easy to apply? Does it even make sense?

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- Easy to apply?

An even simpler form of affine control

- Control consists of nonnegative parameters $\boldsymbol{\lambda^1, \lambda^2, \ldots, \lambda^T}$.

- At time $\boldsymbol{t}$, **all** demands scaled by $\boldsymbol{\lambda^t}$

- Easy to apply.

- But conservative?

Theorem

On a network with $\boldsymbol{m}$ arcs, an optimal control of the above form can be computed in time

$$\boldsymbol{O\left(\frac{m^T}{(T-1)!}\right)}$$

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On a network with $\boldsymbol{m}$ arcs, an optimal control of the above form can be computed in time

$$\boldsymbol{O}\left(\frac{\boldsymbol{m^T}}{\boldsymbol{(T-1)!}}\right)$$

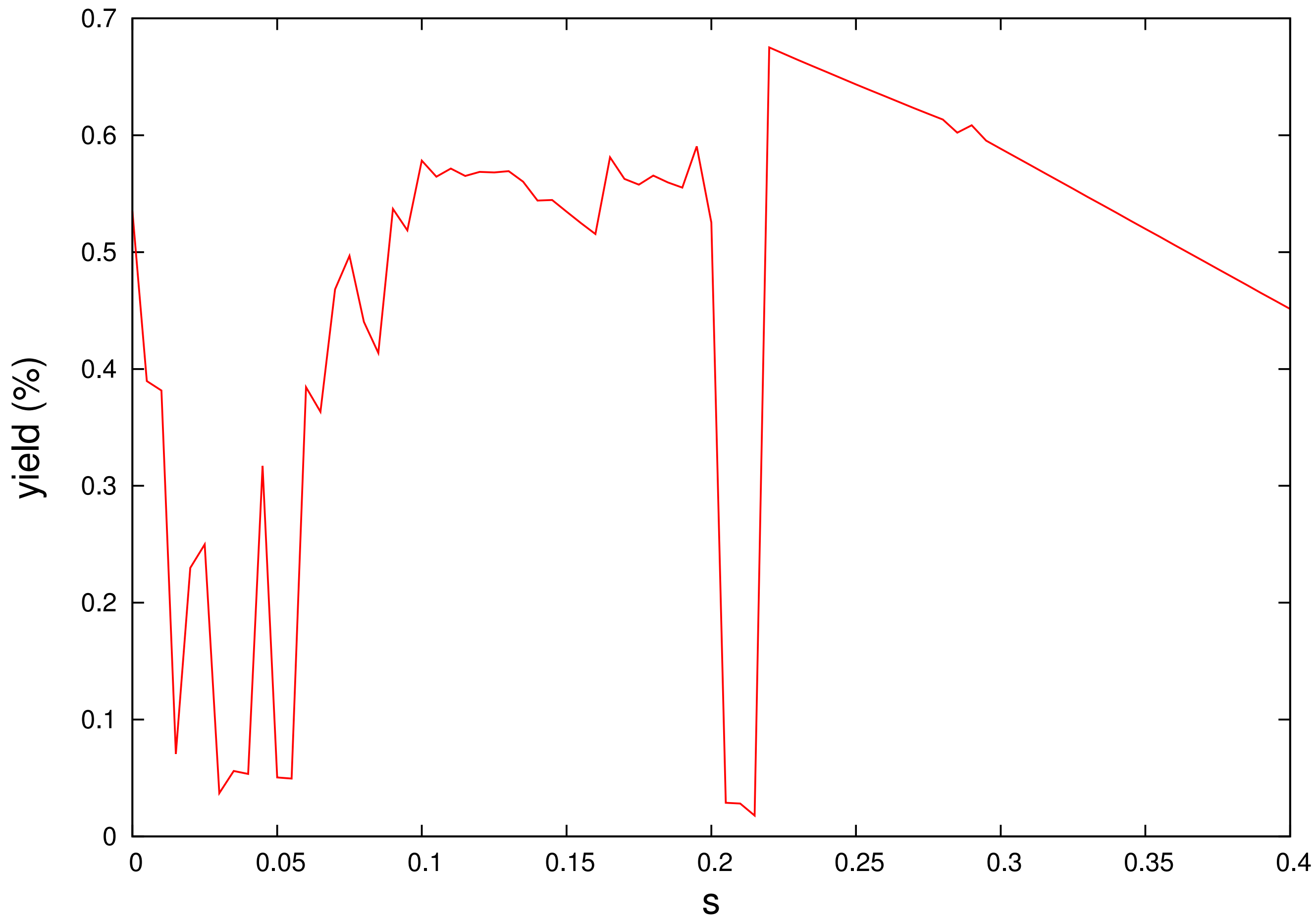
$\longrightarrow$ What is the mathematics to explain "games" of this sort?

One-parameter control: $\mathbf{s}$

- At time $\boldsymbol{t}$, on an island $\boldsymbol{C}$ with max line overload $\boldsymbol{\kappa}^{\boldsymbol{C}}$, scale demands by $\mathbf{1} + \boldsymbol{s}\ (\mathbf{1} - \boldsymbol{\kappa}^{\boldsymbol{C}})$.

One-parameter control: s

- At time t, on an island C with max line overload κ^C, scale demands by $1 + s\,(1 - \kappa^C)$.



$T = 5$, Eastern Interconnect (23 K lines)

One-parameter control: stochastic line failures

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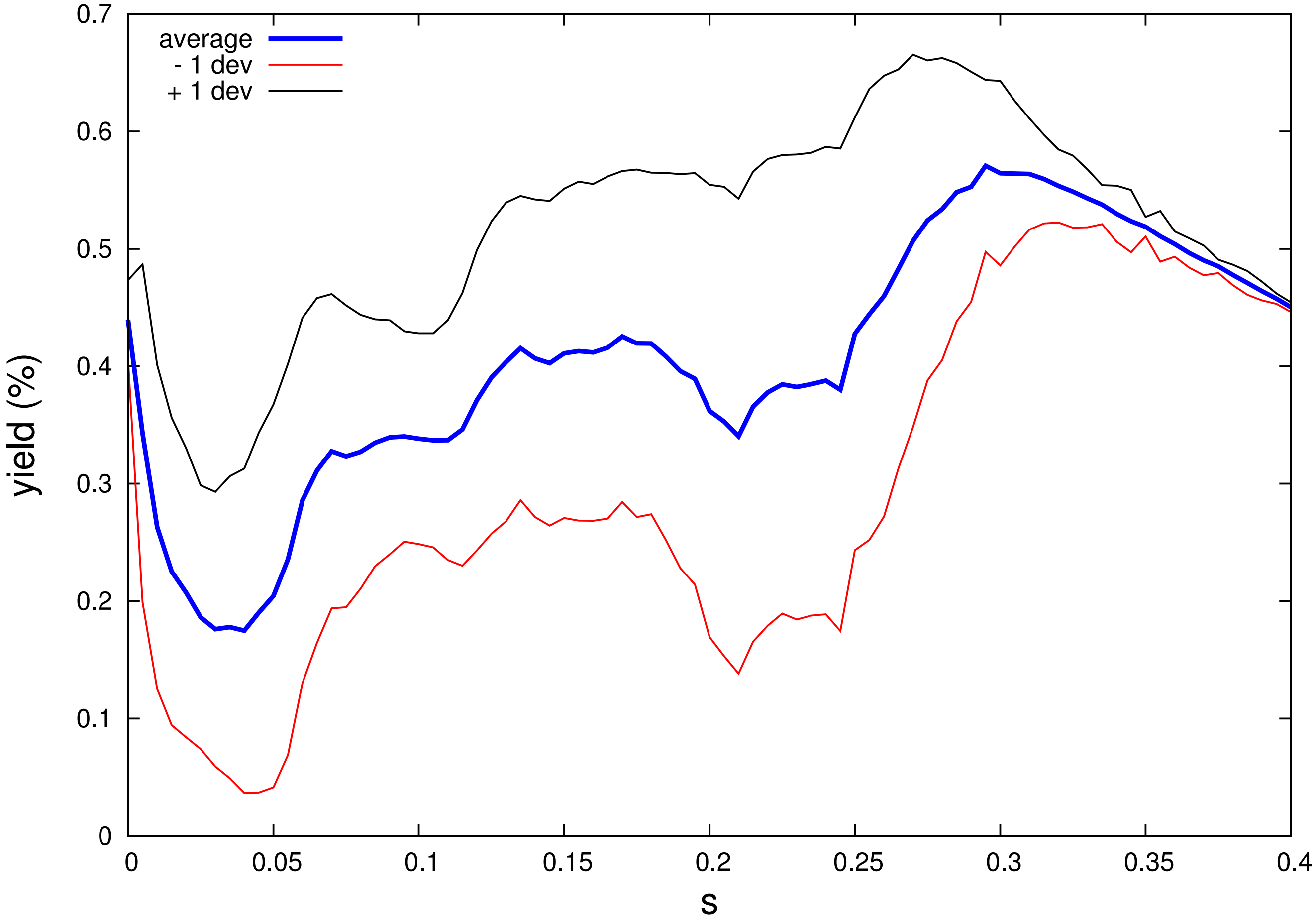

$\boldsymbol{T} = \mathbf{5}$, Eastern Interconnect (23 K lines)

Robustness through noise: the sample average approach

- Let $\hat{\boldsymbol{L}}_{\boldsymbol{j}}$ be the **limit** of line $\boldsymbol{j}$, $\boldsymbol{j = 1, \ldots, m}$ (= number of lines).

Robustness through noise: the sample average approach

- Let $\hat{L}_j$ be the **limit** of line j, $j = 1, \ldots, m$ (= number of lines).
- For $s = 1, \ldots, S$, **sample** values $L_j^{t,s}$, $1 \leq t \leq T$.
- Here, $EL_j^{t,s} = \hat{L}_j$ for all t and s.
- Interpretation: $L_j^{t,s}$ is a noisy estimate for $\hat{L}_j$.

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Optimization problem

Compute a control that maximizes the yield **averaged** across all samples.

Theorem. Can be done in $O(S)$ power flow computations.

Experiment: robust vs. non-robust solutions

(Table shows **yield**)

T	2	3	4	5
Non-robust solution and *non-robust model*	65.46%	65.46%	74.44%	**86.84%**
Non-robust solution and *robust model*	31.92%	30.46%	47.75%	**23.07%**
robust solution and *robust model*	62.19%	62.19%	70.73%	**78.36%**