Estimating Production Functions in Differentiated-Product Industries with Quantity Information and External Instruments*

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July 2023

SUPPLEMENTARY UNPUBLISHED MATERIALS

These supplementary materials provide some additional details and results to accompany the main text and online appendix.

S1 Additional Theoretical Details

S1.1 Regularity Conditions for Consumer's Minimization Problem

The second-order conditions for minimization are satisfied without further assumptions if and only if $\sigma_i^y \in (0,1) \cup (1,\infty)$. To see this, note that a necessary and sufficient condition for $\mathcal{L}^y$ to be convex is that all principal minors of order r of the Hessian matrix of $\mathcal{L}^y$ are non-negative, for $r = 1, \cdots, J$, where J is the number of products in Ω_{it}^y. (See e.g. Theorem 2.3.3 in Sydsaeter et al. (2005).) Chen (2012) shows (Theorem 5.1) that the determinant of the Hessian matrix of a CES function is always zero. This implies that the principal minor of order J of the Hessian for $\mathcal{L}^y$ is zero. Furthermore, every principal minor of degree $1 \leq r < J-1$ corresponds to the determinant of the Hessian matrix of a CES aggregator with $J-r$ varieties and hence is also zero. (Note that the theorem still applies when replacing $p(x)$ by $p(x)+c$, where the additional constant arises due to the excluded varieties that now enter as constant terms within the sum.) We are left only with the principal minors of order one, which correspond to the elements of the diagonal of the Hessian matrix of $\mathcal{L}^y$, which are the second derivatives:

$$\frac{\partial^2 \mathcal{L}^y}{\partial^2 Y_{ijt}} = \frac{-\lambda^y}{\sigma_i^y}\left[\widetilde{Y}_{it}^{\frac{1}{\sigma_i^y}} \varphi_{ijt}^{\frac{\sigma_i^y-1}{\sigma_i^y}} Y_{ijt}^{\frac{-1-\sigma_i^y}{\sigma_i^y}}\right]\left[\left(\frac{\varphi_{ijt}Y_{ijt}}{\widetilde{Y}_{it}}\right)^{\frac{\sigma_i^y-1}{\sigma_i^y}} - 1\right]$$

Given that the second term in brackets is always negative and $\lambda^y > 0$ (see the discussion following (A2)), all principal minors of order one are greater than zero if and only if $\sigma_i^y \in (0,1) \cup (1,\infty)$.

Hence $\mathcal{L}^y$ is convex and the second-order conditions for minimization are satisfied for a critical point satisfying the Lagrangian first order conditions if and only if $\sigma_i^y \in (0,1) \cup (1,\infty)$. For the limiting case $\sigma_i^y \to 1$, $\widetilde{Y}_{it}$ tends to a Cobb-Douglas function with exponents φ_{ijt}. In this case, $\widetilde{Y}_{it}$ is concave if and only if $\sum_{j\in\Omega_{it}^y} \varphi_{ijt} \leq 1$.

In the terminology of Sun and Yang (2006), goods in the bundle Ω_{it}^y are *gross substitutes* when $\sigma_i^y > 1$ and *gross complements* when $0 < \sigma_i^y < 1$. That is, the demand for product j increases in response to an increase in the price of any other variety k, holding everything else constant, if and only if $\sigma_i^y > 1$; it decreases if and only if $0 < \sigma_i^y < 1$. Although our methodology can accommodate either case, we believe that in the sectors we consider in our empirical application it is reasonable to assume $\sigma_i^y > 1$, i.e. that goods are gross substitutes.

S1.2 Construction of CES Price/Quantity Indexes, Input Side

The derivations for the price and quantity indexes for the input side are analogous to the ones from the output side in Appendix A.1. We include them here for the sake of completeness.

S1.2.1 Firm's Minimization Problem

The Lagrangian corresponding to the first stage of the firm's problem is given by:

$$\mathcal{L}^m = \sum_{h\in\Omega_{it}^m} M_{iht} W_{iht}^m - \lambda^m \left(\left[\sum_{h\in\Omega_{it}^m} (\alpha_{iht} M_{iht})^{\frac{\sigma_i^m - 1}{\sigma_i^m}} \right]^{\frac{\sigma_i^m}{\sigma_i^m - 1}} - \widetilde{M}_{it} \right)$$

where λ^m is the Lagrange multiplier. The first order condition with respect to input h, $\frac{\partial \mathcal{L}^m}{\partial M_{iht}} = 0$, implies:

$$\frac{W_{iht}^m}{\alpha_{iht}} = \lambda^m (\alpha_{iht} M_{iht})^{-\frac{1}{\sigma_i^m}} \widetilde{M}_{it}^{\frac{1}{\sigma_i^m}} \tag{A1}$$

Raising both sides of this equation to the power $1-\sigma_i^m$, summing over the $h \in \Omega_{it}^m$, using the definition of $\widetilde{W}_{it}^m$ in (8) in the main text, and rearranging, we have:

$$\lambda^m = \widetilde{W}_{it}^m \tag{A2}$$

Analogously to the output case, it can be shown that (without further assumptions) any point satisfying the first order conditions constitutes an global minimum if and only if $\sigma_i^m \in (0,1) \cup (1,\infty)$. Therefore, our method allows material inputs to be *gross complements*, $0 < \sigma_i^m < 1$, or to be *gross substitutes*, $\sigma_i^m > 1$. Nevertheless, given the type of sectors we consider in our empirical analysis, we assume material inputs to be *gross substitutes*: $\sigma_i^m > 1$.

Plugging (A2) into (A1) and rearranging:

$$M_{iht} = \widetilde{M}_{it} \left(\frac{\widetilde{W}^m_{it}}{W^m_{iht}} \right)^{\sigma^m_i} {\alpha_{iht}}^{\sigma^m_i - 1} \tag{A3}$$

As for revenues,

$$E_{it} = \sum_{h \in \Omega^m_{it}} E_{iht} = \sum_{h \in \Omega^m_{it}} W^m_{iht} M_{iht} = \widetilde{W}^m_{it} \widetilde{M}_{it} \left(\widetilde{W}^m_{it} \right)^{\sigma^m_i - 1} \underbrace{\sum_{h \in \Omega^m_{it}} \left(\frac{W^m_{iht}}{\alpha_{iht}} \right)^{1-\sigma^m_i}}_{= (\widetilde{W}^m_{it})^{1-\sigma^m_i}} = \widetilde{W}^m_{it} \widetilde{M}_{it} \tag{A4}$$

S1.2.2 Price Index Log Change

Using (A3),

$$S^m_{iht} = \frac{W^m_{iht} M_{iht}}{E_{it}} = \frac{W^m_{iht} M_{iht}}{\widetilde{W}^m_{it} \widetilde{M}_{it}} = \left(\frac{\left(\frac{W^m_{iht}}{\alpha_{iht}} \right)}{\widetilde{W}^m_{it}} \right)^{1-\sigma^m_i} \tag{A5}$$

Hence from the definitions in (10) in the main text:

$$\chi^m_{it,t-1} = \frac{\sum_{h \in \Omega^{m*}_{it,t-1}} S^m_{iht}}{\sum_{h \in \Omega^m_{it}} S^m_{iht}} = \frac{\sum_{h \in \Omega^{m*}_{it,t-1}} \left(\frac{W^m_{iht}}{\alpha_{iht}} \right)^{1-\sigma^m_i}}{\sum_{h \in \Omega^m_{it}} \left(\frac{W^m_{iht}}{\alpha_{iht}} \right)^{1-\sigma^m_i}}, \quad \chi^m_{it-1,t} = \frac{\sum_{h \in \Omega^{m*}_{it,t-1}} S^m_{iht-1}}{\sum_{h \in \Omega^m_{it-1}} S^m_{iht-1}} = \frac{\sum_{h \in \Omega^{m*}_{it,t-1}} \left(\frac{W^m_{iht-1}}{\alpha_{iht-1}} \right)^{1-\sigma^m_i}}{\sum_{h \in \Omega^m_{it-1}} \left(\frac{W^m_{iht-1}}{\alpha_{iht-1}} \right)^{1-\sigma^m_i}}$$

Then using the definition of $\widetilde{W}^m_{it}$, (8),

$$\begin{aligned} \frac{\widetilde{W}^m_{it}}{\widetilde{W}^m_{it-1}} &= \frac{\left[\sum_{h \in \Omega^m_{it}} \left(\frac{W^m_{iht}}{\alpha_{iht}} \right)^{1-\sigma^m_i} \right]^{\frac{1}{1-\sigma^m_i}}}{\left[\sum_{h \in \Omega^m_{it-1}} \left(\frac{W^m_{iht-1}}{\alpha_{iht-1}} \right)^{1-\sigma^m_i} \right]^{\frac{1}{1-\sigma^m_i}}} \\ &= \left(\frac{\chi^m_{it-1,t}}{\chi^m_{it,t-1}} \right)^{\frac{1}{1-\sigma^m_i}} \frac{\left(\sum_{h \in \Omega^{m*}_{it,t-1}} \left(\frac{W^m_{iht}}{\alpha_{iht}} \right)^{1-\sigma^m_i} \right)^{\frac{1}{1-\sigma^m_i}}}{\left(\sum_{h \in \Omega^{m*}_{it,t-1}} \left(\frac{W^m_{iht-1}}{\alpha_{iht-1}} \right)^{1-\sigma^m_i} \right)^{\frac{1}{1-\sigma^m_i}}} = \left(\frac{\chi^m_{it-1,t}}{\chi^m_{it,t-1}} \right)^{\frac{1}{1-\sigma^m_i}} \frac{\widetilde{W}^*_{it}}{\widetilde{W}^*_{it-1}} \end{aligned} \tag{A6}$$

where $\widetilde{W}^*_{it}$ is the common-goods price index defined in the main text (footnote 16).

To derive an expression for $\frac{\widetilde{W}^*_{it}}{\widetilde{W}^*_{it-1}}$, note that (A5) implies a similar expression for the expenditure share of common goods:

$$S^{m*}_{iht} = \frac{W^m_{iht} M_{iht}}{\widetilde{W}^*_{it} \widetilde{M}^*_{it}} = \frac{W^m_{iht} M_{iht}}{\widetilde{W}^m_{it} \widetilde{M}_{it}} \cdot \frac{\widetilde{W}^m_{it} \widetilde{M}_{it}}{\widetilde{W}^*_{it} \widetilde{M}^*_{it}} = \left(\frac{\left(\frac{W^m_{iht}}{\alpha_{iht}} \right)}{\widetilde{W}^m_{it}} \right)^{1-\sigma^m_i} \frac{\widetilde{W}^m_{it} \widetilde{M}_{it}}{\widetilde{W}^*_{it} \widetilde{M}^*_{it}}$$

Using (A2),

$$\frac{\widetilde{W}_{it}^{m}\widetilde{M}_{it}}{\widetilde{W}_{it}^{*}\widetilde{M}_{it}^{*}} = \frac{\widetilde{W}_{it}^{m}\widetilde{M}_{it}}{\sum_{h\in\Omega_{it,t-1}^{m*}} W_{iht}^{m} M_{iht}} = \left(\frac{\widetilde{W}_{it}^{m}}{\widetilde{W}_{it}^{*}}\right)^{1-\sigma_i^m}$$

Hence:

$$S_{iht}^{m*} = \left(\frac{\left(\frac{W_{iht}^{m}}{\alpha_{iht}}\right)}{\widetilde{W}_{it}^{*}}\right)^{1-\sigma_i^m} \tag{A7}$$

Divide (A7) by the same equation for the previous year, take logs, and re-arrange:

$$\frac{\ln\left(\frac{\widetilde{W}_{it}^{m*}}{\widetilde{W}_{it-1}^{m*}}\right) - \ln\left(\frac{\frac{W_{iht}^{m}}{\alpha_{iht}}}{\frac{W_{iht-1}^{m}}{\alpha_{iht-1}}}\right)}{\ln\left(\frac{S_{iht}^{m*}}{S_{iht-1}^{m*}}\right)} = \frac{1}{\sigma_i^m - 1}$$

Multiply both sides by $S_{iht}^{m*} - S_{iht-1}^{m*}$ and sum over the common goods:

$$\sum_{h\in\Omega_{it,t-1}^{m*}} \left(S_{iht}^{m*} - S_{iht-1}^{m*}\right) \frac{\ln\left(\frac{\widetilde{W}_{it}^{m*}}{\widetilde{W}_{it-1}^{m*}}\right) - \ln\left(\frac{\frac{W_{iht}^{m}}{\alpha_{iht}}}{\frac{W_{iht-1}^{m}}{\alpha_{iht-1}}}\right)}{\ln\left(\frac{S_{iht}^{m*}}{S_{iht-1}^{m*}}\right)} = \left(\frac{1}{\sigma_i^m - 1}\right) \sum_{h\in\Omega_{it,t-1}^{m*}} \left(S_{iht}^{m*} - S_{iht-1}^{m*}\right) = 0$$

where the second equality follows because $\sum_{h\in\Omega_{it,t-1}^{m*}} S_{iht}^{m*} = \sum_{h\in\Omega_{it,t-1}^{m*}} S_{iht-1}^{m*} = 1$. This implies:

$$\sum_{h\in\Omega_{it,t-1}^{m*}} \left(\frac{S_{iht}^{m*} - S_{iht-1}^{m*}}{\ln S_{iht}^{m*} - \ln S_{iht-1}^{m*}}\right) \ln\left(\frac{\widetilde{W}_{it}^{m*}}{\widetilde{W}_{it-1}^{m*}}\right) = \sum_{h\in\Omega_{it,t-1}^{m*}} \left(\frac{S_{iht}^{m*} - S_{iht-1}^{m*}}{\ln S_{iht}^{m*} - \ln S_{iht-1}^{m*}}\right) \ln\left(\frac{\frac{W_{iht}^{m}}{\alpha_{iht}}}{\frac{W_{iht-1}^{m}}{\alpha_{iht-1}}}\right)$$

Since $\ln\left(\frac{\widetilde{W}_{it}^{m*}}{\widetilde{W}_{it-1}^{m*}}\right)$ does not vary with h, this can be re-written as:

$$\ln\left(\frac{\widetilde{W}_{it}^{m*}}{\widetilde{W}_{it-1}^{m*}}\right) = \sum_{h\in\Omega_{it,t-1}^{m*}} \psi_{iht}^{m} \ln\left(\frac{W_{iht}^{m}}{W_{iht-1}^{m}}\right) - \sum_{h\in\Omega_{it,t-1}^{m*}} \psi_{iht}^{m} \ln\left(\frac{\alpha_{iht}}{\alpha_{iht-1}}\right) \tag{A8}$$

where ψ_{iht}^{m} is as defined in (10) above. Combining (A6) and (A8), we have:

$$\ln\left(\frac{\widetilde{W}_{it}^{m}}{\widetilde{W}_{it-1}^{m}}\right) = \sum_{h\in\Omega_{it,t-1}^{m*}} \psi_{iht}^{m} \ln\left(\frac{W_{iht}^{m}}{W_{iht-1}^{m}}\right) - \sum_{h\in\Omega_{it,t-1}^{m*}} \psi_{iht}^{m} \ln\left(\frac{\alpha_{iht}}{\alpha_{iht-1}}\right) - \frac{1}{\sigma_i^m - 1} \ln\left(\frac{\chi_{it-1,t}^{m}}{\chi_{it,t-1}^{m}}\right) \tag{A9}$$

which is (9) in the main text.

S1.2.3 Quantity Index Log Change

We start by noting that (A3) implies

$$W^m_{iht} = \widetilde{W}^m_{it} \left(\frac{\widetilde{M}_{it}}{M_{iht}} \right)^{\frac{1}{\sigma^m_i}} \alpha_{iht}^{\frac{\sigma^m_i - 1}{\sigma^m_i}}$$

Hence:

$$\ln\left(\frac{W^m_{iht}}{W^m_{iht-1}} \right) = \ln\left(\frac{\widetilde{W}^m_{it}}{\widetilde{W}^m_{it-1}} \right) + \frac{1}{\sigma^y_i} \ln\left(\frac{\widetilde{M}_{it}}{\widetilde{M}_{it-1}} \right) - \frac{1}{\sigma^y_i} \ln\left(\frac{W^m_{iht}}{W^m_{iht-1}} \right) + \frac{\sigma^y_i}{\sigma^y_i - 1} \ln\left(\frac{\alpha_{iht}}{\alpha_{iht-1}} \right)$$

Plugging this into (A9), re-arranging, and using the fact that $\sum_{h \in \Omega^{m*}_{it,t-1}} \psi^m_{iht} = 1$ gives the log change in $\widetilde{M}_{it}$:

$$\ln\left(\frac{\widetilde{M}_{it}}{\widetilde{M}_{it-1}} \right) = \sum_{h \in \Omega^{m*}_{it,t-1}} \psi^m_{iht} \ln\left(\frac{M_{iht}}{M_{iht-1}} \right) + \sum_{h \in \Omega^{m*}_{it,t-1}} \psi^m_{iht} \ln \frac{\alpha_{iht}}{\alpha_{iht-1}} + \frac{\sigma^m_i}{\sigma^m_i - 1} \ln\left(\frac{\chi^m_{it-1,t}}{\chi^m_{it,t-1}} \right)$$

which is (11) in the main text. The fact that $\widetilde{W}^{m*}_{it} \widetilde{M}^*_{it} = E^*_{it}$ can be shown as in (A4), using just common goods.

S1.3 Construction of Alternative Quantity Indexes

On the input side, following standard formulations (see e.g. Dodge (2008)), we define the Laspeyres input quantity index for $t-1$ and t as:

$$\widetilde{M}^{Lasp}_{it,t-1} = \frac{\sum_{h \in \Omega^{m*}_{it,t-1}} W^m_{iht-1} M_{iht}}{\sum_{h \in \Omega^{m*}_{it,t-1}} W^m_{iht-1} M_{iht-1}} \tag{A10}$$

and the Paasche input quantity index as:

$$\widetilde{M}^{Paas}_{it,t-1} = \frac{\sum_{h \in \Omega^{m*}_{it,t-1}} W^m_{iht} M_{iht}}{\sum_{h \in \Omega^{m*}_{it,t-1}} W^m_{iht} M_{iht-1}}, \tag{A11}$$

The Tornqvist quantity index is defined as:

$$\widetilde{M}^{Torn}_{it,t-1} = \prod_{h \in \Omega^{m*}_{it,t-1}} \left(\frac{M_{iht}}{M_{iht-1}} \right)^{\frac{1}{2}(S^{m*}_{iht} + S^{m*}_{iht-1})} \tag{A12}$$

where S^{m*}_{iht} and S^{m*}_{iht-1} are as defined in footnote 22 of the main text.

Note that the Laspeyres quantity index is related to the Paasche price index, and vice-versa. If we define the Laspeyres price index as:

$$\widetilde{W}_{it,t-1}^{m,Lasp} = \frac{\sum_{h\in\Omega_{it,t-1}^{m*}} W_{iht}^{m} M_{iht-1}}{\sum_{h\in\Omega_{it,t-1}^{m*}} W_{iht-1}^{m} M_{iht-1}}. \tag{A13}$$

and the Paasche price index as:

$$\widetilde{W}_{it,t-1}^{m,Paas} = \frac{\sum_{h\in\Omega_{it,t-1}^{m*}} W_{iht}^{m} M_{iht}}{\sum_{h\in\Omega_{it,t-1}^{m*}} W_{iht-1}^{m} M_{iht}} \tag{A14}$$

then the common-input expenditure ratio between t and $t-1$ is the product of the Laspeyres price index and the Paasche quantity index and also the product of the Laspeyres quantity index and the Paasche price index:

$$\frac{E_{it}^{*}}{E_{it-1}^{*}} = \frac{\sum_{h\in\Omega_{it,t-1}^{m*}} W_{iht}^{m} M_{iht}}{\sum_{h\in\Omega_{it,t-1}^{m*}} W_{iht-1}^{m} M_{iht-1}} = \widetilde{M}_{it,t-1}^{Lasp} \times \widetilde{W}_{it,t-1}^{m,Paas} = \widetilde{M}_{it,t-1}^{Paas} \times \widetilde{W}_{it,t-1}^{m,Lasp}$$

The definition of the alternative output quantity indexes is analogous to the definition of the input quantity indexes (A10), (A11) and (A12).

S1.4 Variance Correction for β_k in Levels-Equation Estimation

Our sequential production function estimation belongs to a general class of two-step M-Estimators discussed for instance in Wooldridge (2002, Section 12.4) and previously in Newey (1984). The results there can be applied directly in our setting. Under our assumptions, our first-step estimates $\widehat{\beta}_m$ and $\widehat{\beta}_l$ and their standard errors are consistently estimated. The levels-equation estimate of β_k, call it $\widehat{\widehat{\beta}}_k$, can be calculated by solving:

$$\sum_{t=1}^{T}\sum_{i=1}^{N} \triangle k_{it-1}\left(\left(\widetilde{y}_{it}^{SV} - \widehat{\beta}_m \widetilde{m}_{it}^{SV} - \widehat{\beta}_l l_{it}\right) - \widehat{\widehat{\beta}}_k k_{it}\right) = 0. \tag{A15}$$

As noted in the main text, the consistency of $\widehat{\beta}_m$ and $\widehat{\beta}_l$ is sufficient to guarantee the consistency of $\widehat{\widehat{\beta}}_k$. In the special case when $\mathbb{E}(\triangle k_{it-1}\widetilde{m}_{it}^{SV}) = 0$ and $\mathbb{E}(\triangle k_{it-1}\ell_{it}) = 0$, the first step estimation can be ignored when computing the asymptotic variance of $\widehat{\widehat{\beta}}_k$.[1] If those conditions do not hold, then we need to use a corrected expression for the asymptotic variance of $\widehat{\beta}_k$, which takes into account that $\widehat{\beta}_m$ and $\widehat{\beta}_l$ were estimated in a previous step. A consistent estimate of the corrected asymptotic

[1] The score function corresponding to the levels-equation IV estimation is $s(a_{it}, \beta_k; \beta_m, \beta_l) = \triangle k_{it-1}(\widetilde{y}_{it}^{SV} - \beta_m \widetilde{m}_{it}^{SV} - \beta_l l_{it} - \beta_k k_{it})$, where $a_{it} = (\widetilde{y}_{it}^{SV}, \widetilde{m}_{it}^{SV}, l_{it}, k_{it}, \triangle k_{it-1})$. If $\mathbb{E}(\triangle k_{it-1}\widetilde{m}_{it}^{SV}) = 0$ and $\mathbb{E}(\triangle k_{it-1}\ell_{it}) = 0$ then the gradient of the score function with respect to β_m and β_ℓ is zero and equation 12.37 of Wooldridge (2002) holds, implying that we can ignore the first step in calculating the asymptotic variance of $\widehat{\widehat{\beta}}_k$.

variance for $\widehat{\widehat{\widehat{\beta}}}_k$, call it $\widehat{V}_{\beta_k}$, is given by Newey and McFadden (1994):[2]

$$\widehat{V}_{\beta_k} = \frac{(T \times N)^{-1} \left(\sum_{t=1}^{T} \sum_{i=1}^{N} \left(\widehat{s}_{it} + \widehat{F} \widehat{\psi}_{it} \right)^2 \right)}{\widehat{G}^2} \tag{A16}$$

where

$$\begin{aligned}
\widehat{G} &= -\frac{1}{NT} \sum_{t=1}^{T} \sum_{i=1}^{N} \triangle k_{it-1} k_{it} \\
\widehat{F} &= -\frac{1}{NT} \sum_{t=1}^{T} \sum_{i=1}^{N} \triangle k_{it-1} \left[\widetilde{m}_{it}^{SV}, l_{it}, 0 \right] \\
\widehat{s}_{it} &= \triangle k_{it-1} \left(\widetilde{y}_{it}^{SV} - \widehat{\beta}_m \widetilde{m}_{it}^{SV} - \widehat{\beta}_l l_{it} - \widehat{\widehat{\widehat{\beta}}}_k k_{it} \right) \\
\widehat{\psi}_{it} &= -\left(\widehat{H}' \widehat{W} \widehat{H} \right)^{-1} \widehat{H}' \widehat{W} \widehat{m}_{it}
\end{aligned}$$

and the terms in $\widehat{\psi}_{it}$ are defined as:

$$\begin{aligned}
\widehat{H} &= \frac{1}{NT} \sum_{t=1}^{T} \sum_{i=1}^{N} \left[\triangle \widehat{\widetilde{w}}_{it}^{imp}, \triangle z_{it}, k_{it-2}, \widetilde{m}_{it-2}^{SV}, l_{it-2} \right] \left[\triangle \widetilde{m}_{it}^{SV}, \triangle l_{it}, \triangle k_{it} \right]' \\
\widehat{W} &= \frac{1}{NT} \sum_{t=1}^{T} \sum_{i=1}^{N} \left[\triangle \widehat{\widetilde{w}}_{it}^{imp}, \triangle z_{it}, k_{it-2}, \widetilde{m}_{it-2}^{SV}, l_{it-2} \right] \left[\triangle \widehat{\widetilde{g}}_{it}, \triangle z_{it}, k_{it-2}, \widetilde{m}_{it-2}^{SV}, l_{it-2} \right]' \\
\widehat{m}_{it} &= \left[\triangle \widehat{\widetilde{g}}_{it}, \triangle z_{it}, k_{it-2}, \widetilde{m}_{it-2}^{SV}, l_{it-2} \right]' \left(\triangle \widetilde{y}_{it}^{SV} - \widehat{\beta}_m \triangle \widetilde{m}_{it}^{SV} - \widehat{\beta}_l \triangle l_{it} - \widehat{\beta}_k \triangle k_{it} \right)
\end{aligned}$$

We report the corresponding corrected standard errors when we report $\widehat{\widehat{\widehat{\beta}}}_k$.

[2]See also Proposition 2 of Kripfganz and Schwarz (2019).

S2 Additional Empirical Details

S2.1 Real Exchange Rate Fluctuations

Figure S1 depicts the movements in real exchange rates (where increases reflect real appreciations in the trading partner) for the 12 countries from which rubber and plastics producers purchased the most imports during the period of our analysis. We see that several of the most important import origins had significant RER fluctuations. Venezuela and Mexico, both major oil producers, had large real appreciations in 1995-2000 and large real depreciations subsequently. Indonesia suffered a major crisis accompanied by sharp real devaluation in 1997 (as did Argentina (not pictured) in 2001). Even the US and Eurozone countries, which were less volatile overall, experienced non-trivial variation in the RER relative to Colombia.

S2.2 Real Minimum Wage, 1994-2009

Figure S2 shows the steady increase of the real minimum wage from 1996 to 2009.

S2.3 Comparison to System GMM

For purposes of comparison to the System GMM results using our quantity indexes in Table 5, Table S3 below presents the results from applying standard System GMM, using deflated sales and expenditures for output and inputs as in typical applications. Table S4 presents the corresponding weak-instrument diagnostics. The message is broadly similar to Table 5, except for the capital coefficient, which seems implausibly small, pointing to a potential advantage of using the quantity indexes in System GMM, even when external instruments are not available.

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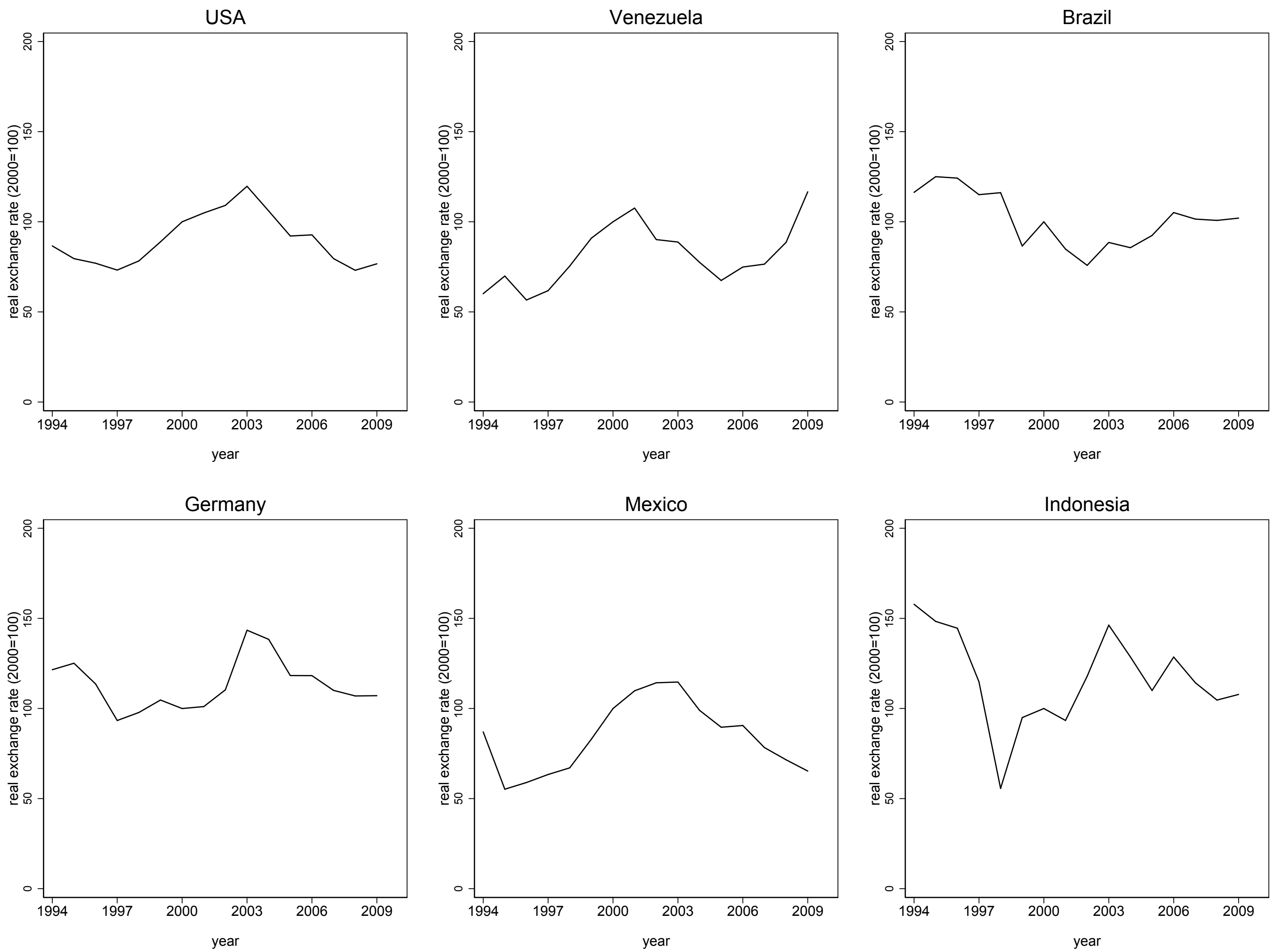
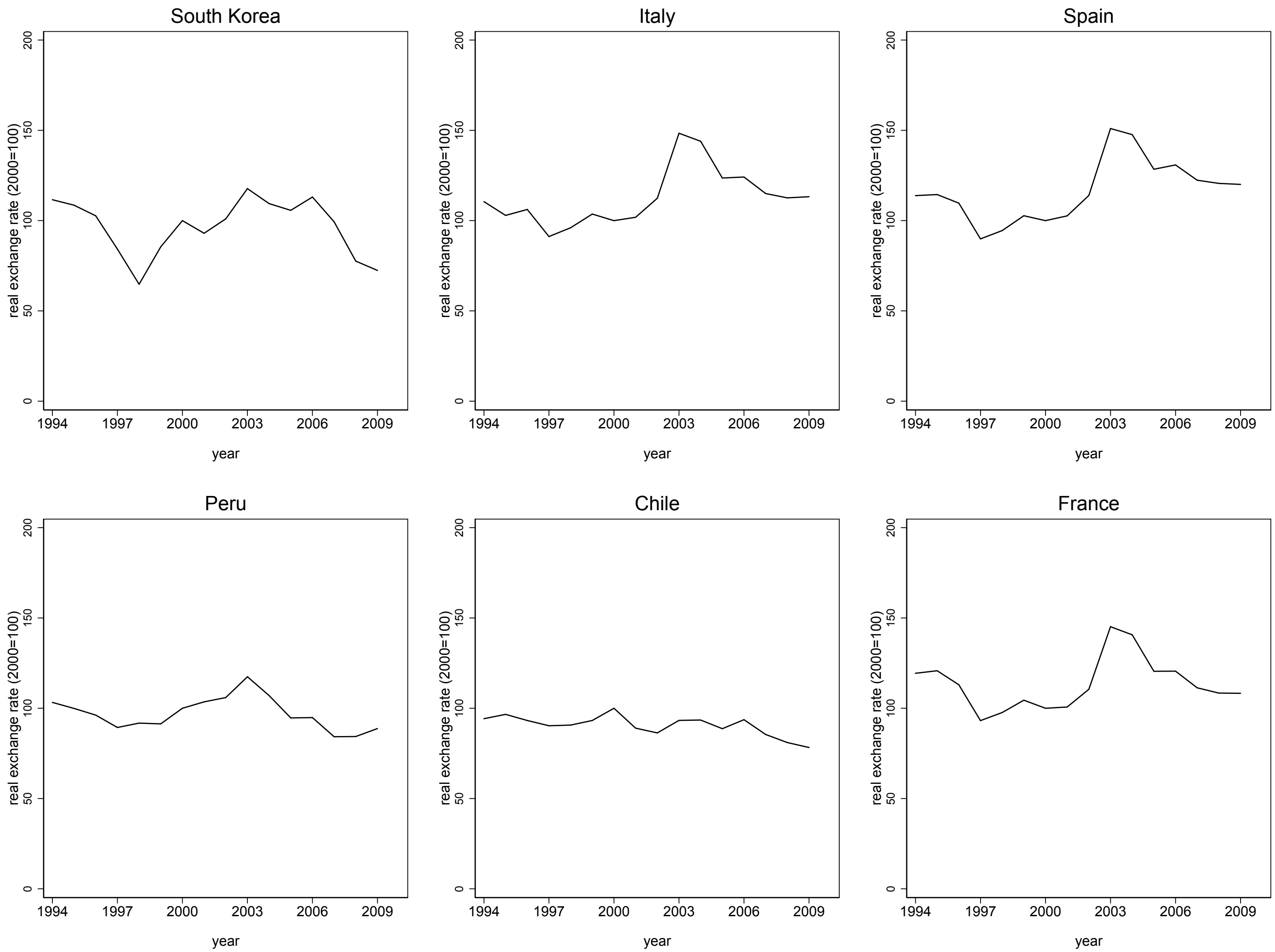
Figure S1. Real Exchange Rate Variation, 1994-2009



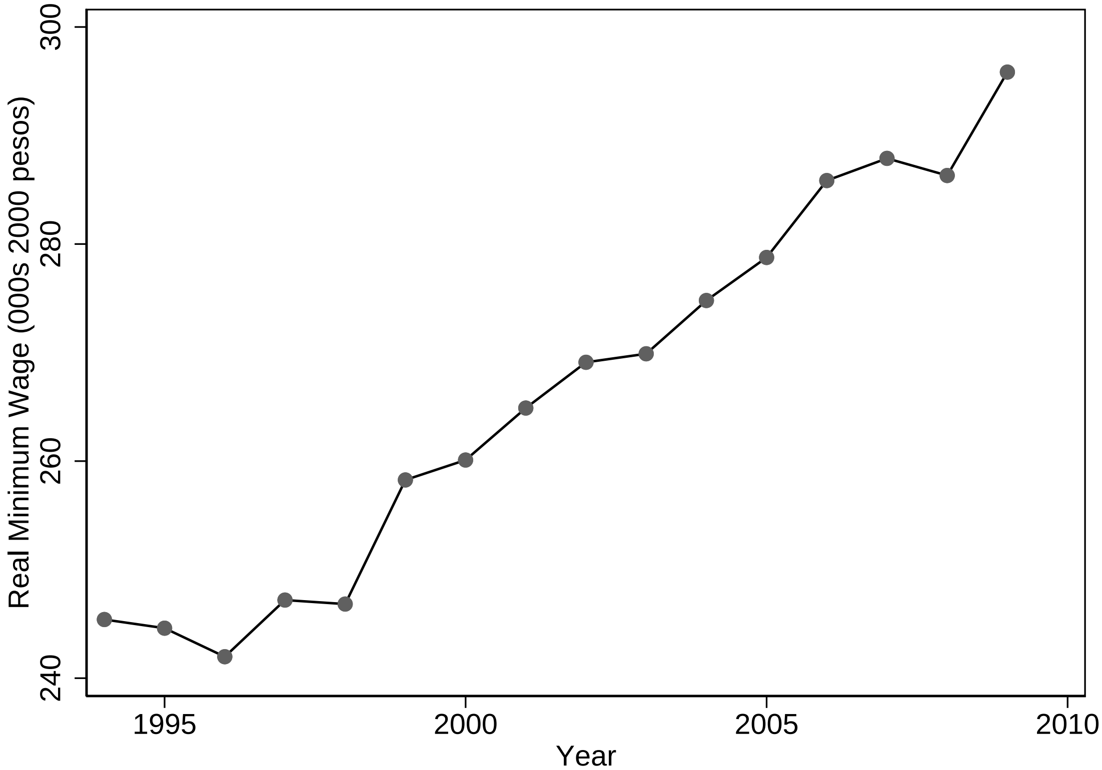
Notes: Figure plots real exchange rate (RER), normalized to 100 in 1994, calculated as in equation (21) in text, for top six import origins for rubber and plastics sectors. An RER increase reflects a real appreciation in the trading partner.

Figure S1. Real Exchange Rate Variation, 1994-2009 (cont.)



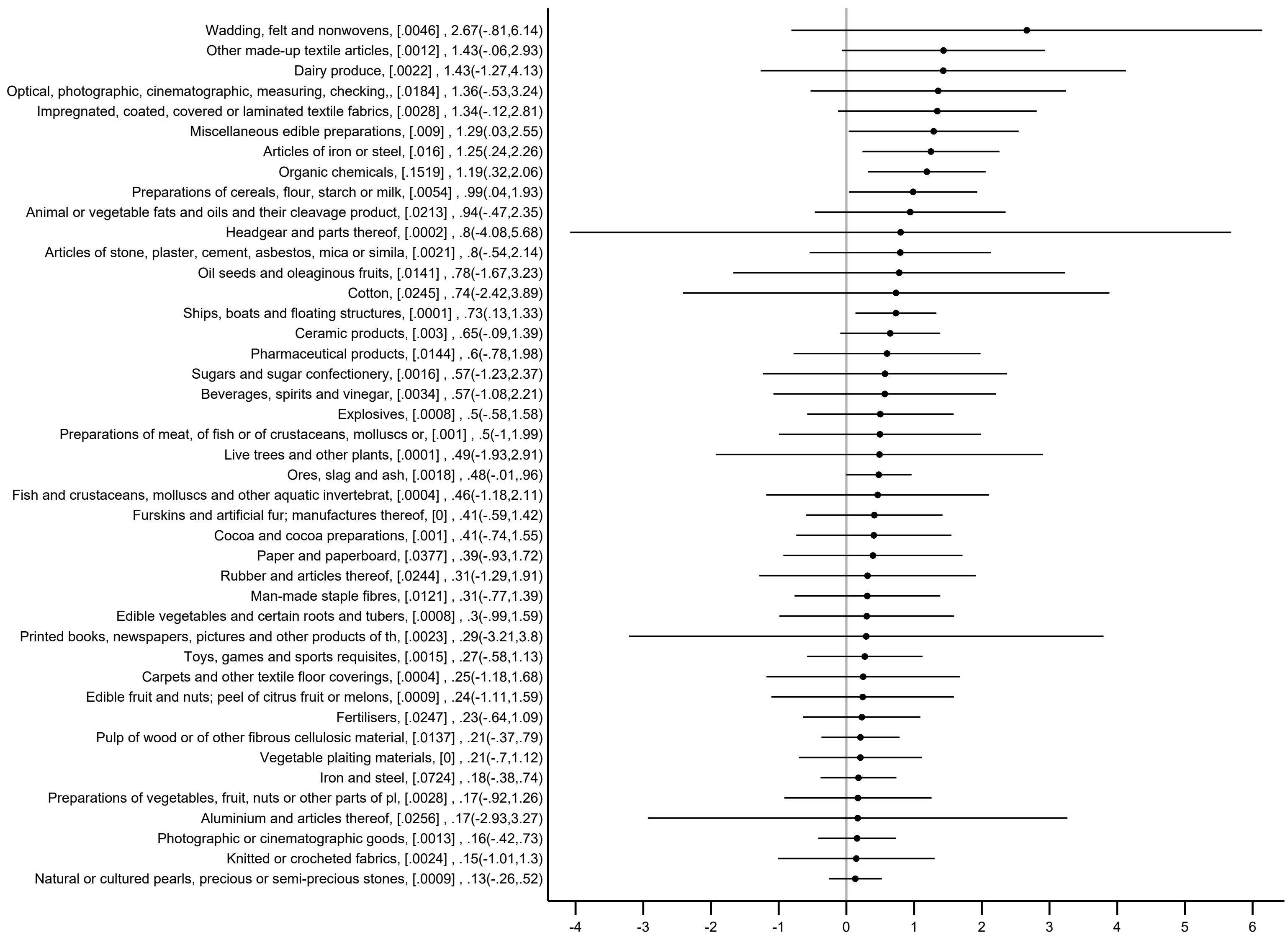
Notes: Figure plots real exchange rate (RER), normalized to 100 in 1994, calculated as in equation (21) in text, for import origins ranked 7-12 for rubber and plastics sectors. (See Fig. S1 for ranks 1-6.) An RER increase reflects a real appreciation in the trading partner.

Figure S2. Real Minimum Monthly Wage, 1994-2009



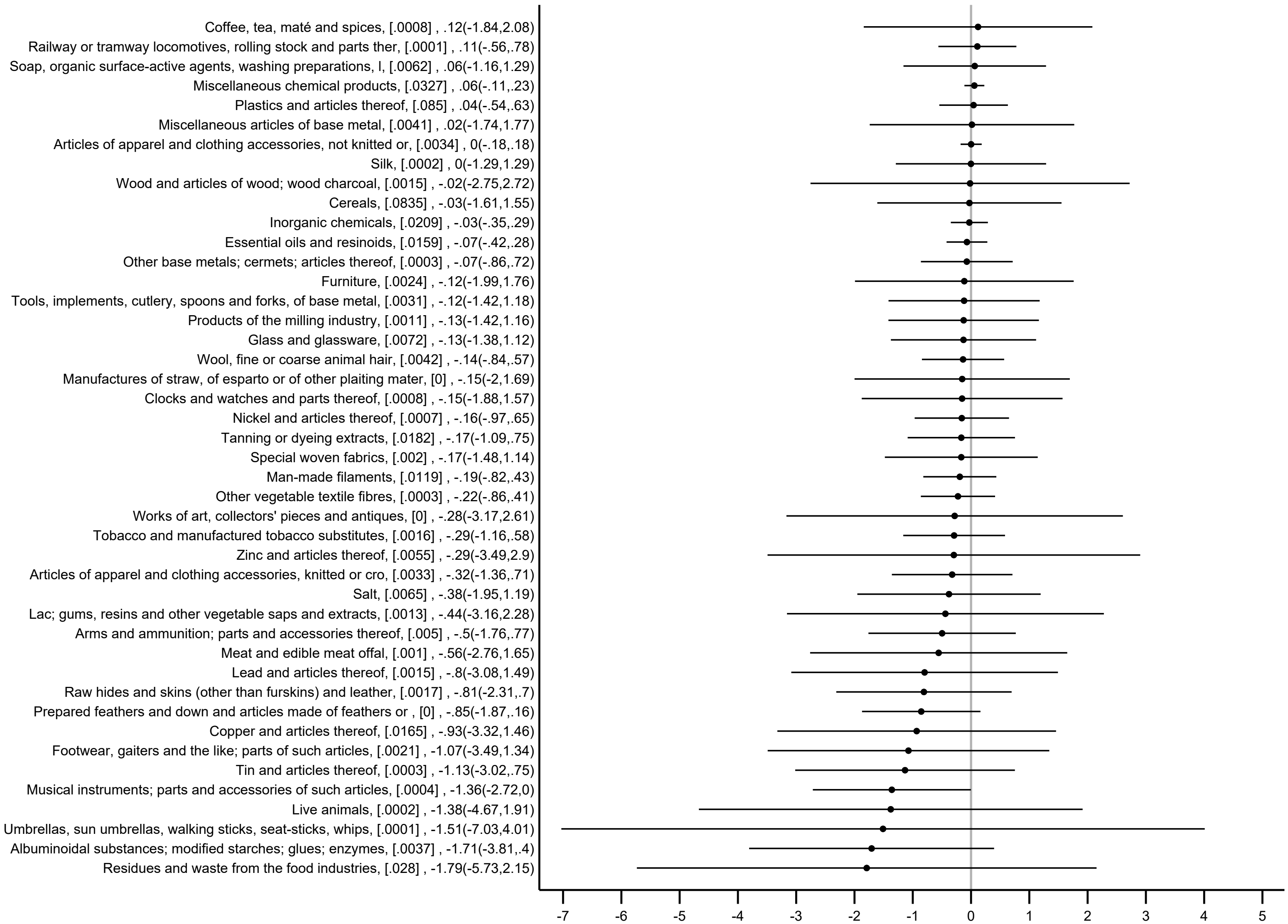
Notes: Figure plots Colombian national real monthly minimum wage, in thousands of 2000 pesos, for 1994-2009. Average 2000 exchange rate is approximately 2,000 pesos/USD.

Figure S3. Coefficients from Import-Price Regressions



Notes: Average sector-specific coefficients from estimating equation (23) in main text. We generate 362 sets of leave-one-out coefficient estimates, then average estimates and standard errors across firms and years. All Harmonized System 2-digit categories except except petroleum products, machinery and equipment (HS2 categories 27, 84 and 85) included; see Section 2.5.1 for details. Import share calculated as imports in HS2 category over total imports for 1994-2009 period. After sector names at left, we list share of total imports (square brackets), the regression coefficient, and 95% confidence interval (parentheses).

Figure S3. Coefficients from Import-Price Regressions (cont.)



Notes: See notes on previous page.

Table S1. Primary Outputs and Inputs, Glass Products Producers

CPC code	Share of total revenues or expenditures	Export/Import share	CPC description
A. Outputs			
3719102	0.21	0.16	Glass bottles for soft drinks
3719103	0.18	0.19	Glass bottles of a capacity not exceeding 1 liter
3711502	0.18	0.53	Safety glass
3711201	0.12	0.17	Unworked flat glass
3719104	0.11	0.18	Glass bottles of a capacity exceeding 1 liter
3711503	0.06	0.28	Safety glass for motor cars, windshields, similar
3719101	0.03	0.22	Small glass jars for perfumery, pharmacy, laboratory
3712204	0.01	0.49	Glass wool sheet
3719309	0.01	0.58	Glass vases
2799704	0.01	0.01	Asphalt fabrics
4299942	0.01	0.11	Wire rods and rings, for brassieres
3719302	0.01	0.24	Glasswares of a kind used for table and kitchen
3712203	0.01	0.27	Fiberglass ducts
3719503	0.01	0.04	Glass ampoules
3712101	0.01	0.46	Fiberglass
3711601	0.01	0.06	Unframed mirror
3712907	0.01	0.38	Fiberglass bathtubs
3712908	0.01	0.09	Fiberglass tanks
3711501	0.00	0.00	Tempered glass
3719903	0.00	0.10	Glass screens
B. Inputs			
3711201	0.30	0.79	Unworked flat glass
3424501	0.22	0.44	Sodium carbonate
3711103	0.10	0.07	Waste and scrap of glass
3633019	0.07	0.93	Plastic fabric
3633007	0.05	0.98	Polyvinyl film
1531201	0.05	0.00	Siliceous sands and gravels
1639902	0.03	0.00	Feldspar
3219702	0.03	0.00	Printed labels
3474002	0.02	0.37	Polyester resins
1512004	0.02	0.00	Crushed or ground limestone
3215308	0.02	0.00	Partitions and dividers of cardboard for boxes
3215302	0.01	0.01	Corrugated cardboard boxes
4151203	0.01	0.36	Angles, shapes and sections of copper
3511104	0.01	0.37	Anticorrosive bases and paints
3712101	0.01	0.45	Fiberglass
3170101	0.01	0.00	Wooden packaging box
4299942	0.01	0.72	Wire rods and rings, for brassieres
3170105	0.01	0.00	Pallets
3424202	0.01	0.07	Sodium sulfate
3641002	0.01	0.00	Unprinted plastic film in tubular form

Notes: Table similar to Tables A1-A2 for producers of glass products (ISIC rev. 2 category 362). See notes for those tables.

Table S2. Summary Statistics, Glass Products

A. Period: 1996-2009	
Number of observations	410
Number of firms	34
Number of workers per firm	122.97
Share of firms that are single-product	0.15
Production value (billions 2000 pesos) per firm	16.41
Earnings per year per firm, permanent workers (millions 2000 pesos)	7.06
B. Period: 2000-2009	
Input variables	
No. inputs per firm	9.43
Share of firms that import	0.65
No. inputs per firm, conditional on importing	9.50
Share of expenditure on imported inputs	0.30
No. imported HS8 categories, conditional on importing	24.74
Output variables	
No. outputs per firm	2.89
Share of firms that export	0.53
No. outputs per firm, conditional on exporting	3.44
Fraction of revenues from exported outputs	0.11
No. exported HS8 categories, conditional on exporting	5.31

Notes: Sample is producers of glass products (ISIC rev. 2 category 362). Exports and imports available in EAM data only in 2000-2009. Average 2000 exchange rate is approximately 2,000 pesos/USD.

Table S3. System GMM, Using Sales and Expenditures

	log output index ($\triangle\widetilde{y}_{it}^{SV}$)		
	(1)	(2)	(3)
log sales$_{it-1}$	0.626***	0.549***	0.515***
	(0.080)	(0.080)	(0.060)
log expenditures$_{it}$	0.548***	0.573***	0.490***
	(0.088)	(0.073)	(0.057)
log expenditures$_{it-1}$	-0.211**	-0.202***	-0.127**
	(0.086)	(0.067)	(0.049)
log labor$_{it}$ (ℓ_{it})	0.347**	0.339**	0.353***
	(0.140)	(0.148)	(0.075)
log labor$_{it-1}$ (ℓ_{it-1})	-0.326**	-0.269*	-0.236***
	(0.145)	(0.141)	(0.077)
log capital$_{it}$ (k_{it})	0.042	0.061	0.014
	(0.071)	(0.074)	(0.063)
log capital$_{it-1}$ (k_{it-1})	-0.025	-0.030	0.022
	(0.061)	(0.063)	(0.055)
Observations	4,247	4,247	4,247
Lag limit	3	4	All
Hansen test	120.700	171.000	347.600
Hansen p-value	0.141	0.279	1.000

Notes: Table presents estimates of standard System GMM model (Blundell and Bond, 2000), using sales and expenditures for output and inputs, and using the "two-step" procedure described in Roodman (2009), with initial weighting matrix defined in Doornik et al. (2012) and finite-sample correction from Windmeijer (2005). The Stata command is xtabond2 (Roodman, 2009), with options h(2), twostep, and robust. The difference equation includes lags to $t-3$ in Column 1, lags to $t-4$ in Column 2, and all available lags in Column 3. The numbers of instruments are as indicated in Appendix Table S4. The Hansen test of overidentifying restrictions is appropriate in the non-homoskedastic case, but should be interpreted with caution, as it is weakened by the presence of many instruments. See Section 5 for further details. Robust standard errors in parentheses. *10% level, **5% level, ***1% level.

Table S4. Weak IV Diagnostics for System GMM, Using Revenues and Expenditures

	Differences			Levels	
	Dep. var.: $\triangle$log sales$_{it}$				Dep. var.: log sales$_{it}$
	(1)	(2)	(3)		(4)
$\triangle$log sales$_{it-1}$	0.277***	0.255***	0.183***	log sales$_{it-1}$	0.680***
	(0.091)	(0.078)	(0.060)		(0.108)
$\triangle$log expenditure$_{it}$	0.387***	0.401***	0.397***	log expenditure$_{it}$	0.556***
	(0.092)	(0.081)	(0.053)		(0.172)
$\triangle$log expenditure$_{it-1}$	-0.115*	-0.096*	-0.073	log expenditure$_{it-1}$	-0.274***
	(0.065)	(0.058)	(0.045)		(0.103)
$\triangle$ log labor ($\triangle\ell_{it}$)	0.339**	0.453***	0.341***	log labor(ℓ_{it})	-0.432*
	(0.144)	(0.123)	(0.070)		(0.239)
$\triangle$ log labor ($\triangle\ell_{it-1}$)	0.069	0.039	0.046	log labor(ℓ_{it-1})	0.481**
	(0.116)	(0.102)	(0.062)		(0.209)
$\triangle$ log capital ($\triangle k_{it}$)	-0.003	-0.017	-0.004	log capital(k_{it})	0.016
	(0.081)	(0.072)	(0.053)		(0.127)
$\triangle$ log capital ($\triangle k_{it-1}$)	-0.146*	-0.103	-0.084*	log capital(k_{it-1})	0.010
	(0.084)	(0.070)	(0.046)		(0.125)
N	4,247	4,247	4,247		4,247
R-squared	0.203	0.217	0.264		0.961
Lag Limit	3	4	All		NA
Number of excluded instruments	108	156	420		56
SW F-stat log sales$_{it}$	2.070	2.060	2.233		3.970
SW F-stat log expenditure$_{it}$	2.034	2.161	2.473		1.845
SW F-stat log expenditure$_{it-1}$	2.334	2.499	3.869		2.094
SW F-stat log labor (ℓ_{it})	1.643	1.504	1.985		1.238
SW F-stat log labor (ℓ_{it-1})	2.334	2.499	3.869		1.392
SW F-stat log capital (k_{it})	2.120	2.227	1.970		1.339
SW F-stat log capital (k_{it-1})	2.208	2.022	1.855		1.400
KP LM test (underidentification)	124.000	160.100	444.000		51.840
KP LM p-value	0.069	0.271	0.149		0.402
KP Wald test (weak instruments)	1.462	1.447	1.835		0.968

Notes: Table reports IV estimates corresponding to differences (Columns 1-3) and levels (Column 4) equations of System GMM using sales and expenditures (Table S3), with weak-instrument diagnostic statistics. Robust standard errors in parentheses. *10% level, **5% level, ***1% level.

Table S5. Monte Carlo Simulation: TSIV, Differences (Step 1), First Stage

	DGP1		DGP2		DGP3		DGP4	
	$\triangle\widetilde{m}_{it}^{SV}$	$\triangle k_{it}$	$\triangle\widetilde{m}_{it}^{SV}$	$\triangle k_{it}$	$\triangle\widetilde{m}_{it}^{SV}$	$\triangle k_{it}$	$\triangle\widetilde{m}_{it}^{SV}$	$\triangle k_{it}$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Mean coefficient $\widetilde{m}_{it-2}^{SV}$	-0.107	0.020	-0.122	0.013	-0.091	0.028	-0.086	0.019
	(0.004)	(0.005)	(0.002)	(0.004)	(0.002)	(0.004)	(0.001)	(0.003)
Mean std. error $\widetilde{m}_{it-2}^{SV}$	0.039	0.054	0.029	0.060	0.026	0.054	0.025	0.051
	(0.004)	(0.005)	(0.002)	(0.004)	(0.002)	(0.004)	(0.001)	(0.003)
Mean coefficient k_{it-2}	0.065	-0.031	0.038	-0.047	0.023	-0.054	0.021	-0.050
	(0.003)	(0.004)	(0.001)	(0.002)	(0.001)	(0.002)	(0.001)	(0.002)
Mean std. error k_{it-2}	0.028	0.039	0.014	0.029	0.012	0.026	0.012	0.024
	(0.003)	(0.004)	(0.001)	(0.002)	(0.001)	(0.002)	(0.001)	(0.002)
Mean coefficient $\triangle\widehat{\widetilde{w}}_{it}^{imp}$	-0.537	0.016	-0.425	0.008	-0.434	-0.008	-0.429	0.007
	(0.050)	(0.068)	(0.027)	(0.054)	(0.028)	(0.058)	(0.026)	(0.052)
Mean std. error $\triangle\widehat{\widetilde{w}}_{it}^{imp}$	0.046	0.063	0.026	0.053	0.026	0.053	0.026	0.053
	(0.005)	(0.007)	(0.002)	(0.003)	(0.001)	(0.003)	(0.001)	(0.003)
Mean R-squared	0.020	0.010	0.041	0.022	0.042	0.023	0.040	0.022
	(0.003)	(0.001)	(0.003)	(0.002)	(0.003)	(0.002)	(0.003)	(0.002)
Mean F - statistic	90.819	34.510	196.166	85.574	195.149	86.546	192.040	89.260
	(16.139)	(5.882)	(29.523)	(19.588)	(29.442)	(23.672)	(28.385)	(19.951)
Mean F - SW	2038.723	97.334	3052.729	263.593	2904.559	266.689	2147.679	266.264
	(583.469)	(19.661)	(340.280)	(53.826)	(316.686)	(59.610)	(225.227)	(49.561)
Mean KP LM statistic	97.293		205.699		206.345		203.568	
	(10.172)		(12.257)		(12.750)		(15.392)	
Mean KP Wald F-statistic	34.215		84.909		86.001		88.401	
	(5.859)		(19.741)		(23.760)		(19.838)	

Notes: Table presents the first stage of step 1 of TSIV procedure for the four DGPs in our Monte Carlo simulation. See Section 5.2 and Appendix D for details. Table reports means of statistics across 100 simulated samples for each DGP. In parentheses are standard deviations of statistics across the 100 samples. N=15,000 for each sample. Dependent variables are indicated at the tops of columns. SW refers to Sanderson and Windmeijer (2016) and KP to Kleibergen and Paap (2006). The F-statistic is the standard F for a test that the coefficients on the excluded instruments (indicated at left) are zero. The KP statistics (LM test for under-identification and Wald F test for weak instruments) are not specific to a particular dependent variable.

Table S6. Monte Carlo Simulation: TSIV, Levels (Step 2), First Stage

	Dep. var.: log capital (k_{it})			
	DGP 1	DGP 2	DGP 3	DGP 4
	(1)	(2)	(3)	(4)
Mean coefficient $\triangle k_{it-2}$	0.496	0.490	0.486	0.486
	(0.046)	(0.023)	(0.021)	(0.020)
Mean standard error $\triangle k_{it-2}$	0.039	0.022	0.022	0.022
	(0.007)	(0.003)	(0.002)	(0.003)
Mean R-squared	0.009	0.038	0.037	0.037
	(0.002)	(0.006)	(0.007)	(0.006)
Mean KP LM test	37.560	93.910	94.114	92.324
	(9.274)	(16.577)	(16.518)	(15.594)
Mean KP Wald - F test	173.851	531.539	519.421	517.743
	(56.981)	(131.070)	(125.108)	(132.209)

Notes: Table presents the first-stage of step 2 of TSIV procedure for the four DGPs in our Monte Carlo simulation. See Section 5.2 and Appendix D for details. Table reports means of statistics across 100 simulated samples for each DGP. In parentheses are standard deviations of statistics across the 100 samples. N=15,000 for each sample.

Table S7. Monte Carlo Simulation: TSIV, Steps 1 & 2, Second Stages

	Dep. var.: $\triangle$ log output quantity ($\triangle y_{it}$)							
	DGP1		DGP2		DGP3		DGP4	
	Step 1	Step 2	Step 1	Step 2	Step 1	Step 2	Step 1	Step 2
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Materials:								
Mean elasticity	0.646	. . .	0.655	. . .	0.651	. . .	0.657	. . .
	(0.048)	. . .	(0.066)	. . .	(0.070)	. . .	(0.059)	. . .
Mean standard error	0.048	. . .	0.062	. . .	0.062	. . .	0.063	. . .
	(0.002)	. . .	(0.002)	. . .	(0.002)	. . .	(0.002)	. . .
Capital:								
Mean elasticity	0.253	0.253	0.248	0.248	0.249	0.250	0.246	0.246
	(0.037)	(0.036)	(0.033)	(0.033)	(0.033)	(0.034)	(0.030)	(0.030)
Mean standard error	0.035	0.036	0.030	0.030	0.030	0.030	0.031	0.031
	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Mean R-squared	0.729	0.950	0.542	0.839	0.542	0.838	0.545	0.836
	(0.022)	(0.014)	(0.017)	(0.035)	(0.016)	(0.040)	(0.015)	(0.041)

Notes: Table presents the second stages of steps 1 and 2 of TSIV procedure for the four DGPs in our Monte Carlo simulation. See Section 5.2 and Appendix D for details. Corresponding Monte Carlo first-stage estimates are in Tables S5 and S6. Table reports means of the statistics across 100 simulated samples for each DGP. In parentheses are standard deviations of statistics across the 100 samples. N=15,000 for each sample. The true values for the elasticities are 0.65 for materials and 0.25 for capital.