Robust Models of Epidemics, and Emergency Resource Allocation

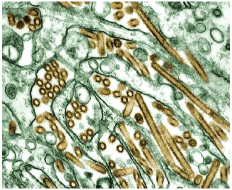
Daniel Bienstock, joint with A. Cecilia Zenteno

Columbia University

USC Epstein, February 2013

Motivation

- Virus mutates continuously $\rightarrow$ epidemic



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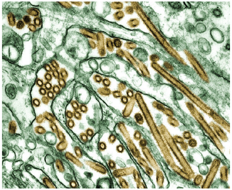

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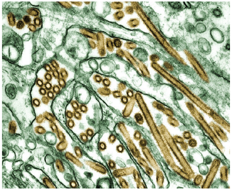

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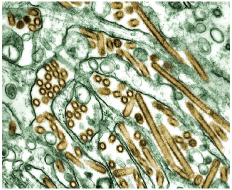

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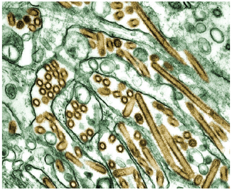

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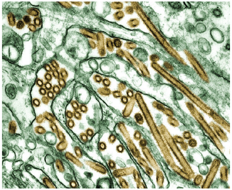

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  - Energy plants
  - Water plants
  - Supply chains
  - Hospitals and clinics

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- **When** and **how many**?

Agenda

1. Disease Modeling

2. Hiring Restrictions & Implementation

3. System Utilization Measure

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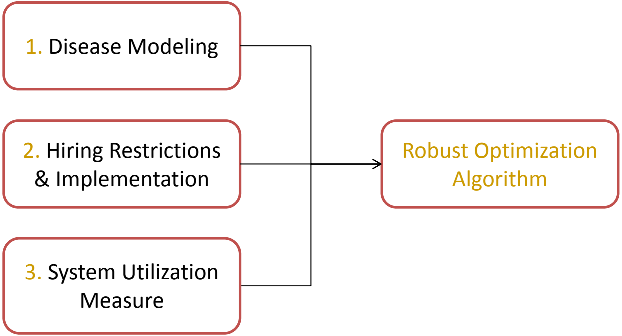
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Robust Optimization Algorithm

1. A model for influenza

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  - Deterministic
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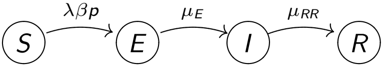

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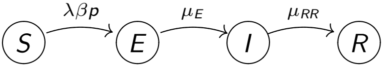

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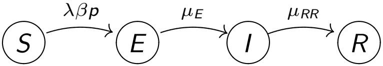

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Keep track of absenteeism $\rightarrow$ **separate** accounting of workers.

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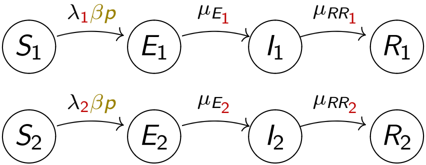

→ General population

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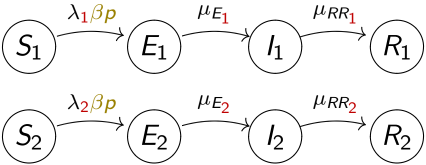

→ General population

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$$\beta = \frac{\lambda_1 I^1 + \lambda_2 I^2}{\lambda_1 N^1 + \lambda_2 N^2}$$

Discrete time SEIR model

Model for subgroup j transition $t \rightarrow t+1$:

$$
\begin{aligned}
S^j_{t+1} &= S^j_t e^{-\lambda_j * \beta_t * p} \\
E^j_{t+1} &= E^j_t e^{-\mu_{E_j}} + S^j_t (1 - e^{-\lambda_j * \beta_t * p}) \\
I^j_{t+1} &= I^j_t \; e^{-\mu_{RR_j}} + E^j_t (1 - e^{-\mu_{E_j}}) \\
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[LJS Allen et al, 1991; Larson, 2007]

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- $\uparrow$ Severity $\Rightarrow$ Average # contacts $\downarrow$

$$\lambda_t^j = \Lambda^j \frac{S_t^j + E_t^j + R_t^j}{N_t^j}$$

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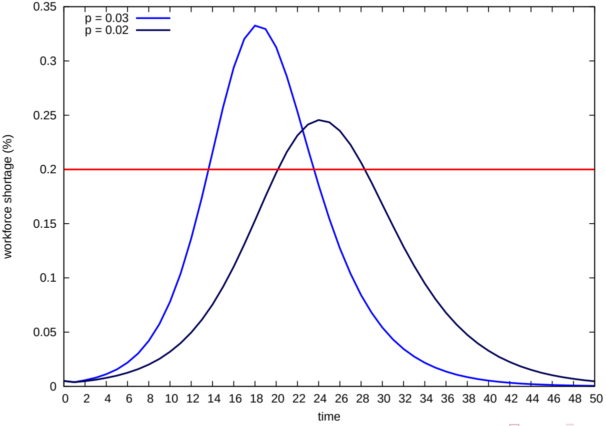
- Incubation and recovery rates (μ_E, μ_{RR}) are "easy"
- Contagion rate $\lambda\beta p$ **?**
- Focus uncertainty on probability of contagion **p**

Planning under uncertainty

Leave SEIR parameters fixed, except probability of contagion, p.



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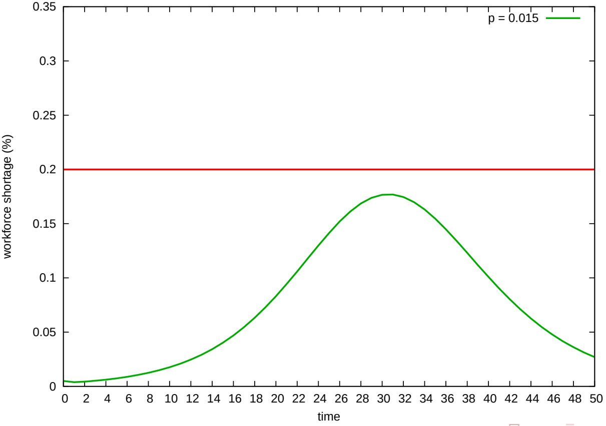

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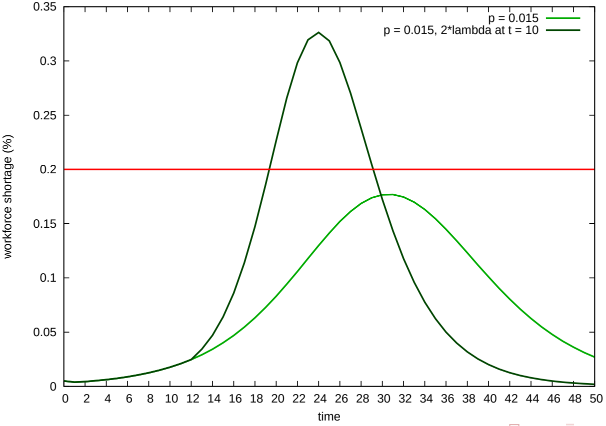

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- When is the surge strategy rolled out?

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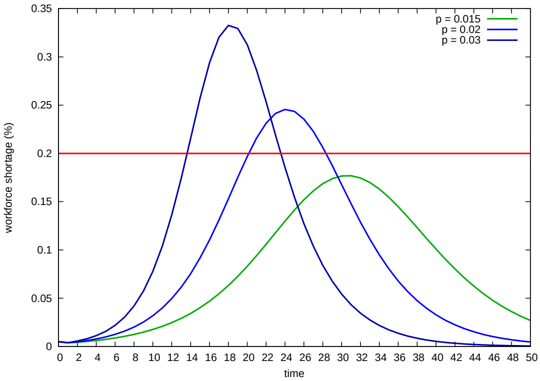
- Assumption: Epidemic is *correctly* declared

A technical detail

- Epidemics with different "p" declared at different times



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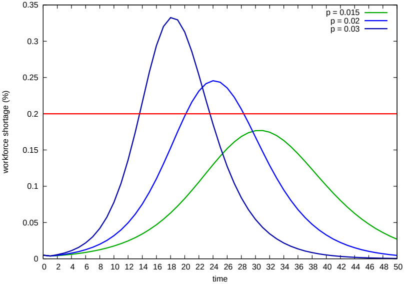

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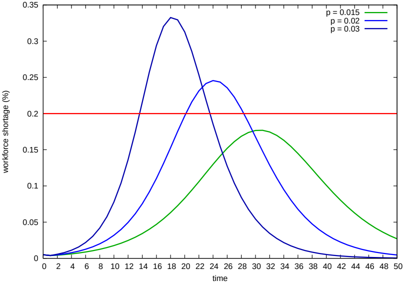
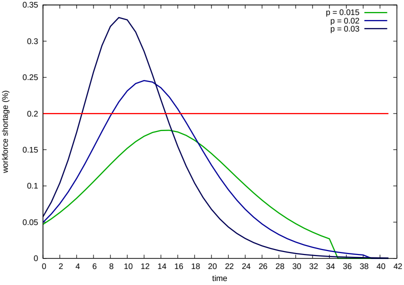
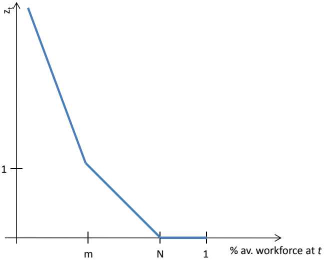




3. Quantifying the impact - Utilization measures

Total "social" cost: sum of per day costs
Two specific settings:

- Min workforce level to operate
  m - threshold



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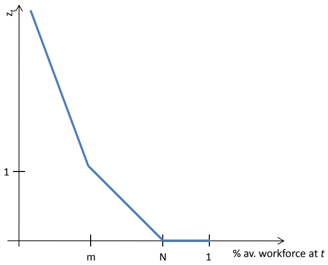

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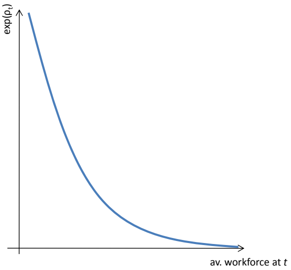

$$\text{Cost} = \sum_t e^{K(\rho_t - 1)^+} - 1$$

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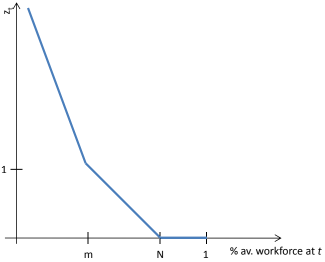

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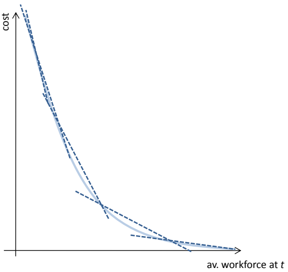

Convex piecewise-linear functions

★ First Optimization Model

Assumption: Size of surge staff corps is small relative to population; so staff deployment does not alter epidemic

Key modeling variables:

$\forall$ time periods $t' > t$, the quantities of surge staff that

- are first deployed at time t, and
- are susceptible, or exposed, or infected at time t'

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Robust Optimization Problem

$$V^* \quad = \quad \min_{h \in H} \max_{p \in P} V(h|p)$$

Objective: Strategy resilient against *all* scenarios

$H \leftarrow$ set of feasible surge strategies
$P \leftarrow$ uncertainty set

Some formulation details

- at time t, variable $\mathbf{a_t}$ = total number of available staff

  = original staff, non-infective at time t (known from SEIR model)

  + surge staff, non-infective at time t

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Discrete time SEIR model applied to surge staff

For each given time t', track of condition of staff deployed at t':

$\mathbf{h_{t'}}$: quantity deployed at t'

$\mathbf{s^s_{t,t'}}$: quantity deployed at t' and susceptible at t,

$\mathbf{e^s_{t,t'}}$: quantity deployed at t' and exposed at t,

$$s^s_{t',t'} = h_{t'}$$

and for all $t' \leq t < t' + K$,

$$s^s_{t+1,t'} = s^s_{t,t'} e^{-\lambda_s * \beta_t * p}$$

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- Likewise, constraints to keep track of (convex) costs are linear

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- and constraint: $\kappa_t \ge s_i \Gamma_t + b_i$, for $1 \le i \le I_t$ ($s_i \ge 0$ for all i)

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$$
\begin{aligned}
V(h|p) := \min \quad & c^T x \\
\text{s.t.} \quad & A_p x = h && \text{(SEIR eqs)} \\
& C_p x \geq d_p && \text{(piecewise-linear approx)} \\
& x \geq 0, \quad x \in H
\end{aligned}
$$

$x \leftarrow$ groups $SE(IR)$ + objective function aux. variables
$H \leftarrow$ set of feasible strategies

Solving the problem

Our problem:

$$V^* = \min_{h \in H} \max_{p \in P} V(h|p)$$

(An **infinite LP**.) How to solve?

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(Today) Approximate model: use finite $\mathbf{Q} \subset P$

$$\begin{aligned} V^* \approx \min \quad & c^T x \\ \text{s.t.} \quad & A_p x = h, \quad \forall p \in \mathbf{Q} \\ & C_p x \geq d_p, \quad \forall p \in \mathbf{Q} \\ & h \in H \\ & x \geq 0 \end{aligned}$$

- Example: $P = [0.01, 0.013]$,
  $\mathbf{Q}$ = subset of P at integral multiples of 0.0001

Theoretical justification

Suppose we know that $p \geq p_0 > 0$.

For each **epsilon** $\mathbf{> 0}$ small enough there is a $\boldsymbol{\delta = O(\epsilon)}$ s.t.:

If $|\mathbf{p} - \mathbf{p}'| < \delta$ then $\mathbf{V(h|p') \leq (1 + \epsilon)V(h|p)}$ for any h

Benders' Decomposition

- Generalized Benders' Decomposition

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Optimization problem

$$
\begin{aligned}
V^* &= \min_{h \in H} \max_{p \in P} V(h|p) \\
&= \min_{z, h \in H} z \\
& \quad \text{s.t.} \quad z \geq V(h|p) \quad \forall\, p \in P
\end{aligned}
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Benders' Decomposition

- Generalized Benders' Decomposition
- Idea: replace $V(h|p)$ by cuts obtained from the dual

Optimization problem

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Approximation

$$\begin{aligned} V^* &\approx \min_{h \in H} \max_{p \in P} V(h|p) \\ &= \min_{z, h \in H} z \\ &\quad \text{s.t.} \quad z \geq \alpha_p^T h + \pi_p^T d_p, \ (\text{dual } \forall p \in \mathbf{Q}) \end{aligned}$$

$\mathbf{Q}$ is a relatively small subset of P - so separation problem is fast

Basic Algorithm

Iterate:

1. Solve Master Problem; let $\hat{\mathbf{h}}$ be the computed surge and $\hat{\mathbf{z}}$ be the estimate of its worst-case cost.
2. **Sample:** compute the worst-case data realization for $\hat{\mathbf{h}}$.
3. If the cost of $\hat{\mathbf{h}}$ under this resolution is at most $\hat{\mathbf{z}}$, **STOP**.
4. Otherwise, add to the master a duality cut violated by $\hat{\mathbf{h}}, \hat{\mathbf{z}}$, and **goto 1**.

Algorithmic enhancements

- "Powers of two" approximation to finite grid

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- “Pre-Benders”' cuts

Alternative Optimization Problem 1

- Intervals or tranches $I_1, \ldots I_1$ of $[0, 1]$; a time period J
- At time $= 1$, it is known that $p \in I_1$.
- At time J there is a switch. For $t \geq J$, $p \in I_h$ (known at $t = J$)

Decision maker:

- Rolls out a surge at $t = 1$ that covers periods $1 \leq t < J$,
- At $t = 1$ announces m surge plans to cover periods $J \leq t \leq T$
- At time $= J$, switches to one of the announced plans

Alternative Optimization Problem 2

- There is a known interval I such that $p_t \in I$ for all t J
- At time $= t$, $p_t = \mu + \delta_t$, and is observed
- Here $\mu =$ midpoint of I, and $\delta_t =$ zero mean stochastic, small

Decision maker:

- At time 1, announces "expected" surge quantities h_t for all t, and a multiplier $\lambda \geq 0$
- At time t, corrects h_t by $\lambda \frac{\sum_{j<t}(p_t - \mu)}{t}$
- (up to a maximum allowable)

More general uncertainty sets

- Flexible algorithm - more general uncertainty sets

More general uncertainty sets

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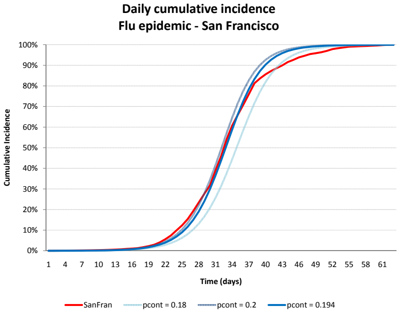

More general uncertainty sets

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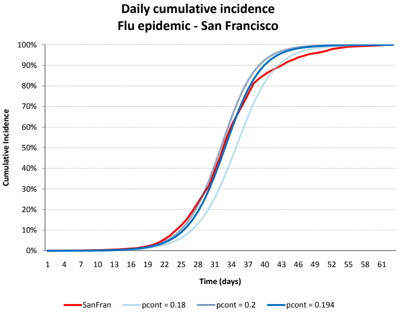
- Sudden weather changes [Lowen et al, 2007]
- Public Health measures could change course of epidemic
- Analyze the impact of multiple values of p during 1 epidemic



Numerical example

- **Demography**

	General Population	High Risk Population
Size	900,000	20,000
Initial infected	5	0
Contact rates (per day)	30	35
Incubation rate (μE)	10/19	10/19
Removal rate (μR)	10/41	10/41
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- **Social Contact Model**
  - Non-homogeneous contact
  - Contact rate decreases 30% when epidemic is declared

Comparisons:

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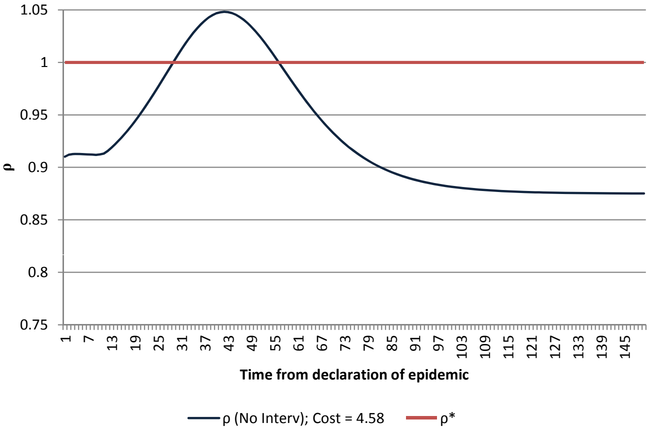
- **The robust strategy**

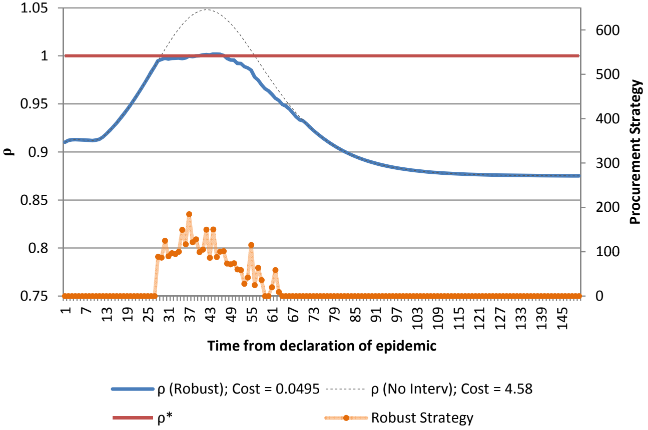
No surge staff deployment

Most costly data realization: $(p_1, p_2, d) = (0.0109, 0.0135, 140)$
Cost: 4.58



Same data realization, but using Robust Strategy

$(p_1, p_2, d) = (0.0109, 0.0135, 140)$
Cost: 0.0495



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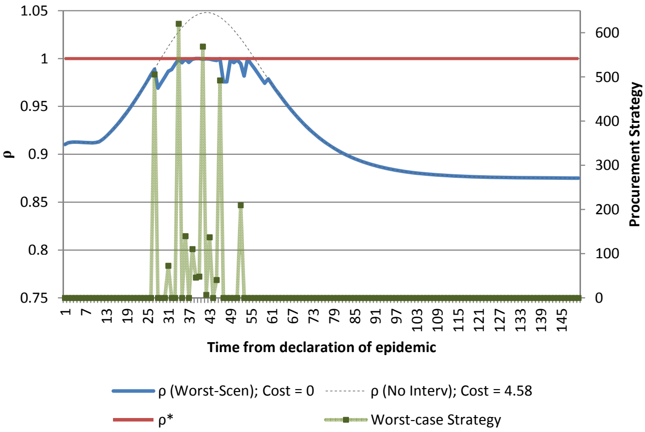
$(p_1, p_2, d) = (0.0109, 0.0135, 140)$, Cost: 0



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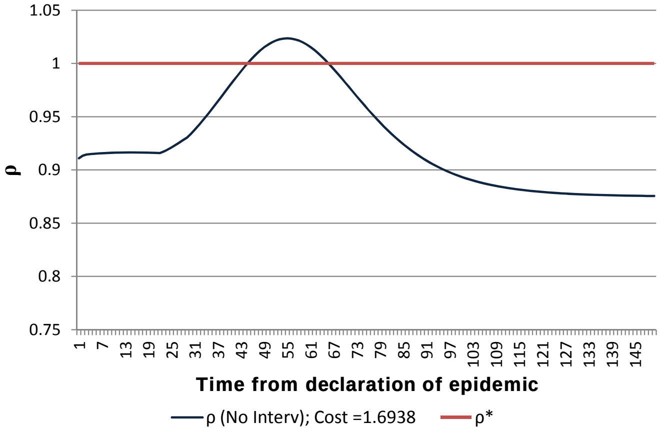
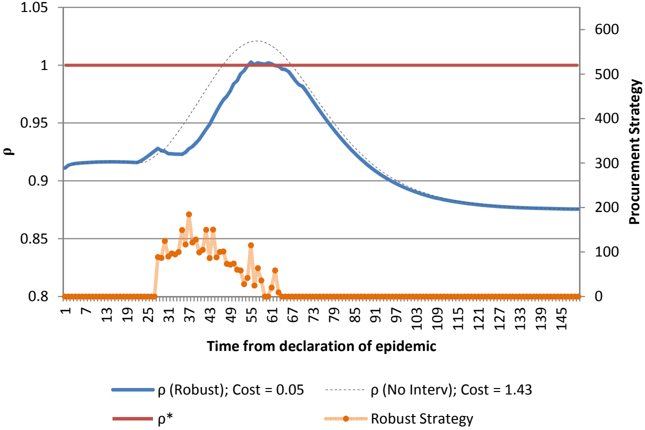
Using this data realization, "do nothing" cost: 1.69



Same data: $(p_1, p_2, d) = (0.01168, 0.0135, 140)$

... against the Robust Strategy, Cost: 0.05



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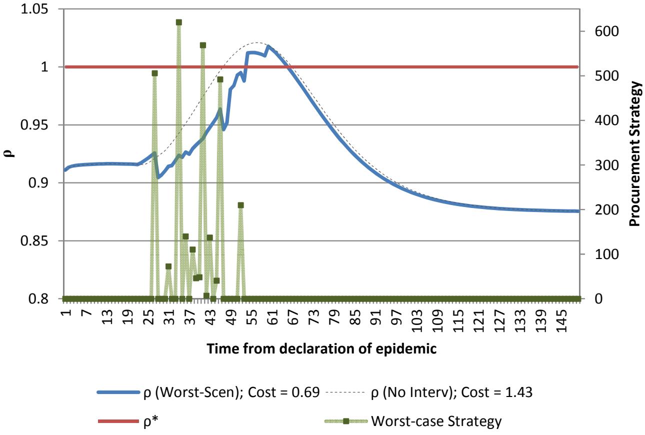
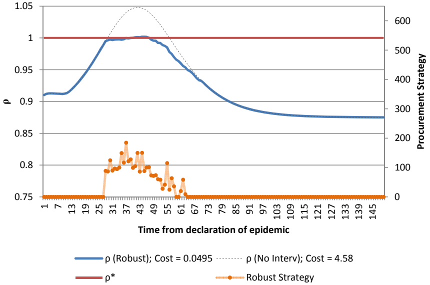
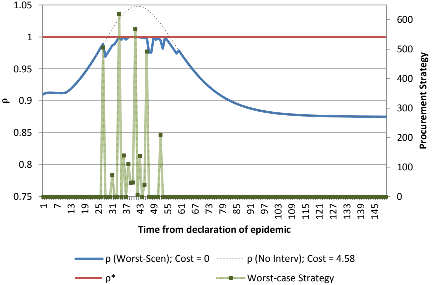
Cost: 0.69



Example - Comparing strategies

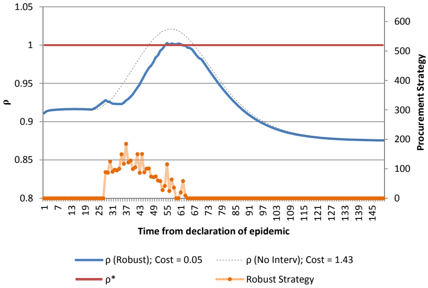
Scenario I

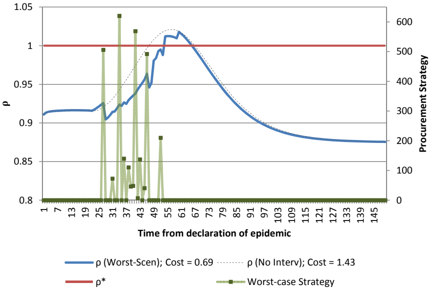




Example - Comparing strategies

Scenario II





Example - Comparing strategies

		No Intervention	Robust Strategy	Worst-Case Strategy
No intervention:	Cost	4.581	0.050	0.000
worst tuple	Maximum ρ	1.002	1.048	1.000
(0.01092, 0.0135, 140)	Critical days ($\rho > 1$)	28	8	0
Robust Strategy:	Cost	1.694	0.052	0.686
worst tuple	Maximum ρ	1.024	1.003	1.017
(0.01168, 0.0135, 140)	Critical days ($\rho > 1$)	21	7	12
Worst-case Strategy:	Cost	1.430	0.050	0.710
worst tuple	Maximum ρ	1.021	1.002	1.018
(0.01172, 0.0135, 140)	Critical days ($\rho > 1$)	20	8	13

Example - Comparing strategies

Take-away: Planning against most expensive scenario is **not** enough!

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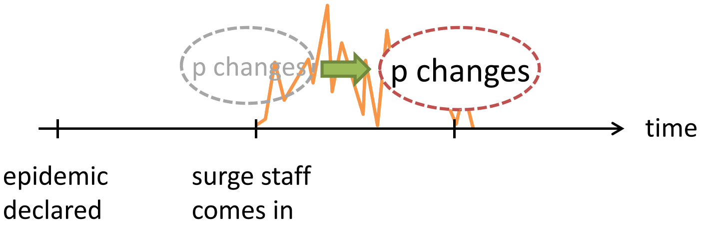

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  - Worst case:$(p_1, p_2, d) = (0.01168, 0.0135, 140)$

	Worst case given Robust Policy								
Scenarios	**Original**	**Hypothetical 1**				**Hypothetical 2**			
p_1	0.01168	0.01168				0.01168			
p_2	0.0135	0.014				0.015			
day epidemic is declared	113	113				113			
day deployment starts	140	140				140			
day p changes	140	150	155	160	165-	150	155	160	165
cost Robust Policy	**0.0508**	**0.3268**	**0.07295**	**0**	**0**	**2.1737**	**1.4068**	**0.6282**	**0.0294**
cost No Intervention	4.58	1.609	0.762	0.087	0	4.133	2.669	1.243	0.146

Example - Numeric Performance

- 350 days
- max-cost tuple : grid search
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- $\sim$ 175 iterations
- 5% duality gap
- $\sim$ 15min CPU time

Example - Numeric Performance

- 350 days
- max-cost tuple : grid search
- VBA (UI) + C (SEIR) + AMPL (Gurobi solver)

Enhancement:

- 8 pre-Benders' iterations + 1 Benders' cut
- Duality gap 0.0052%
- $<$ 1 min CPU time