Operations Research Problems in Power Engineering

Daniel Bienstock

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2016

Power engineering for non-power engineers

References

Andersson: *Modelling and Analysis of Electric Power Systems*

Bergen, Vittal: *Power Systems Analysis*

Glover, Sarma, Overbye: *Power System Analysis and Design*

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Rebours, Kirschen: *What is spinning reserve?*

Power engineering for non-power engineers

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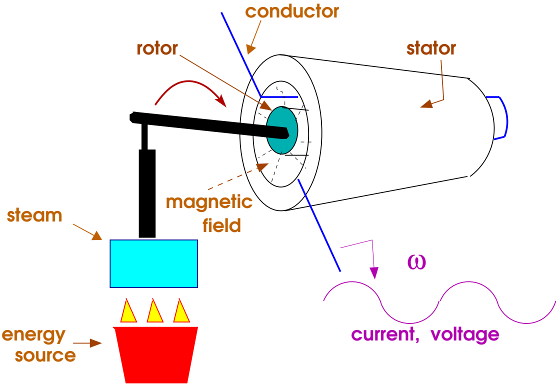

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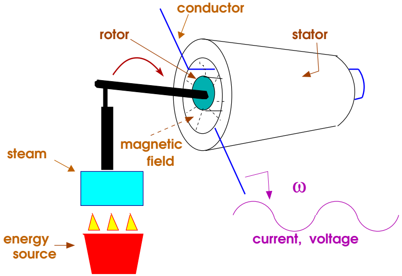
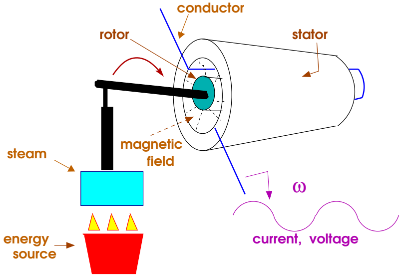


A generator produces **current** at a certain **voltage**.

Power engineering for non-power engineers



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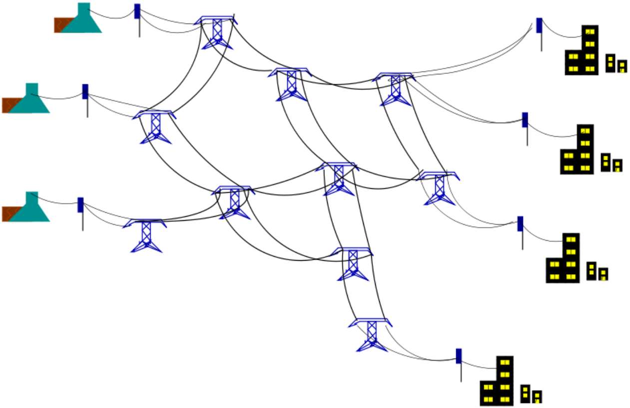
Ohm's law: power = current x voltage



AC Power Flows

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Real-time:

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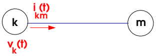
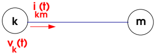


- Voltage at bus k: $v_k(t) \; = \; V_k^{max} \cos(\omega t + \theta_k^V)$
- Current injected at k into km: $i_{km}(t) \; = \; I_{km}^{max} \cos(\omega t + \theta_{km}^I)$.

AC Power Flows

Real-time:



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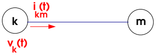
- Current injected at k into km: $i_{km}(t) \;=\; I_{km}^{max}\cos(\omega t + \theta_{km}^I)$.
- Power injected at k into km: $p_{km}(t) \;=\; v_k(t) i_{km}(t)$.

Averaged over period T:

- $p_{km} \;\doteq\; \frac{1}{T}\int_0^T p(t) \;=\; \frac{1}{2} V_k^{max} I_{km}^{max} \cos(\theta_k^V - \theta_{km}^I)$.



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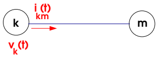

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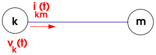

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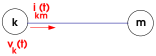

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- $p_{km} \;=\; |V_k||I_{km}| \cos(\theta_k^V - \theta_{km}^I) \;=\; \mathcal{R}\mathrm{e}(V_k I_{km}^*)$
- $q_{km} \doteq Im(V_{km} I_{km}^*)$ and $S_{km} \doteq p_{km} + jq_{km}$

- $V_k \doteq \frac{V_k^{max}}{\sqrt{2}} e^{j\theta_k^V}, \quad I_{km} \doteq \frac{I_{km}^{max}}{\sqrt{2}} e^{j\theta_{mk}^I}$ (voltage, current)

$$p_{km} = \mathcal{R}e(V_k I_{km}^*), \quad q_{km} = Im(V_{km} I_{km}^*) \qquad (1)$$

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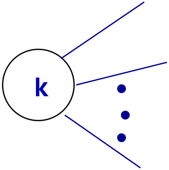
Network Equations



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$$I_{km} \;=\; \boldsymbol{y_{\{k,m\}}}(V_k - V_m), \; \boldsymbol{y_{\{k,m\}}} = \textit{admittance of } km. \tag{4}$$

Network Equations

$$\sum_{km \in \delta(k)} p_{km} \;=\; \hat{P}_k, \quad \sum_{km \in \delta(k)} q_{km} \;=\; \hat{Q}_k \quad \forall\, k \tag{5}$$

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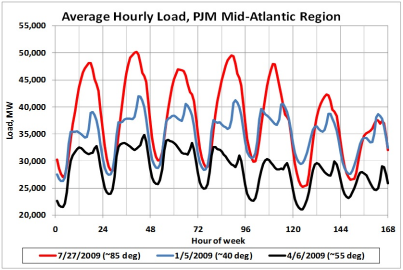
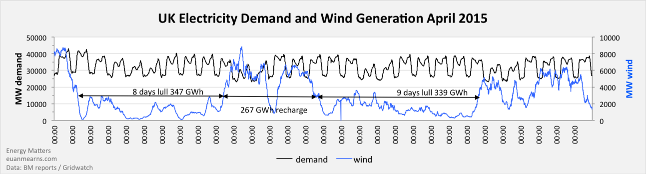
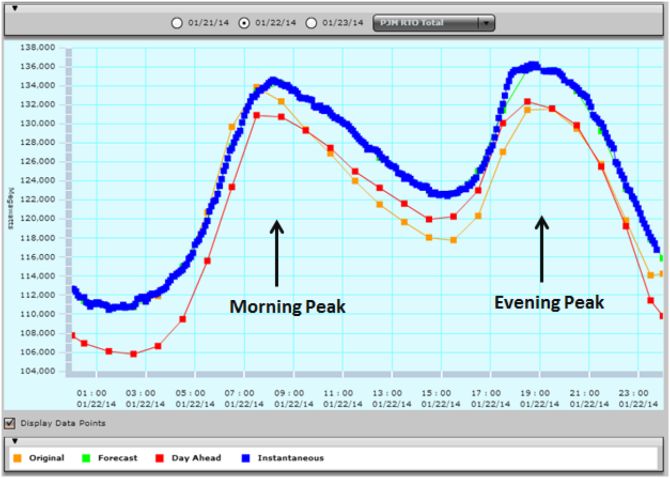
Generator: $\hat{P}_k, |V_k|$ given. Other buses: $\hat{P}_k, \hat{Q}_k$ given.

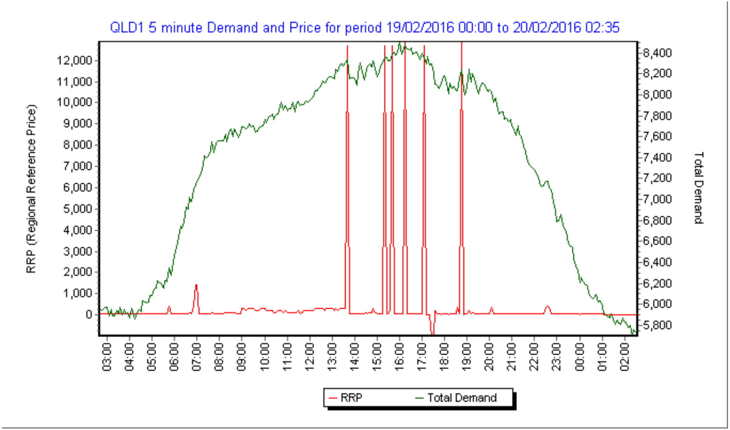
Managing changing demands





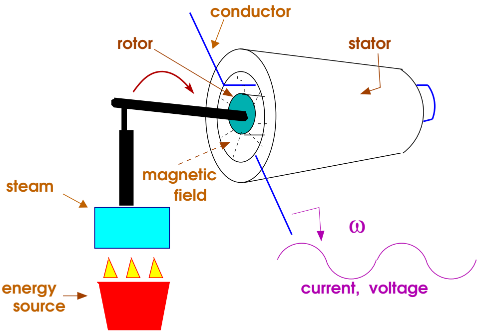


01/21/14
01/22/14
01/23/14
PJM RTO Total
138,000
136,000
134,000
132,000
130,000
128,000
126,000
124,000
122,000
120,000
118,000
116,000
114,000
112,000
110,000
108,000
106,000
104,000
Megawatts
Morning Peak
Evening Peak
01 : 00 01/22/14
03 : 00 01/22/14
05 : 00 01/22/14
07 : 00 01/22/14
09 : 00 01/22/14
11 : 00 01/22/14
13 : 00 01/22/14
15 : 00 01/22/14
17 : 00 01/22/14
19 : 00 01/22/14
21 : 00 01/22/14
23 : 00 01/22/14
Display Data Points
Original
Forecast
Day Ahead
Instantaneous



QLD1 5 minute Demand and Price for period 19/02/2016 00:00 to 20/02/2016 02:35
RRP (Regional Reference Price)
Total Demand
12,000
11,000
10,000
9,000
8,000
7,000
6,000
5,000
4,000
3,000
2,000
1,000
0
8,400
8,200
8,000
7,800
7,600
7,400
7,200
7,000
6,800
6,600
6,400
6,200
6,000
5,800
03:00
04:00
05:00
06:00
07:00
08:00
09:00
10:00
11:00
12:00
13:00
14:00
15:00
16:00
17:00
18:00
19:00
20:00
21:00
22:00
23:00
00:00
01:00
02:00
RRP
Total Demand

What happens when there is a generation/load mismatch



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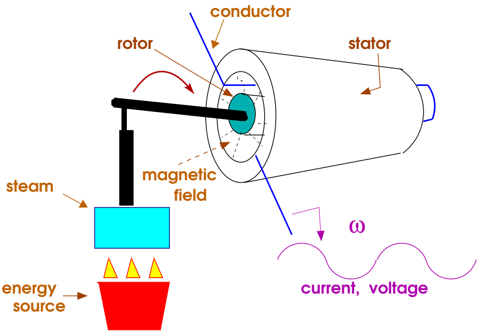

Frequency response:

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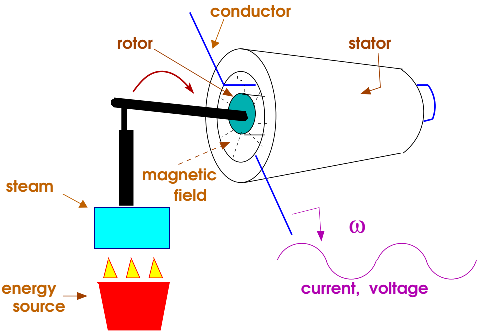

Frequency response:

mismatch $\boldsymbol{\Delta P}$

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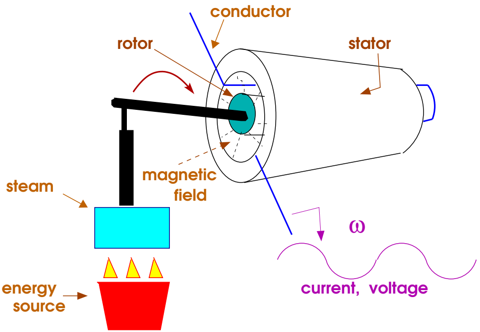

Frequency response:

mismatch $\boldsymbol{\Delta P}$ $\Rightarrow$ **frequency change** $\boldsymbol{\Delta\omega} \approx -c\,\Delta P$

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THE ENERGY CHALLENGE

Wind Energy Bumps Into Power Grid's Limits

Mike Groll/Associated Press

The Maple Ridge Wind farm near Lowville, N.Y. It has been forced to shut down when regional electric lines become congested.

By MATTHEW L. WALD
Published: August 26, 2008

When the builders of the Maple Ridge Wind farm spent $320 million to put nearly 200 wind turbines in upstate New York, the idea was to get paid for producing electricity. But at times, regional electric lines have been so congested that Maple Ridge has been forced to shut down even with a brisk wind blowing.

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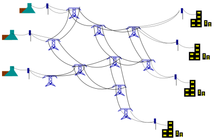
CIGRE -International Conference on Large High Voltage Electric Systems '09

- Large unexpected fluctuations in wind power can cause additional flows through the transmission system (grid)
- Large power deviations in renewables must be balanced by other sources, which may be far away
- Flow reversals may be observed – control difficult
- A solution – expand transmission capacity! Difficult (expensive), takes a long time
- Problems **already observed** when renewable penetration high

CIGRE -International Conference on Large High Voltage Electric Systems '09

- "Fluctuations" – 15-minute timespan
- Due to turbulence ("storm cut-off")
- Variation of the same order of magnitude as mean
- Most problematic when renewable penetration starts to exceed $20 - 30\%$
- Many countries are getting into this regime

Optimal power flow (economic dispatch, tertiary control)



- Used periodically to handle the next time window (e.g. 15 minutes, one hour)
- Choose generator outputs
- Minimize cost (quadratic)
- Satisfy demands, meet generator and network constraints
- **Constant load (demand) estimates for the time window**

OPF:

$$\min \ c(p) \quad \text{(a quadratic)}$$

s.t.

$$B\theta = p - d \tag{6}$$

$$|y_{ij}(\theta_i - \theta_j)| \ \leq \ u_{ij} \quad \text{for each line } ij \tag{7}$$

$$P_g^{min} \ \leq \ p_g \ \leq \ P_g^{max} \quad \text{for each bus } g \tag{8}$$

Notation:

$p \ = \$ vector of generations $\ \in \mathcal{R}^n, \quad d \ = \$ vector of loads $\ \in \mathcal{R}^n$

$B \ \in \mathcal{R}^{n \times n}, \quad$ (bus susceptance matrix)

$$\forall i, j : \quad B_{ij} = \begin{cases} -y_{ij}, & ij \ \in \ \mathcal{E} \text{ (set of lines)} \\ \sum_{k;\{k,j\} \in \mathcal{E}} y_{kj}, & i = j \\ 0, & \text{otherwise} \end{cases}$$

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How does the grid handle short-term fluctuations in **demand** (d)?
Secondary frequency control:

- Deployed a few seconds after ongoing change – "minute-by-minute" control
- Generator output varies up or down **proportionally** to **aggregate** change

$$
\begin{aligned}
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\text{s.t.} & \\
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$$

How does the grid handle short-term fluctuations in **demand** (d)?
Secondary frequency control:

- Deployed a few seconds after ongoing change – "minute-by-minute" control
- Generator output varies up or down **proportionally** to **aggregate** change

How does the grid handle short-term fluctuations in renewable output?
Answer: Same mechanism, now used to handle aggregate wind power change

“Participation factors”

For each generator i, a parameter α_i with

- $\sum_i \alpha_i \ = \ 1$
- $\alpha_i \geq 0$
- $\alpha_i > 0$ only for selected generators

Assuming real-time generation/demand mismatch Δ, real-time output of generator i:

$$p_i = \overline{p}_i - \alpha_i \Delta$$

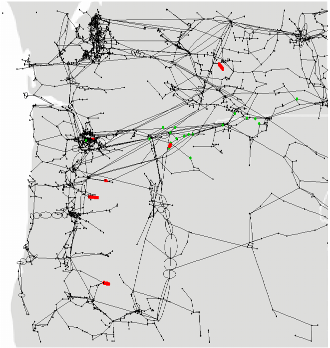
where $\overline{p}_i$ = OPF computed output for generator i.

Note: the α_i are precomputed e.g. uniform or based on economic considerations.

Experiment

Bonneville Power Administration data, Northwest US

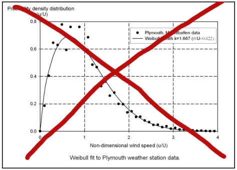
- data on wind fluctuations at planned farms
- with standard OPF, 7 lines exceed limit $\geq 8\%$ of the time



Wind model?

Need to model variation in wind power between dispatches

Wind at farm attached to bus i of the form $\mu_i + \boldsymbol{w_i}$. Weibull distribution?



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- Typical wind farm spans a significant geographical zone with many turbines
- Real-time variations due to turbulence
- Turbulence is local ($\approx 50m$ radius) and arguably local effects are indepenent
- Working model: real-time variations in a farm's output modeled as a normal variable

Line limits and line tripping

If power flow in a line exceeds its limit, the line becomes compromised and may 'trip'. But process is complex and time-averaged:

- Thermal limit is most common
- Thermal limit may be in terms of terminal equipment, not line itself
- Wind strength and wind direction contributes to line temperature
- IEEE Standard 738 attempts to account for *everything*
- In 2003 U.S. blackout event, many critical lines tripped due to thermal reasons, but well short of their line limit

Background

- When a power line overheats it becomes exposed to several risk factors

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- If the line overheats enough, it may sag and experience a contact/arc, which will cause a trip
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- What is risky? What is a critical temperature?
- 2003 event: critical temperatures estimates were sometimes incorrect.

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- Note: power lines can be more than 100 miles long.
- How can we account for data uncertainty, errors, unavailability?

Line trip model

summary: exceeding limit for too long is bad, but complicated

want: "fraction time a line exceeds its limit is small"

proxy: prob(violation on line i) $< \epsilon$ for each line i

Goals

- simple control
- aware of limits
- not too conservative
- computationally practicable with a simple algorithm

Control

For each generator i, two parameters:

- $\overline{p_i}$ = risk-aware mean output
- α_i = risk-aware participation factor

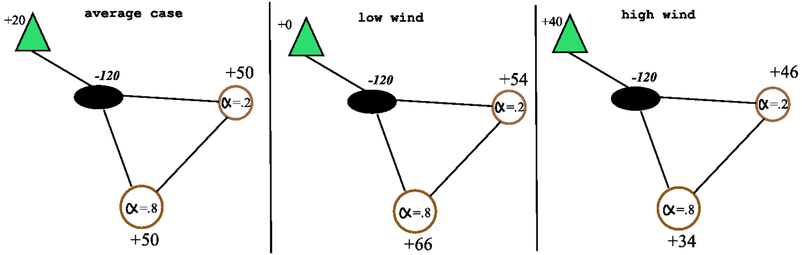
Real-time output of generator i:

$$p_i = \overline{p}_i - \alpha_i \sum_j \Delta\omega_j$$

where $\Delta\omega_j$ = change in output of renewable j (from mean).

$$\sum_i \alpha_i = 1$$

Set up
control



Computing line flows

wind power at bus i: $\mu_i + \boldsymbol{w}_i$

DC approximation

- $B\boldsymbol{\theta} = \overline{p} - d$
  $+(\mu + \boldsymbol{w} - \alpha \sum_{i \in G} \boldsymbol{w}_i)$
- $\boldsymbol{\theta} = B^{+}(\bar{p} - d + \mu) + B^{+}(I - \alpha e^{T})\boldsymbol{w}$
- flow is a linear combination of bus power injections:

$$\boldsymbol{f_{ij}} = y_{ij}(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j)$$

Computing line flows

$$\boldsymbol{f_{ij}} = y_{ij} \left((B_i^+ - B_j^+)^T (\bar{p} - d + \mu) + (A_i - A_j)^T \boldsymbol{w} \right),$$

$$A = B^+ (I - \alpha e^T)$$

Given distribution of wind can calculate moments of line flows:

- $E\,\boldsymbol{f_{ij}} = y_{ij} (B_i^+ - B_j^+)^T (\bar{p} - d + \mu)$
- $var(\boldsymbol{f_{ij}}) := s_{ij}^2 \geq y_{ij}^2 \sum_k (A_{ik} - A_{jk})^2 \sigma_k^2$
  (assuming independence)
- and higher moments if necessary

Chance constraints to deterministic constraints

- chance constraint: $P(\boldsymbol{f_{ij}} > f_{ij}^{max}) < \epsilon_{ij}$ **and** $P(\boldsymbol{f_{ij}} < -f_{ij}^{max}) < \epsilon_{ij}$
- from moments of $\boldsymbol{f_{ij}}$, can get conservative approximations using e.g. Chebyshev's inequality

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- for Gaussian wind, can do better, since $\boldsymbol{f_{ij}}$ is Gaussian :

$$|E\boldsymbol{f_{ij}}| + var(\boldsymbol{f_{ij}})\phi^{-1}(1 - \epsilon_{ij}) \leq f_{ij}^{max}$$

Formulation:

Choose mean generator outputs and control to minimize expected cost, with the probability of line overloads kept small.

$$\min_{\overline{p},\alpha} \mathbb{E}[c(\overline{p})]$$

$$\text{s.t.} \sum_{i \in G} \alpha_i = 1, \;\; \alpha \geq 0$$

$$B\delta = \alpha, \delta_n = 0$$

$$\sum_{i \in G} \overline{p}_i + \sum_{i \in W} \mu_{\mathbf{i}} = \sum_{i \in D} d_i$$

$$\overline{f}_{ij} = y_{ij}(\overline{\theta}_i - \overline{\theta}_j),$$

$$B\overline{\theta} = \overline{p} + \mu - d, \;\; \overline{\theta}_n = 0$$

$$s_{ij}^2 \geq y_{ij}^2 \sum_{k \in W} \sigma_{\mathbf{k}}^{\mathbf{2}} (B_{ik}^+ - B_{jk}^+ - \delta_i + \delta_j)^2$$

$$|\overline{f}_{ij}| + \; s_{ij}\phi^{-1}(1 - \epsilon_{ij}) \leq f_{ij}^{max}$$

Big cases

Polish 2003-2004 winter peak case

- 2746 buses, 3514 branches, 8 wind sources
- 5% penetration and $\sigma = .3\mu$ each source

CPLEX: the optimization problem has

- 36625 variables
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- 128538 nonzeros, 87 dense columns

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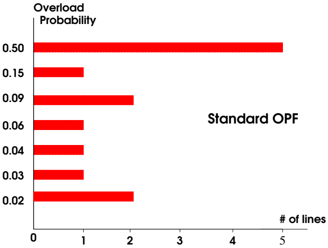
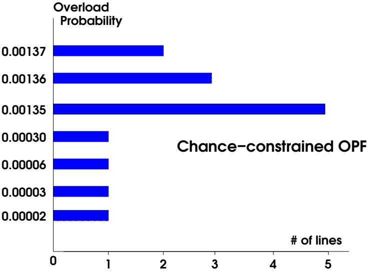
→ **basic cutting-plane algorithm works well**

Experiment: Polish grid, 20% wind penetration, 50 farms





Bienstock, Chertkov, Harnett, SIAM Review '15

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- Extension to unit commitment problem
- Better risk model for line temperature
  Bienstock, Blanchet and Li '15

The heat equation on a 1-dimensional line

- Line modeled as one-dimensional object parameterized by x, $0 \leq x \leq L$.
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- $I(t)$ = current at time t, $T(x,t)$ = temperature at x at time t.
- *Heat equation:*

$$\frac{\partial T(x,t)}{\partial t} = \kappa \frac{\partial^2 T(x,t)}{\partial x^2} + \alpha I^2(t) - \nu (T(x,t) - T^{ext}(x,t)),$$

where $\kappa \geq 0$, $\alpha \geq 0$ and $\nu \geq 0$ are (line dependent) constants, and $T^{ext}(x,t)$ is the ambient temperature at (x,t)

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$\boldsymbol{h}(\boldsymbol{x}) \;=\;$ a random variable, at x.

Back to the stochastic heat equation

$$\frac{\partial T(x,t)}{\partial t} = \alpha I^2(t) - \nu(T(x,t) - G(\boldsymbol{h}(\boldsymbol{x})).$$

Recall: $x \in [0, L]$, $t \in [0, \tau]$

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$$\boldsymbol{H}(\boldsymbol{t}) \triangleq \frac{1}{L}\int_0^L T(x,t)dx \quad \text{(average internal line temperature at t)}$$

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3. At time $\tau/2$, we **observe the value** of $\boldsymbol{R}$. Assuming $\boldsymbol{R} = r_i$, then for all $t \in [\tau/2, \tau]$, we set $I(t) = I_{2,i}$.

Goals:

(a) $P(\boldsymbol{H}(\boldsymbol{\tau}) > k) < \epsilon$.

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3. At time $\tau/2$, we **observe the value** of $\boldsymbol{R}$. Assuming $\boldsymbol{R} = r_i$, then for all $t \in [\tau/2, \tau]$, we set $I(t) = I_{2,i}$.

Goals:

(a) $P(\boldsymbol{H}(\boldsymbol{\tau}) > k) < \epsilon$. k **smaller than** critical temperature

(b) $I_1 \leq L(\tau/2)$.

Adaptive control

Simplification:

$\boldsymbol{R}$ is a discrete random variable. $P(\boldsymbol{R} = r_i) = p_i, i = 1, 2, \ldots, n$ (known).

1. At time $\tau = 0$, we compute values l_1, and $l_{2,i}$ for $i = 1, 2, \ldots, n$. These values are used as follows:
2. For all $t \in [0, \tau/2]$, we set $l(t) = l_1$.
3. At time $\tau/2$, we **observe the value** of $\boldsymbol{R}$. Assuming $\boldsymbol{R} = r_i$, then for all $t \in [\tau/2, \tau]$, we set $l(t) = l_{2,i}$.

Goals:

(a) $P(\boldsymbol{H}(\boldsymbol{\tau}) > k) < \epsilon$. k **smaller than** critical temperature

(b) $l_1 \leq L(\tau/2)$.

(c) What about performance?

We want to maximize:

- "Total" current: $\frac{\tau}{2}\ I_1\ + \frac{\tau}{2}\ I_{2,i}\quad ?$

We want to maximize:

- "Total" current: $\frac{\tau}{2}\ I_1\ + \frac{\tau}{2}\ I_{2,i}$?
- "Average" current? Square current?

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- "Total" current: $\frac{\tau}{2}\, I_1 \; + \frac{\tau}{2}\, I_{2,i}$?
- "Average" current? Square current?

$\boldsymbol{F(I_1, I_2)}$ **:** a monotonely increasing function of I_1, I_2

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Goals:

(a) $P(\boldsymbol{H}(\boldsymbol{\tau}) > k) < \epsilon$. k **smaller than** critical temperature

(b) $I_1 \leq L(\tau/2)$.

(c) Maximize:

$$\sum_{i=1}^{n} F(I_1, I_{2,i}) p_i$$

$$\begin{array}{ll} \max & \displaystyle\sum_{i=1}^{n} F(I_1, I_{2,i}) p_i \\ \text{s.t.} & P(\boldsymbol{H}(\boldsymbol{\tau}) > k) \leq \epsilon \end{array}$$

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& \boldsymbol{H}(\boldsymbol{\tau}) \ \leq \ u
\end{aligned}
$$

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Recall:

$$
\boldsymbol{H}(\boldsymbol{\tau}) \ = \ \int_0^{\tau} e^{-\nu(\tau - s)} \alpha I^2(s) ds + \boldsymbol{R}(1 - e^{-\nu\tau}) + Ce^{-\nu\tau},
$$

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$$

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So chance constraint is of the form:

$$
\sum_{i=1}^{n} \mathbb{I}\{ v_1 \ l_1^2 \ + \ v_2 \ l_2^2(i) \ > u \ - \ r_i(1 - e^{-\nu\tau}) \ - \ C e^{-\nu\tau} \} p_i \ \leq \ \epsilon.
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$$
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So chance constraint s of the form:

$$
\sum_{i=1}^{n} \mathbb{I}\{\underbrace{v_1 \ I_1^2}_{z_1} \ + \ \underbrace{v_2 \ I_2^2(i)}_{z_2(i)} > \underbrace{u \ - \ r_i(1 - e^{-\nu\tau}) \ - \ Ce^{-\nu\tau}}_{w_i}\} p_i \ \le \ \epsilon.
$$

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{n} \tilde{F}(z_1, z_2(i))\ p_i \\
\text{s.t.} \quad & \sum_{i=1}^{n} \mathbb{I}\{z_1 + z_2(i) > w_i\} p_i \ \leq \ \epsilon \\
& z_1 + z_2(i) \ \leq \ u_i \quad (w_i < u_i) \\
& z_1 \ \leq \ \bar{k} \\
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Lemma: At optimality,

$$z_1 \ + \ z_2(i) \ = \ w_i \text{ or } u_i, \quad \text{all } i$$

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$\longrightarrow$ Use **binary** variable

$$y_i = \begin{cases} 0 & \text{when} \quad z_1 + z_2(i) = w_i \\ 1 & \text{when} \quad z_1 + z_2(i) = u_i \end{cases}$$

Continuous knapsack problem

$$\max \sum_{i=1}^{n} \tilde{F}(z_1, w_i - z_i)p_i(1 - y_i) + \tilde{F}(z_1, u_i - z_i)p_i y_i$$

$$\text{s.t.} \sum_{i=1}^{n} u_i p_i y_i \ \leq \ \epsilon$$

$$0 \ \leq \ z_1 \ \leq \ \bar{k}$$

$$y_i \ = \ 0 \text{ or } 1, \quad \text{all } i.$$

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$$\max_{z_1 \in [0, \bar{k}]} M(z_1)$$

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Practicable!

Identifying Risky Contingencies of Transmission Systems

(Joint with S. Harnett, T. Kim and S. Wright)

- **N - 1** criterion widely used. But is it enough?
- How about **N - K**, for **K** "larger"? Everybody knows that:
  - It is *too* slow. A very difficult combinatorial problem.

N	$K = 2$	$K = 3$	$K = 4$
1000	499500	166167000	41417124750
4000	7998000	10658668000	10650673999000
8000	31996000	85301336000	170538695998000
10000	49995000	166616670000	416416712497500

  - It is too conservative. It is not conservative enough.
    (T. Boston) during Hurricane Sandy, **N - 142** was observed.
  - Perhaps **N - K** does not necessarily capture all interesting events?

Example: August 14 2003

U.S. - Canada report on blackout:
"Because it had been hot for several days in the Cleveland-Akron area, more air conditioners were running to overcome the persistent heat, and consuming relatively high levels of reactive power – further straining the area's limited reactive generation capabilities."

- A **system-wide** condition that impedes the system

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- A **system-wide** condition that impedes the system
- Not a cause, but a contributor
- Look for events that combine both effects?

A continuous interdiction model

- A fictitious adversary is trying to interdict the transmission system.
- This adversary negatively alters the physical parameters of equipment, e.g. transmission lines, so as to impede transmission.
- The adversary has a budget available (both system-wide and per-line).
    - On line *km*, reactance x_{km} increased to $(1 + \lambda_{km})x_{km}$

A blast from the past: Bienstock and Verma 2007

- **DC approximation** to power flows.
- Adversary **increases reactances** of lines.
- **Limit** on total percentage-increase of reactances, and on per-line increase.
- Adversary maximizes the maximum **line overload**:

$$\max_{x,\theta} \quad \max_{km} \left\{ \frac{|\theta_k - \theta_m|}{u_{km} \; x_{km}} \right\} \qquad = \max_{km} \frac{|\text{flow on line } km|}{\text{limit of line } km}$$

$$\text{s.t.} \qquad B_x \theta \; = \; d$$

$$x \text{ within budget}$$

- Continuous, but non-smooth problem.

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- Adversary **increases reactances** of lines.
- **Limit** on total percentage-increase of reactances, and on per-line increase.
- Adversary maximizes the maximum **line overload**:

$$\max_{x,\theta,\alpha} \quad \sum_{km} (\alpha^+_{km} - \alpha^-_{km}) \frac{(\theta_k - \theta_m)}{u_{km} \; \boldsymbol{x_{km}}}$$

$$\text{s.t.} \quad \boldsymbol{B_x} \theta = d$$

$\boldsymbol{x}$ within budget

$$\sum_{km} (\alpha^+_{km} + \alpha^-_{km}) = 1, \quad \alpha^+, \alpha^- \geq 0.$$

- Continuous, smooth, **nonconvex**.

And what happens?

- Efficient computation of gradient and Hessian of objective
- Local optimization algorithm implemented using LOQO
- Algorithm scales well (2007): CPU times of ~ 1 hour for studying systems with thousands of lines.
- Optimal * attack concentrated on a handful of lines
- Plus system-wide attack impacting many lines

single = 20	total = 60	single = 10	total = 30	single = 10	total = 40
Range	**Count**	**Range**	**Count**	**Range**	**Count**
$[1, 1]$	8	$[1, 1]$	1	$[1, 1]$	14
$(1, 2]$	72	$(1, 2]$	405	$(1, 2]$	970
$(2, 3]$	4	$(2, 9]$	0	$(2, 5]$	3
$(5, 6]$	1	$(9, 10]$	**3**	$(5, 6]$	0
$(6, 7]$	1			$(6, 7]$	1
$(7, 8]$	**4**			$(7, 9]$	0
$(8, 20]$	0			$(9, 10]$	**2**

"single" = max multiplicative increase of a line's reactance

"total" = max total multiplicative increase of line reactances

Today: the AC power flows setting

Adversary increases impedances, so as to **maximize:**

- Phase angle differences across ends of a lines
- Voltage deviations (loss)
- Lost load following recourse actions

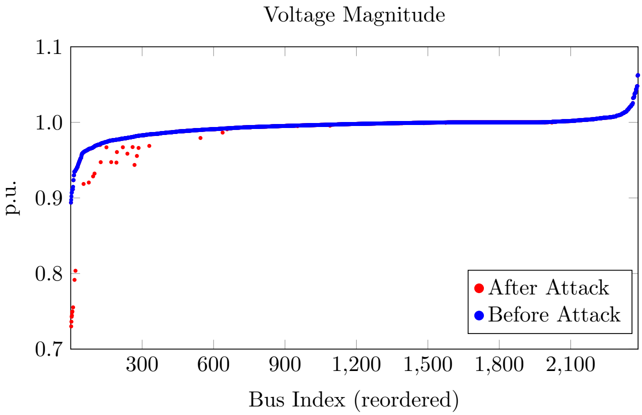
Generically:

$$\begin{aligned} \textbf{max} \quad & \mathcal{F}(\boldsymbol{x}) \\ \text{s.t.} \quad & \boldsymbol{x} \in \mathcal{B} \end{aligned}$$

- $\boldsymbol{x}$ = impedances, $\boldsymbol{\mathcal{B}}$ = budget constraints
- $\mathcal{F}(\boldsymbol{x})$= meausure of phase angle differences, voltage loss, load loss
- Challenge: $\mathcal{F}(\boldsymbol{x})$ is implicitly defined (bilevel optimization problem)

Voltage attack on 2383-bus Polish

"Double the reactance of at most three lines"



→ Primarily 4 lines interdicted

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