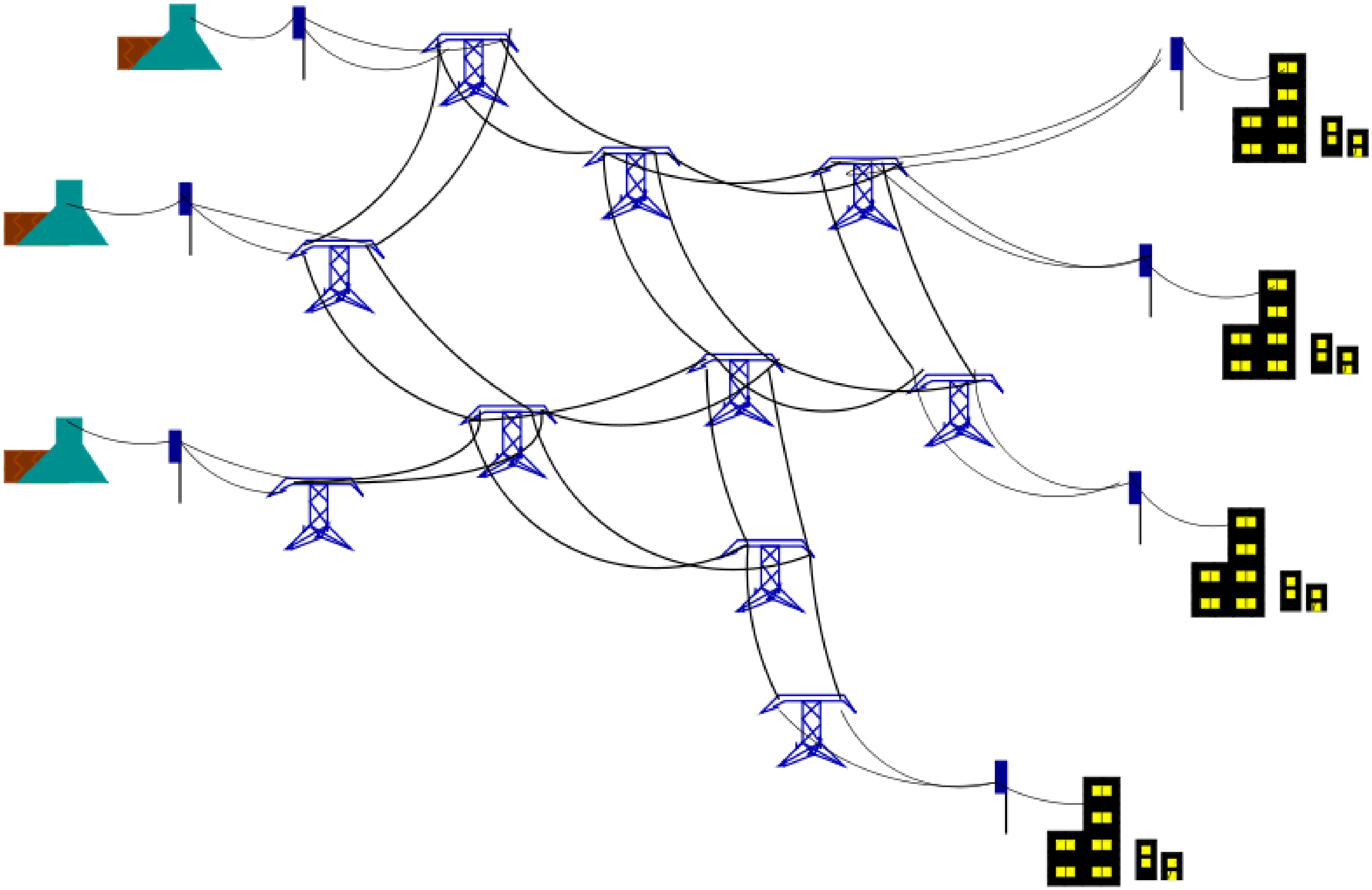
Chance-constrained generator dispatch
+ how the grid works

Daniel Bienstock, Columbia University

“The grid”:



Brief introduction

"The grid":

- Transmission network + distribution networks
- Transmission: used for long-distance transmission of power at high voltages
- Distribution: used for local conveyance of power at low(er) voltages
- This talk: focused on transmission
- What is power? What is voltage? What is transmission?

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- What is power and how is it generated? What is voltage? What is transmission?
- High-school classical physics:
  voltage = potential energy per unit charge,
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- What is power and how is it generated? What is voltage? What is transmission?
- High-school classical physics:
  voltage = potential energy per unit charge,
  electrical current = charge per unit time (per unit area)
- Generators generate **current** at a given **voltage**
  Voltage $\times$ current = power

Brief introduction

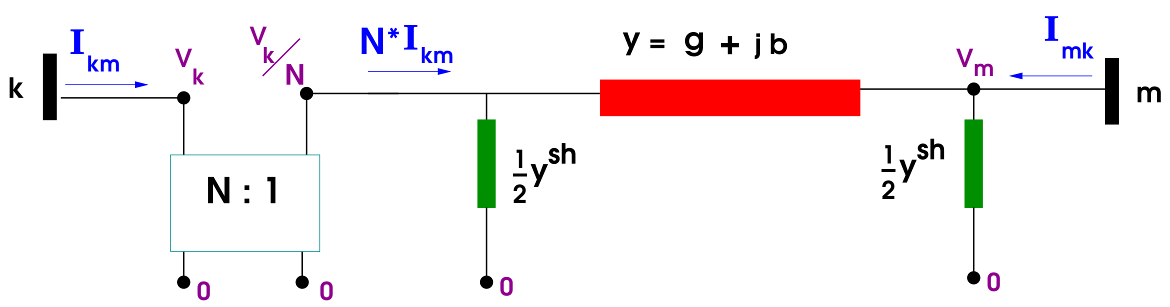
- Grid modeled as a network; nodes = "buses", edges = "lines"
- **Steady-state** operation: each bus k has a voltage (potential energy)

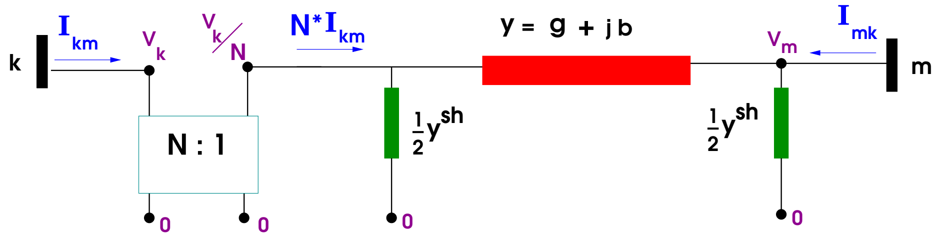
$$V_k \; = \; |V_k| e^{j\theta_k}$$

- Each line $\{k, m\}$ has physical attributes: e.g. *resistance* r, *reactance* x, *shunt admittance* y^{sh}

$$\begin{aligned} z \; &\doteq \; r + jx, \quad \text{(series impedance)} \\ y \; &\doteq \; z^{-1} = g + jb, \quad \text{(admittance)} \\ g = \frac{r}{r^2 + x^2} \quad &\text{and} \quad b = -\frac{x}{r^2 + x^2}, \end{aligned}$$

- A **transformer** with $N \; \doteq \; \tau e^{j\sigma}$ scales voltages by N.





$$(N = \tau e^{j\sigma})$$

$$V = \begin{pmatrix} V_k \\ V_m \end{pmatrix} = \begin{pmatrix} |V_k|e^{j\theta_k} \\ |V_m|e^{j\theta_m} \end{pmatrix} = \begin{pmatrix} e_k + jf_k \\ e_m + jf_m \end{pmatrix} \quad \text{(voltages at } k \text{ and } m\text{)}$$

$$I = \begin{pmatrix} I_{km} \\ I_{mk} \end{pmatrix} \quad \text{(complex current injections at } k \text{ and } m\text{)}$$

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Then

$$S_{km} = V_k I_{km}^*, \quad S_{mk} = V_m I_{mk}^* \quad \text{and} \quad I = \mathbb{Y}V,$$

where

$$\mathbb{Y} = \begin{pmatrix} (y + \frac{y^{sh}}{2})\frac{1}{\tau^2} & -y\frac{1}{\tau e^{-j\sigma}} \\ -y\frac{1}{\tau e^{j\sigma}} & y + \frac{y^{sh}}{2} \end{pmatrix}.$$

Very nice math, but how does the grid operate?

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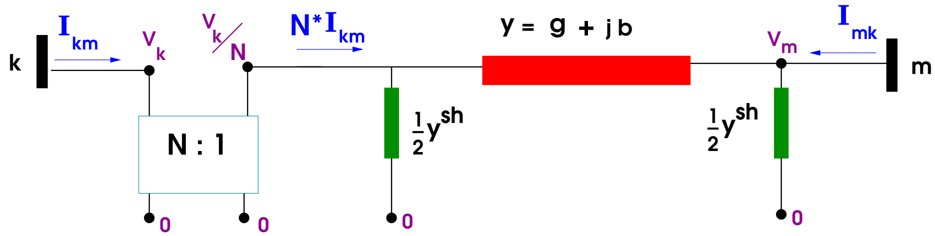
A *simplified* view:

- **Unit commitment problem**
  Run e.g. every twelve hours, to decide which (large) generators to operate
  Uses a simplifed model of the physics and the grid, plus demand *estimates*
  A linear mixed-integer program

- **OPF = Optimal power flow**
  Once generators have been picked, OPF is used to approximately minimize generation cost
  And also to verify stable operation
  Uses a more accurate model of the physics and grid
  A nonconvex continuous optimization problem

- **OPF = Optimal power flow**
  Run as often as every five minutes, to minimize generation cost
  Uses estimates of demands over the next time window
  Simplified model: linear approximation to the physics

- **Primary + secondary frequency control**
  Used for real-time management of small demand oscillations



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Optimal Power Flow

- Primary goal: economic and secure operation
- Accurate physics modeling, but expensive. Linearized version run most commonly
- Inputs for the computation: the current state of the grid, and estimates of demands ("loads") in the next time window
- First proposed by Carpentier (EDF) in 1962

$$S_{km} = P_{km} + jQ_{km}$$

$$P_{km} = |V_k|^2 g - |V_k||V_m| g \cos\theta_{km} - |V_k||V_m| b \sin\theta_{km}$$

(active power injected by k into km)

$$Q_{km} = -|V_k|^2 b + |V_k||V_m| b \cos\theta_{km} - |V_k||V_m| b \sin\theta_{km}$$

(reactive power injected by k into km)

$$(\theta_{km} \doteq \theta_k - \theta_m)$$

$$S_{km} \;=\; P_{km} \;+\; jQ_{km}$$

$$\begin{aligned} P_{km} \;&=\; |V_k|^2 g - |V_k||V_m| g\cos\theta_{km} - |V_k||V_m| b\sin\theta_{km} \\ &\;\; \text{(active power injected by } k \text{ into } km\text{)} \end{aligned}$$

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$$(\theta_{km} \;\doteq\; \theta_k - \theta_m)$$

$$P_k \;\doteq\; \sum_{km} P_{km} \quad \text{total active power injection by } k$$

$$Q_k \;\doteq\; \sum_{km} Q_{km} \quad \text{total reactive power injection by } k$$

OPF problem, simple version

Choose $|V_k|$ and θ_k for each bus k, so that

$$\min \quad \sum_{g \in \mathbb{G}} F_g(P_g)$$

$$\text{s.t.} \quad L_k \;\leq\; P_k \;\leq\; U_k \qquad \text{all } k$$

$$V_k^{min} \;\leq\; |V_k| \;\leq\; V_k^{max} \qquad \text{all } k$$

$$|S_{km}| \;\leq\; S_{km}^{max} \qquad \text{all } km$$

$$|\theta_{km}| \;\leq\; \theta_{km}^{max} \qquad \text{all } km, \text{ sometimes}$$

F_g convex quadratic, usually.

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\end{aligned}$$

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In principle, this is a difficult, nonconvex optimization problem

How does the industry handle this problem?

- Techniques borrowed from convex optimization, i.e. logarithmic barrier methods
- Sequential linearization
- Other heuristics
- If everything fails, change the problem
- Some software is quite old
- Works very well on routine problems – may run in (tens of) seconds
- May not work well on grids under distress

$$P_{km} = |V_k|^2 g - |V_k||V_m| g \cos\theta_{km} - |V_k||V_m| b \sin\theta_{km}$$

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Practical adaptation for routine operation

- $r = 0$ for each line (zero resistance).
  So $g = \frac{r}{r^2+x^2} = 0$, $b = -\frac{x}{r^2+x^2} = -x^{-1}$
- $|V_k| = 1$ for all buses k (after scaling)
- $\theta_k - \theta_m \approx 0$ for all lines km, so $\sin(\theta_k - \theta_m) \approx \theta_k - \theta_m$
- Only focus on active power

$$P_{km} = |V_k|^2 g - |V_k||V_m| g \cos\theta_{km} - |V_k||V_m| b \sin\theta_{km}$$

$$\approx \frac{\theta_k - \theta_m}{x} = y(\theta_k - \theta_m)$$

$$P_{km} \;=\; |V_k|^2 g - |V_k||V_m| g\cos\theta_{km} - |V_k||V_m| b \sin\theta_{km}$$

"DC Approximation"

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$$\begin{aligned} P_{km} \;&=\; |V_k|^2 g - |V_k||V_m| g\cos\theta_{km} - |V_k||V_m| b \sin\theta_{km} \\ &\approx\; \frac{\theta_k - \theta_m}{x} \;=\; y(\theta_k - \theta_m) \end{aligned}$$

So, get

$$\begin{aligned} \min \quad & \sum_{g\in\mathbb{G}} F_g(P_g) \\ \text{s.t.} \quad & \sum_{km} y_{km}(\theta_k - \theta_m) \;=\; P_k \quad \text{all } k \\ & L_k \;\le\; P_k \;\le\; U_k \quad \text{all } k, \quad |y_{km}(\theta_k - \theta_m)| \;\le\; U_{km}^{max} \quad \text{all } km \end{aligned}$$

For the optimization jockes: OPF using rectangular coordinates

$$V = \begin{pmatrix} V_k \\ V_m \end{pmatrix} = \begin{pmatrix} |V_k|e^{j\theta_k} \\ |V_m|e^{j\theta_m} \end{pmatrix} = \begin{pmatrix} e_k + jf_k \\ e_m + jf_m \end{pmatrix} \quad \text{(voltages at } k \text{ and } m\text{)}$$

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$\rightarrow$ P_{km} and Q_{km} are *bilinear* functions of e_k, e_m, f_k, f_m, e.g.

$$P_{km} = e_k g(e_k - e_m) - e_k b(f_k - f_m) + f_k g(f_k - f_m) + f_k b(e_k - e_m)$$

in the *no shunt, no transformer case.*

OPF in rectangular coordinates, simple case

Choose $|e_k|$ and f_k for each bus k, so that

$$
\begin{aligned}
K^{OPF} \;=\; \min \quad & \sum_{g \in \mathbb{G}} F_g(P_g) \\
\text{s.t.} \quad & w^T A_k w \;=\; P_k, \qquad \text{all } k \\
& w^T B_k w \;=\; Q_k, \qquad \text{all } k \\
& \text{box constraints on } P_k,\, Q_k, \text{ for all } k \\
& V_k^{min} \;\leq\; w^T M_k w \;\leq\; V_k^{max} \qquad \text{all } k
\end{aligned}
$$

Here $w = (e_1, e_2, \ldots, e_n, f_1, f_2, \ldots, f_n)^T$.

OPF in rectangular coordinates, II

$$
\begin{aligned}
K^{OPF} \;=\; \min \quad & w^T F w \\
\text{s.t.} \quad & L_k \;\le\; w^T A^k w \;\le\; U_k, \qquad k = 1, 2, \ldots m \\
& w \in \mathbb{R}^n.
\end{aligned}
$$

Here, $F \succeq 0$.

OPF in rectangular coordinates, II

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Here, $F \succeq 0$. A **quadratically constrained, quadratic program**.

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\end{aligned}
$$

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Write $W \ = \ ww^T \ \in \mathbb{R}^{n \times n}$. Then $W \succeq 0$, rank 1. So:

$$
\begin{aligned}
K^{OPF} \ = \ \min \quad & \sum_{i,j} F_{ij} W_{ij} \\
\text{s.t.} \quad & L_k \ \leq \ \sum_{ij} A^k_{ij} W_{ij} \ \leq \ U_k, \qquad k = 1, 2, \ldots m
\end{aligned}
$$

A *linear* program ? A **quadratically constrained, quadratic program**.

OPF in rectangular coordinates, III

$$
\begin{aligned}
K^{OPF} \ = \ \min \quad & w^T F w \\
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Here, $F \succeq 0$.

Back to DC Approximation

$$
\begin{aligned}
\min \quad & \sum_{g \in \mathbb{G}} F_g(P_g) \\
\text{s.t.} \quad & \sum_{km} y_{km}(\theta_k - \theta_m) \ = \ P_k \quad \text{all } k \\
& L_k \ \le \ P_k \ \le \ U_k \quad \text{all } k, \quad |y_{km}(\theta_k - \theta_m)| \ \le \ U_{km}^{max} \quad \text{all } km
\end{aligned}
$$

Q: how do we handle wind power?

Back to DC Approximation

$$
\begin{array}{ll}
\min & \displaystyle\sum_{g \in \mathbb{G}} F_g(P_g) \\
\text{s.t.} & \displaystyle\sum_{km} y_{km}(\theta_k - \theta_m) \; = \; P_k \quad \text{all } k \\
& L_k \;\leq\; P_k \;\leq\; U_k \quad \text{all } k, \quad |y_{km}(\theta_k - \theta_m)| \;\leq\; U_{km}^{max} \quad \text{all } km
\end{array}
$$

Q: how do we handle wind power?

One option:

- model each wind farm as another node (bus) in the transmission system
- model average wind output of a farm as a negative load
- manage real-time variations using frequency control
- e.g. secondary control:

$$
\text{load change } \Delta \;\Rightarrow\; \text{generator } g \text{ changes its output by } \alpha_g \Delta
$$

$$
\sum_g \alpha_g \;=\; 1, \quad \alpha_g = \text{“participation factor”, computed based on economics}
$$

THE ENERGY CHALLENGE

Wind Energy Bumps Into Power Grid's Limits

Mike Groll/Associated Press

The Maple Ridge Wind farm near Lowville, N.Y. It has been forced to shut down when regional electric lines become congested.

By MATTHEW L. WALD
Published: August 26, 2008

When the builders of the Maple Ridge Wind farm spent $320 million to put nearly 200 wind turbines in upstate New York, the idea was to get paid for producing electricity. But at times, regional electric lines have been so congested that Maple Ridge has been forced to shut down even with a brisk wind blowing.

CIGRE -International Conference on Large High Voltage Electric Systems '09

- Large unexpected fluctuations in wind power can cause additional flows through the transmission system (grid)
- Large power deviations in renewables must be balanced by other sources, which may be far away
- Flow reversals may be observed – control difficult
- A solution – expand transmission capacity! Difficult (expensive), takes a long time
- Problems **already observed** when renewable penetration high

OPF:

$$\min \; c(p) \quad \text{(a quadratic)}$$

s.t.

$$B\theta = p - d \tag{1}$$

$$|y_{ij}(\theta_i - \theta_j)| \;\leq\; u_{ij} \quad \text{for each line } ij \tag{2}$$

$$P_g^{min} \;\leq\; p_g \;\leq\; P_g^{max} \quad \text{for each bus } g \tag{3}$$

Notation:

$p \;=\;$ vector of generations $\in \mathbb{R}^n$, $\quad d \;=\;$ vector of loads $\in \mathbb{R}^n$

$B \;\in \mathbb{R}^{n \times n}$, $\quad$ (bus susceptance matrix)

$$\forall i, j : \quad B_{ij} = \begin{cases} -y_{ij}, & ij \;\in\; \mathcal{E} \text{ (set of lines)} \\ \sum_{k;\{k,j\} \in \mathcal{E}} y_{kj}, & i = j \\ 0, & \text{otherwise} \end{cases}$$

OPF:

$$\min \ c(p) \quad \text{(a quadratic)}$$

s.t.

$$B\theta = p - d \tag{4}$$

$$|y_{ij}(\theta_i - \theta_j)| \ \leq \ u_{ij} \quad \text{for each line } ij \tag{5}$$

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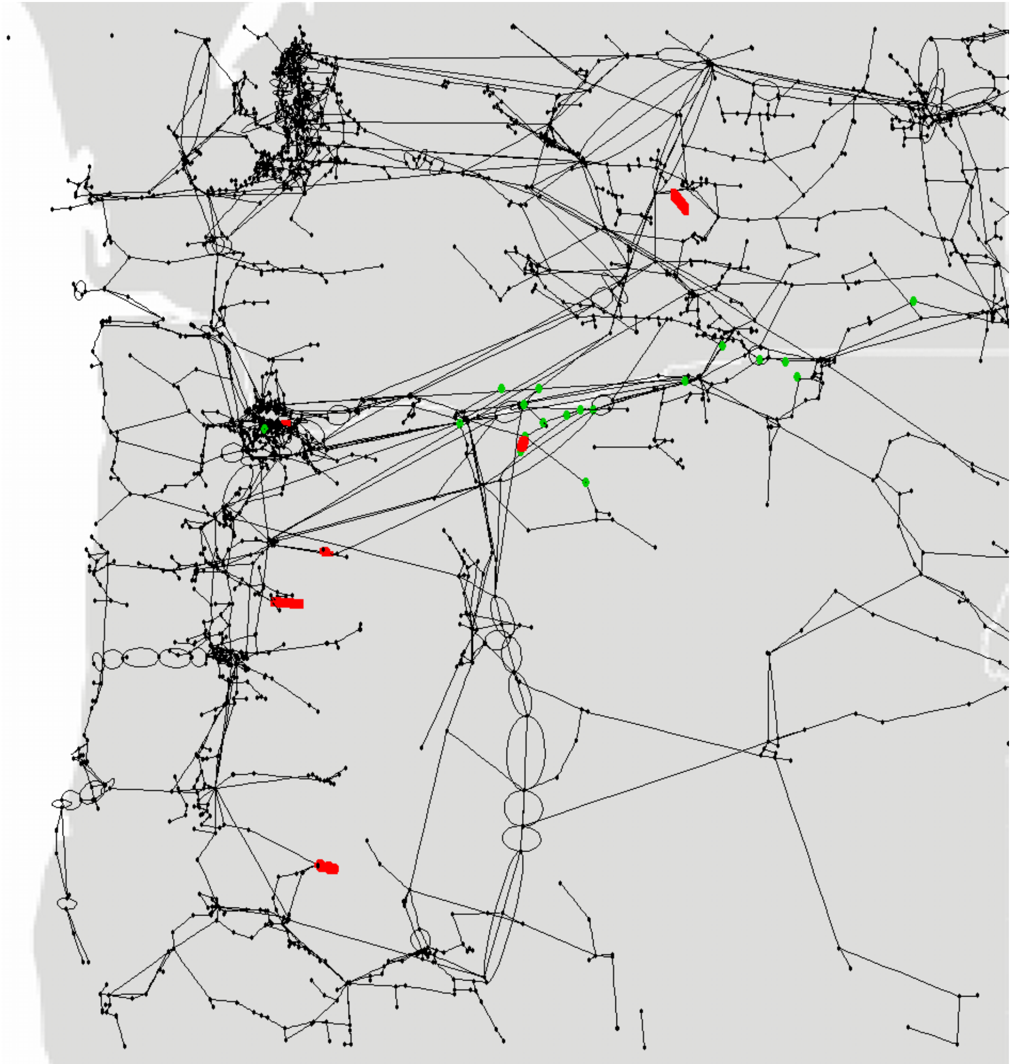
Secondary response:

"Load" change $\Delta \Rightarrow$ generator g changes its output by $\alpha_g \Delta$

Experiment Bonneville Power Administration data, Northwest US

- data on wind fluctuations at planned farms
- with standard OPF, 7 lines exceed limit $\geq 8\%$ of the time



Modeling risk: line limits and line tripping

If power flow in a line exceeds its limit, the line becomes compromised and may 'trip'. But process is complex and time-averaged:

Modeling risk: line limits and line tripping

If power flow in a line exceeds its limit, the line becomes compromised and may 'trip'. But process is complex and time-averaged:

- Thermal limit is most common
- Thermal limit may be in terms of terminal equipment, not line itself
- Wind strength and wind direction contributes to line temperature
- IEEE Standard 738 computes line temperature as a function of power flow and **numerous** exogenous parameters (wind, temperature, humidity, air pressure, date, time of day, latitude and longitude, ...)
- In 2003 U.S. blackout event, many critical lines tripped due to thermal reasons, but well short of their line limit

Modeling risk: line limits and line tripping

summary: exceeding limit for too long is bad, but precise model difficult

want: "fraction of time a line exceeds its limit is small"

proxy: prob(violation on line km) $< \epsilon_{km}$

- ϵ_{km} small, a parameter we control
- must have a working model for wind behavior

OPF:

$$\min \; c(p) \quad \text{(a quadratic)}$$

s.t.

$$B\theta = p - d \tag{7}$$

$$|y_{ij}(\theta_i - \theta_j)| \; \leq \; u_{ij} \quad \text{for each line } ij \tag{8}$$

$$P_g^{min} \; \leq \; p_g \; \leq \; P_g^{max} \quad \text{for each bus } g \tag{9}$$

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Line flows under wind power

wind power at bus i: $\mu_i + \boldsymbol{w}_i$

DC approximation $\Rightarrow$

- $B\boldsymbol{\theta} = \overline{p} - d$
  $+(\mu + \boldsymbol{w} - \alpha \sum_{i \in G} \boldsymbol{w}_i)$
- $\boldsymbol{\theta} = B^{+}(\bar{p} - d + \mu) + B^{+}(I - \alpha e^T)\boldsymbol{w}$
- flow is a linear combination of bus power injections:

$$\boldsymbol{f_{ij}} = \beta_{ij}(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j)$$

Line flows under wind power

$$\boldsymbol{f_{ij}} = \beta_{ij}\left((B_i^+ - B_j^+)^T(\bar{p} - d + \mu) + (A_i - A_j)^T\boldsymbol{w}\right),$$
$$A = B^+(I - \alpha e^T)$$

Given distribution of wind can calculate moments of line flows:

- $E\boldsymbol{f_{ij}} = \beta_{ij}(B_i^+ - B_j^+)^T(\bar{p} - d + \mu)$
- $var(\boldsymbol{f_{ij}}) := s_{ij}^2 \geq \beta_{ij}^2 \sum_k (A_{ik} - A_{jk})^2\sigma_k^2$ (assuming independence)
- and higher moments if necessary

Chance constraints to deterministic constraints

- chance constraint:
  $P(\boldsymbol{f_{ij}} > f_{ij}^{max}) < \epsilon_{ij}$ **and** $P(\boldsymbol{f_{ij}} < -f_{ij}^{max}) < \epsilon_{ij}$
- from moments of $\boldsymbol{f_{ij}}$, can get conservative approximations using e.g. Chebyshev's inequality
- for Gaussian wind, can do better, since $\boldsymbol{f_{ij}}$ is Gaussian :

$$|E\boldsymbol{f_{ij}}| + var(\boldsymbol{f_{ij}})\phi^{-1}\left(1-\epsilon_{ij}\right) \leq f_{ij}^{max}$$

Chance-constrained DC OPF:

Choose mean generator outputs and control to minimize expected cost, with the probability of line overloads kept small.

$$
\begin{aligned}
&\min_{\overline{p},\alpha} \mathbb{E}[c(\overline{p})] \\
\text{s.t.} &\sum_{i\in G} \alpha_i = 1,\ \alpha \geq 0 \\
&B\delta = \alpha, \delta_n = 0 \\
&\sum_{i\in G} \overline{p}_i + \sum_{i\in \mathcal{F}} \color{red}{\mu_{\mathbf{i}}} = \sum_{i\in D} d_i \\
&\overline{f}_{ij} = \beta_{ij}(\overline{\theta}_i - \overline{\theta}_j), \\
&B\overline{\theta} = \overline{p} + \color{red}{\mu} - d,\ \overline{\theta}_n = 0 \\
&s_{ij}^2 \geq \beta_{ij}^2 \sum_{k\in\mathcal{F}} \color{red}{\sigma_{\mathbf{k}}^{\mathbf{2}}}(B_{ik}^{+} - B_{jk}^{+} - \delta_i + \delta_j)^2 \\
&|\overline{f}_{ij}| + s_{ij}\phi^{-1}\left(1 - \epsilon_{ij}\right) \leq f_{ij}^{max}
\end{aligned}
$$

An experiment:

Polish 2003-2004 winter peak case

- 2746 buses, 3514 branches, 8 wind sources
- 5% penetration and $\sigma = .3\mu$ each source

The optimization problem has:

- 36625 variables
- 38507 constraints, 6242 conic constraints
- 128538 nonzeros, 87 dense columns

CPLEX:

- total time on 16 threads = 3393 seconds
- "optimization status 6"
- solution is wildly infeasible

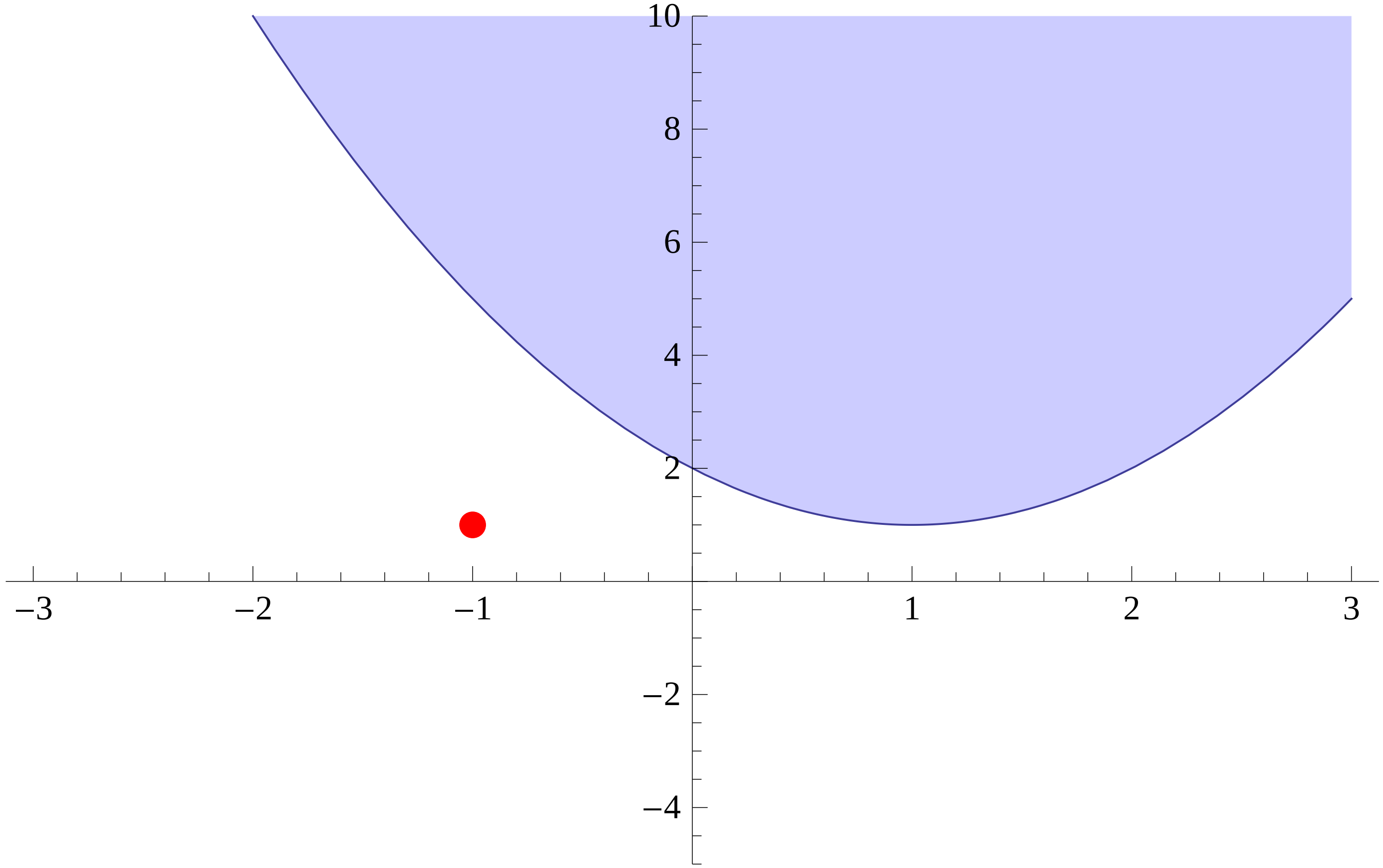
Gurobi:

- time: 31.1 seconds
- "Numerical trouble encountered"

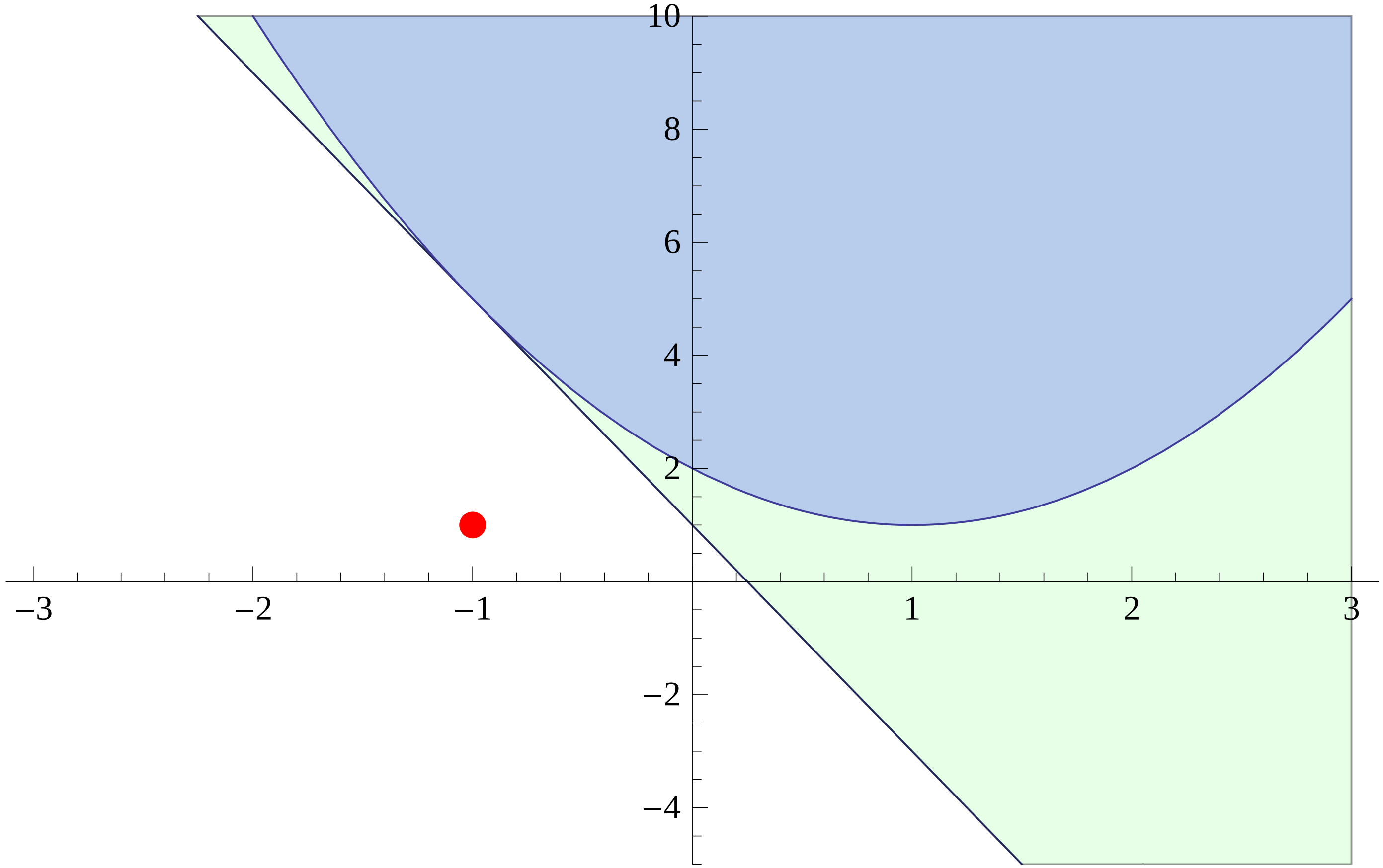
Cutting-plane algorithm:

```
remove all conic constraints
repeat until convergence:
    solve linearly constrained problem
    if no conic constraints violated: return
    find separating hyperplane for maximum violation
    add linear constraint to problem
```

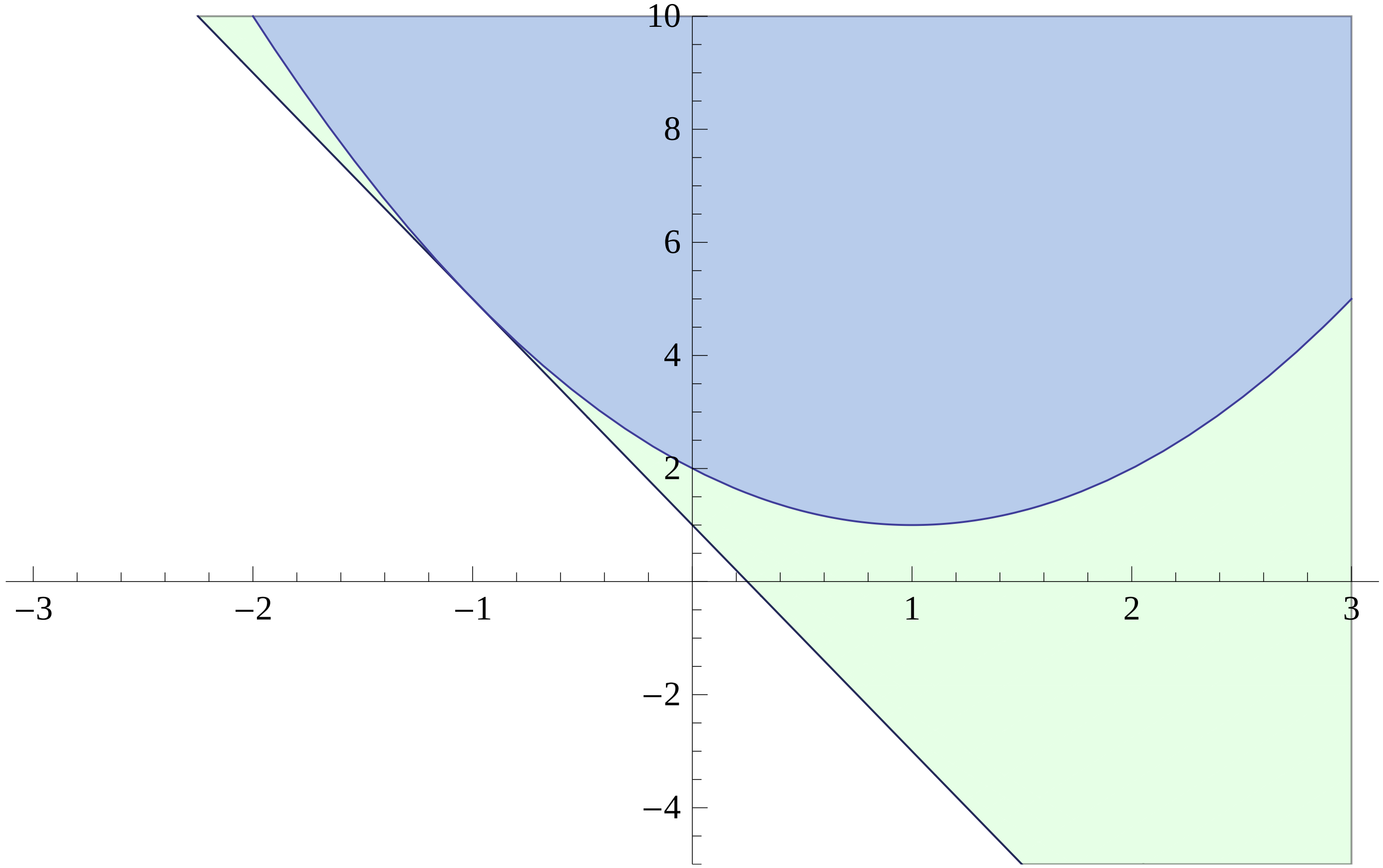
Candidate solution violates conic constraint



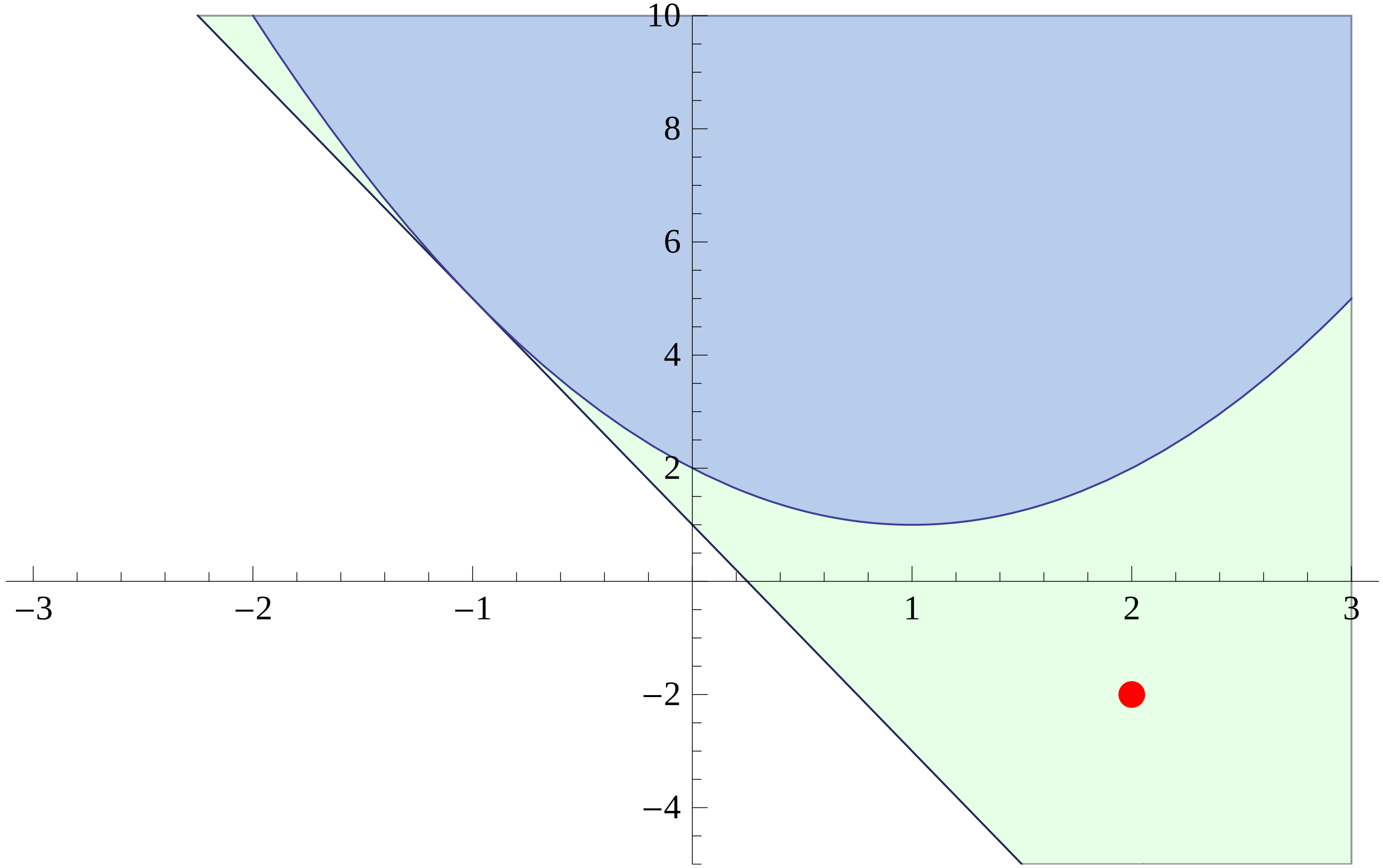
Separate: find a linear constraint also violated



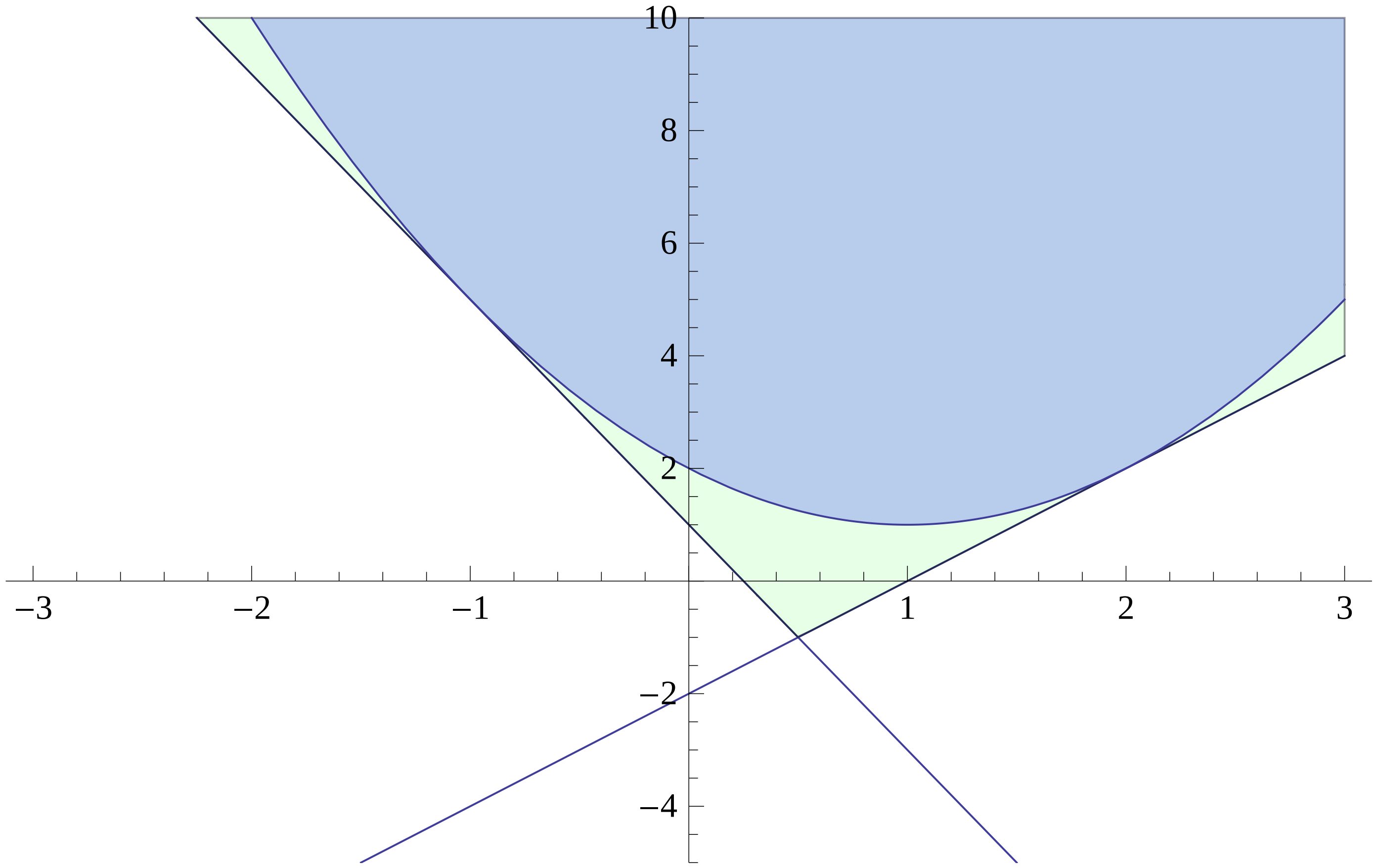
Solve again with linear constraint



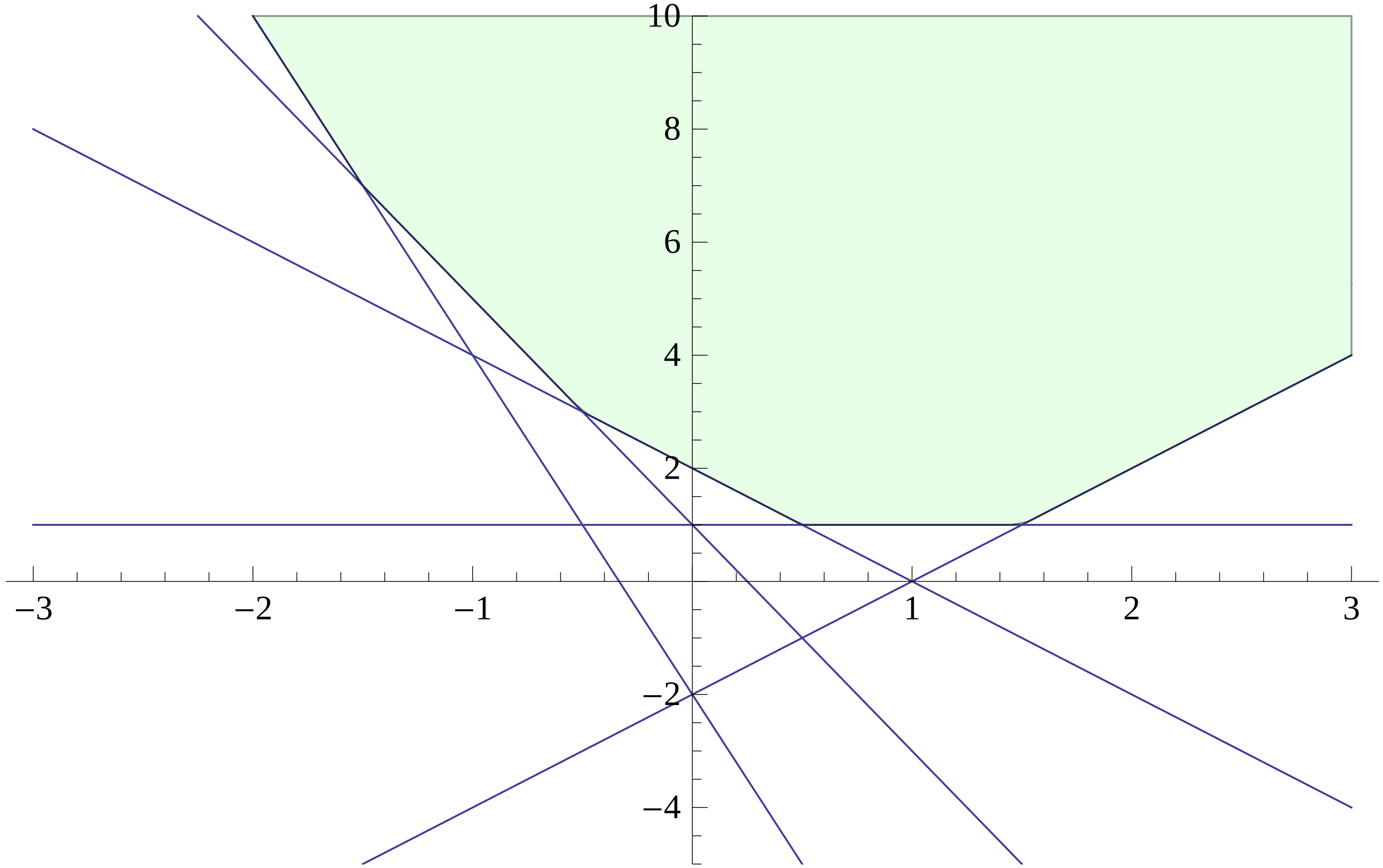
New solution still violates conic constraint



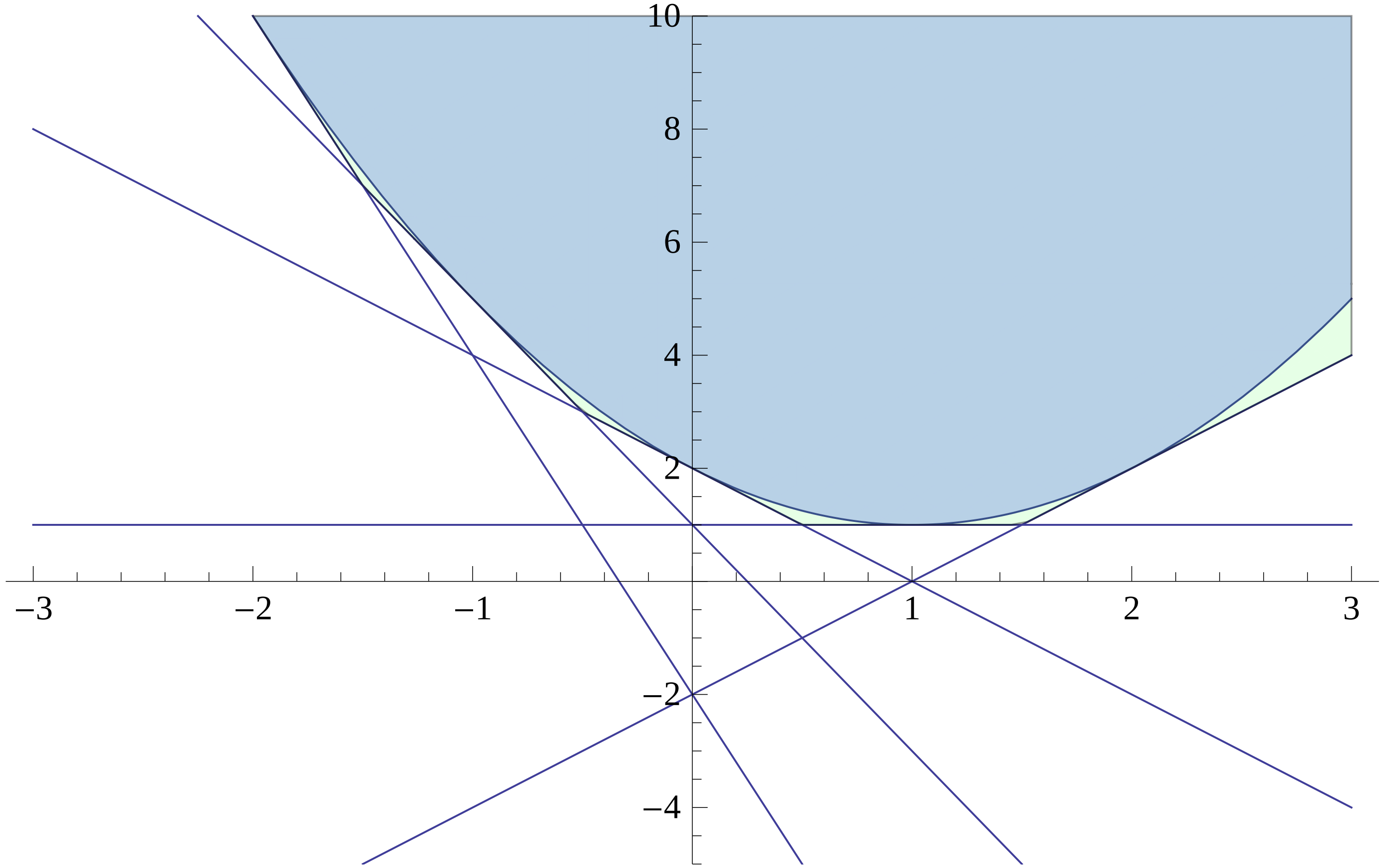
Separate again



We might end up with many linear constraints



... which approximate the conic constraint



Polish 2003-2004 case

CPLEX: “opt status 6”

Gurobi: “numerical trouble”

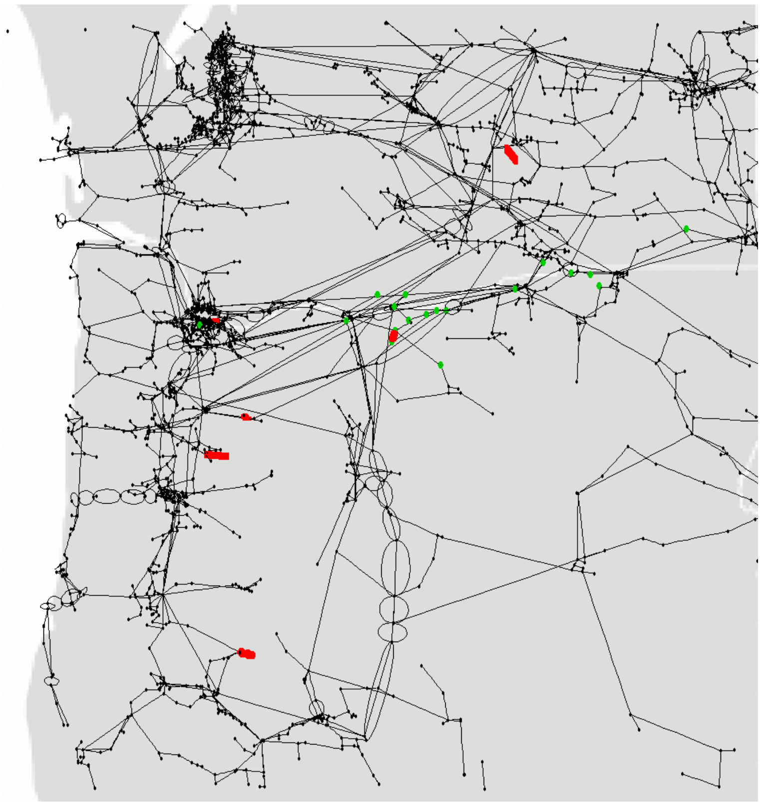
Example run of cutting-plane algorithm:

Iteration	Max rel. error	Objective
1	1.2e-1	7.0933e6
4	1.3e-3	7.0934e6
7	1.9e-3	7.0934e6
10	1.0e-4	7.0964e6
12	8.9e-7	7.0965e6

Total running time: 32.9 seconds

Back to motivating example: BPA case

- standard OPF: cost 235603, 7 lines unsafe $\geq$ 8% of the time
- CC-OPF: cost 237297, every line safe $\geq$ 98% of the time
- run time = 9.5 seconds (one cutting plane!)



Summary:

- Specialized cutting-plane algorithm proves effective
- Commercial solvers do not
- Algorithm efficient even in cases with thousands of buses/lines

Summary:

- Specialized cutting-plane algorithm proves effective
- Commercial solvers do not
- Algorithm efficient even in cases with thousands of buses/lines

Current work:

- Handle imprecise estimations in a robst way
- Extension to nonlinear power flow models
- Perhaps: interaction with some utilities

Need for robustness!

$$
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&\min_{\overline{p},\alpha} \mathbb{E}[c(\overline{p})] \\
\text{s.t.} &\sum_{i\in G} \alpha_i = 1,\ \ \alpha \geq 0 \\
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\end{aligned}
$$

Robustness: what do we want

1. We do not want to go crazy

2. When data errors are **big** we want our solutions to degrade in a controlled manner

3. When data errors are **small** we want our solutions to degrade **very little** from nominal behavior