# Estimating Production Functions in Differentiated-Product Industries with Quantity Information and External Instruments* 

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## Supplementary Unpublished Materials

These supplementary materials provide some additional details and results to accompany the main text and online appendix.

## S1 Additional Theoretical Details

## S1.1 Regularity Conditions for Consumer's Minimization Problem

The second-order conditions for minimization are satisfied without further assumptions if and only if $\sigma_{i}^{y} \in(0,1) \cup(1, \infty)$. To see this, note that a necessary and sufficient condition for $\mathcal{L}^{y}$ to be convex is that all principal minors of order $r$ of the Hessian matrix of $\mathcal{L}^{y}$ are non-negative, for $r=1, \cdots, J$, where $J$ is the number of products in $\Omega_{i t}^{y}$. (See e.g. Theorem 2.3.3 in Sydsaeter et al. (2005).) Chen (2012) shows (Theorem 5.1) that the determinant of the Hessian matrix of a CES function is always zero. This implies that the principal minor of order $J$ of the Hessian for $\mathcal{L}^{y}$ is zero. Furthermore, every principal minor of degree $1 \leq r<J-1$ corresponds to the determinant of the Hessian matrix of a CES aggregator with $J-r$ varieties and hence is also zero. (Note that the theorem still applies when replacing $p(x)$ by $p(x)+c$, where the additional constant arises due to the excluded varieties that now enter as constant terms within the sum.) We are left only with the principal minors of order one, which correspond to the elements of the diagonal of the Hessian matrix of $\mathcal{L}^{y}$, which are the second derivatives:

$$
\frac{\partial^{2} \mathcal{L}^{y}}{\partial^{2} Y_{i j t}}=\frac{-\lambda^{y}}{\sigma_{i}^{y}}\left[\widetilde{Y}_{i t}^{\frac{1}{\sigma_{i}^{y}}} \varphi_{i j t}^{\frac{\sigma_{i}^{y}-1}{\sigma_{i}^{y}}} Y_{i j t}^{\frac{-1-\sigma_{i}^{y}}{\sigma_{i}^{y}}}\right]\left[\left(\frac{\varphi_{i j t} Y_{i j t}}{\widetilde{Y}_{i t}}\right)^{\frac{\sigma_{i}^{y}-1}{\sigma_{i}^{y}}}-1\right]
$$

Given that the second term in brackets is always negative and $\lambda^{y}>0$ (see the discussion following (A2)), all principal minors of order one are greater than zero if and only if $\sigma_{i}^{y} \in(0,1) \cup(1, \infty)$.

Hence $\mathcal{L}^{y}$ is convex and the second-order conditions for minimization are satisfied for a critical point satisfying the Lagrangian first order conditions if and only if $\sigma_{i}^{y} \in(0,1) \cup(1, \infty)$. For the limiting case $\sigma_{i}^{y} \rightarrow 1, \widetilde{Y}_{i t}$ tends to a Cobb-Douglas function with exponents $\varphi_{i j t}$. In this case, $\widetilde{Y}_{i t}$ is concave if and only if $\sum_{j \in \Omega_{i t}^{y}} \varphi_{i j t} \leq 1$.

In the terminology of Sun and Yang (2006), goods in the bundle $\Omega_{i t}^{y}$ are gross substitutes when $\sigma_{i}^{y}>1$ and gross complements when $0<\sigma_{i}^{y}<1$. That is, the demand for product $j$ increases in response to an increase in the price of any other variety $k$, holding everything else constant, if and only if $\sigma_{i}^{y}>1$; it decreases if and only if $0<\sigma_{i}^{y}<1$. Although our methodology can accommodate either case, we believe that in the sectors we consider in our empirical application it is reasonable to assume $\sigma_{i}^{y}>1$, i.e. that goods are gross substitutes.

## S1.2 Construction of CES Price/Quantity Indexes, Input Side

The derivations for the price and quantity indexes for the input side are analogous to the ones from the output side in Appendix A.1. We include them here for the sake of completeness.

## S1.2.1 Firm's Minimization Problem

The Lagrangian corresponding to the first stage of the firm's problem is given by:

$$
\mathcal{L}^{m}=\sum_{h \in \Omega_{i t}^{m}} M_{i h t} W_{i h t}^{m}-\lambda^{m}\left(\left[\sum_{h \in \Omega_{i t}^{m}}\left(\alpha_{i h t} M_{i h t}\right)^{\frac{\sigma_{i}^{m}-1}{\sigma_{i}^{m}}}\right]^{\frac{\sigma_{i}^{m}}{\sigma_{i}^{n-1}}}-\widetilde{M}_{i t}\right)
$$

where $\lambda^{m}$ is the Lagrange multiplier. The first order condition with respect to input $h, \frac{\partial \mathcal{L}^{m}}{\partial M_{i h t}}=0$, implies:

$$
\begin{equation*}
\frac{W_{i h t}^{m}}{\alpha_{i h t}}=\lambda^{m}\left(\alpha_{i h t} M_{i h t}\right)^{-\frac{1}{\sigma_{i}^{m}}} \widetilde{M}_{i t}^{\frac{1}{\sigma_{i}^{m}}} \tag{A1}
\end{equation*}
$$

Raising both sides of this equation to the power $1-\sigma_{i}^{m}$, summing over the $h \in \Omega_{i t}^{m}$, using the definition of $\widetilde{W}_{i t}^{m}$ in (8) in the main text, and rearranging, we have:

$$
\begin{equation*}
\lambda^{m}=\widetilde{W}_{i t}^{m} \tag{A2}
\end{equation*}
$$

Analogously to the output case, it can be shown that (without further assumptions) any point satisfying the first order conditions constitutes an global minimum if and only if $\sigma_{i}^{m} \in(0,1) \cup(1, \infty)$. Therefore, our method allows material inputs to be gross complements, $0<\sigma_{i}^{m}<1$, or to be gross substitutes, $\sigma_{i}^{m}>1$. Nevertheless, given the type of sectors we consider in our empirical analysis, we assume material inputs to be gross substitutes: $\sigma_{i}^{m}>1$.

Plugging (A2) into (A1) and rearranging:

$$
\begin{equation*}
M_{i h t}=\widetilde{M}_{i t}\left(\frac{\widetilde{W}_{i t}^{m}}{W_{i h t}^{m}}\right)^{\sigma_{i}^{m}} \alpha_{i h t}{ }^{\sigma_{i}^{m}-1} \tag{A3}
\end{equation*}
$$

As for revenues,

$$
\begin{equation*}
E_{i t}=\sum_{h \in \Omega_{i t}^{m}} E_{i h t}=\sum_{h \in \Omega_{i t}^{m}} W_{i h t}^{m} M_{i h t}=\widetilde{W}_{i t}^{m} \widetilde{M}_{i t}\left(\widetilde{W}_{i t}^{m}\right)^{\sigma_{i}^{m}-1} \underbrace{\sum_{h \in \Omega_{i t}^{m}}\left(\frac{W_{i h t}^{m}}{\alpha_{i h t}}\right)^{1-\sigma_{i}^{m}}}_{=\left(\widetilde{W}_{i t}^{m}\right)^{1-\sigma_{i}^{m}}}=\widetilde{W}_{i t}^{m} \widetilde{M}_{i t} \tag{A4}
\end{equation*}
$$

## S1.2.2 Price Index Log Change

Using (A3),

$$
\begin{equation*}
S_{i h t}^{m}=\frac{W_{i h t}^{m} M_{i h t}}{E_{i t}}=\frac{W_{i h t}^{m} M_{i h t}}{\widetilde{W}_{i t}^{m} \widetilde{M}_{i t}}=\left(\frac{\left(\frac{W_{i h t}^{m}}{\alpha_{i h t}}\right)}{\widetilde{W}_{i t}^{m}}\right)^{1-\sigma_{i}^{m}} \tag{A5}
\end{equation*}
$$

Hence from the definitions in (10) in the main text:

$$
\chi_{i t, t-1}^{m}=\frac{\sum_{h \in \Omega_{i t-1}^{m *}} S_{i h t}^{m}}{\sum_{h \in \Omega_{i t}^{m}} S_{i h t}^{m}}=\frac{\sum_{h \in \Omega_{i t, t-1}^{m *}}\left(\frac{W_{i h t}^{m}}{\alpha_{i h t}}\right)^{1-\sigma_{i}^{m}}}{\sum_{h \in \Omega_{i t}^{m}}\left(\frac{W_{i h t}^{m}}{\alpha_{i h t}}\right)^{1-\sigma_{i}^{m}}}, \chi_{i t-1, t}^{m}=\frac{\sum_{h \in \Omega_{i t-t-1}^{m *}} S_{i h t-1}^{m}}{\sum_{h \in \Omega_{i t-1}^{m}} S_{i h t-1}^{m}}=\frac{\sum_{h \in \Omega_{i t, t-1}^{m *}}\left(\frac{W_{i h t-1}^{m}}{\alpha_{i h t-1}}\right)^{1-\sigma_{i}^{m}}}{\sum_{h \in \Omega_{i t-1}^{m}}\left(\frac{W_{i h t-1}^{m}}{\alpha_{i h t-1}}\right)^{1-\sigma_{i}^{m}}}
$$

Then using the definition of $\widetilde{W}_{i t}^{m}$, (8),

$$
\begin{align*}
\frac{\widetilde{W}_{i t}^{m}}{\widetilde{W}_{i t-1}^{m}} & =\frac{\left[\sum_{h \in \Omega_{i t}^{m}}\left(\frac{W_{i h t}^{m}}{\alpha_{i h t}}\right)^{1-\sigma_{i}^{m}}\right]^{\frac{1}{1-\sigma_{i}^{m}}}}{\left[\sum_{h \in \Omega_{i t-1}^{m}}\left(\frac{W_{i h t-1}^{m}}{\alpha_{i h t-1}}\right)^{1-\sigma_{i}^{m}}\right]^{\frac{1}{1-\sigma_{i}^{m}}}} \\
& =\left(\frac{\chi_{i t-1, t}^{m}}{\chi_{i t, t-1}^{m}}\right)^{\frac{1}{1-\sigma_{i}^{m}}} \frac{\left(\sum_{h \in \Omega_{i t, t-1}^{m *}}\left(\frac{W_{i h t}^{m}}{\alpha_{i h t}}\right)^{1-\sigma_{i}^{m}}\right)^{\frac{1}{1-\sigma_{i}^{m}}}}{\left(\sum_{h \in \Omega_{i t, t-1}^{m *}}\left(\frac{W_{i h t-1}^{m}}{\alpha_{i h t-1}}\right)^{1-\sigma_{i}^{m}}\right)^{\frac{1}{1-\sigma_{i}^{m}}}}=\left(\frac{\chi_{i t-1, t}^{m}}{\chi_{i t, t-1}^{m}}\right)^{\frac{1}{1-\sigma_{i}^{m}}} \frac{\widetilde{W}_{i t}^{*}}{\widetilde{W}_{i t-1}^{*}} \tag{A6}
\end{align*}
$$

where $\widetilde{W}_{i t}^{*}$ is the common-goods price index defined in the main text (footnote 16).
To derive an expression for $\frac{\widetilde{W}_{i t}^{*}}{\widetilde{W}_{i t-1}^{*}}$, note that (A5) implies a similar expression for the expenditure share of common goods:

$$
S_{i h t}^{m *}=\frac{W_{i h t}^{m} M_{i h t}}{\widetilde{W}_{i t}^{*} \widetilde{M}_{i t}^{*}}=\frac{W_{i h t}^{m} M_{i h t}}{\widetilde{W}_{i t}^{m} \widetilde{M}_{i t}} \cdot \frac{\widetilde{W}_{i t}^{m} \widetilde{M}_{i t}}{\widetilde{W}_{i t}^{*} \widetilde{M}_{i t}^{*}}=\left(\frac{\left(\frac{W_{i h t}^{m}}{\alpha_{i h t}}\right)}{\widetilde{W}_{i t}^{m}}\right)^{1-\sigma_{i}^{m}} \frac{\widetilde{W}_{i t}^{m} \widetilde{M}_{i t}}{\widetilde{W}_{i t}^{*} \widetilde{M}_{i t}^{*}}
$$

Using (A2),

$$
\frac{\widetilde{W}_{i t}^{m} \widetilde{M}_{i t}}{\widetilde{W}_{i t}^{*} \widetilde{M}_{i t}^{*}}=\frac{\widetilde{W}_{i t}^{m} \widetilde{M}_{i t}}{\sum_{h \in \Omega_{i t, t-1}^{m *}} W_{i h t}^{m} M_{i h t}}=\left(\frac{\widetilde{W}_{i t}^{m}}{\widetilde{W}_{i t}^{*}}\right)^{1-\sigma_{i}^{m}}
$$

Hence:

$$
\begin{equation*}
S_{i h t}^{m *}=\left(\frac{\left(\frac{W_{i h t}^{m}}{\alpha_{i h t}}\right)}{\widetilde{W}_{i t}^{*}}\right)^{1-\sigma_{i}^{m}} \tag{A7}
\end{equation*}
$$

Divide (A7) by the same equation for the previous year, take logs, and re-arrange:

$$
\frac{\ln \left(\frac{\widetilde{W}_{i t}^{m *}}{\widehat{W}_{i t-1}^{m *}}\right)-\ln \left(\frac{\frac{W_{i h t}^{m}}{\alpha_{i h t}}}{\frac{W_{i h t-1}^{i h t-1}}{\alpha_{i h t-1}}}\right)}{\ln \left(\frac{S_{i h t}^{m *}}{S_{i h t-1}^{m h}}\right)}=\frac{1}{\sigma_{i}^{m}-1}
$$

Multiply both sides by $S_{i h t}^{m *}-S_{i h t-1}^{m *}$ and sum over the common goods:

$$
\sum_{h \in \Omega_{i t, t-1}^{m *}}\left(S_{i h t}^{m *}-S_{i h t-1}^{m *}\right) \frac{\ln \left(\frac{\widetilde{W}_{i t}^{m *}}{\widehat{W}_{i t-1}^{m *}}\right)-\ln \left(\frac{\frac{W_{i h t}^{m}}{\alpha_{i n t}}}{\frac{W_{i h t-1}}{\alpha_{i h t-1}}}\right)}{\ln \left(\frac{S_{i h t}^{m *}}{S_{i h t-1}^{m *}}\right)}=\left(\frac{1}{\sigma_{i}^{m}-1}\right) \sum_{h \in \Omega_{i t, t-1}^{m *}}\left(S_{i h t}^{m *}-S_{i h t-1}^{m *}\right)=0
$$

where the second equality follows because $\sum_{h \in \Omega_{i t, t-1}^{m *}} S_{i h t}^{m *}=\sum_{h \in \Omega_{i t, t-1}^{m *}} S_{i h t-1}^{m *}=1$. This implies:

$$
\sum_{h \in \Omega_{i t, t-1}^{m *}}\left(\frac{S_{i h t}^{m *}-S_{i h t-1}^{m *}}{\ln S_{i h t}^{m *}-\ln S_{i h t-1}^{m *}}\right) \ln \left(\frac{\widetilde{W}_{i t}^{m *}}{\widetilde{W}_{i t-1}^{m *}}\right)=\sum_{h \in \Omega_{i t, t-1}^{m *}}\left(\frac{S_{i h t}^{m *}-S_{i h t-1}^{m *}}{\ln S_{i h t}^{m *}-\ln S_{i h t-1}^{m *}}\right) \ln \left(\frac{\frac{W_{i h t}^{m}}{\alpha_{i h t}}}{\frac{W_{i h t-1}^{m t}}{\alpha_{i h t-1}}}\right)
$$

Since $\ln \left(\frac{\widetilde{W}_{i t}^{m *}}{\widetilde{W}_{i t-1}^{m *}}\right)$ does not vary with $h$, this can be re-written as:

$$
\begin{equation*}
\ln \left(\frac{\widetilde{W}_{i t}^{m *}}{\widetilde{W}_{i t-1}^{m *}}\right)=\sum_{h \in \Omega_{i t, t-1}^{m * *}} \psi_{i h t}^{m} \ln \left(\frac{W_{i h t}^{m}}{W_{i h t-1}^{m}}\right)-\sum_{h \in \Omega_{i t, t-1}^{m *}} \psi_{i h t}^{m} \ln \left(\frac{\alpha_{i h t}}{\alpha_{i h t-1}}\right) \tag{A8}
\end{equation*}
$$

where $\psi_{i h t}^{m}$ is as defined in (10) above. Combining (A6) and (A8), we have:

$$
\begin{equation*}
\ln \left(\frac{\widetilde{W}_{i t}^{m}}{\widetilde{W}_{i t-1}^{m}}\right)=\sum_{h \in \Omega_{i t, t-1}^{m * *}} \psi_{i h t}^{m} \ln \left(\frac{W_{i h t}^{m}}{W_{i h t-1}^{m}}\right)-\sum_{h \in \Omega_{i t, t-1}^{m *}} \psi_{i h t}^{m} \ln \left(\frac{\alpha_{i h t}}{\alpha_{i h t-1}}\right)-\frac{1}{\sigma_{i}^{m}-1} \ln \left(\frac{\chi_{i t-1, t}^{m}}{\chi_{i t, t-1}^{m}}\right) \tag{A9}
\end{equation*}
$$

which is (9) in the main text.

## S1.2.3 Quantity Index Log Change

We start by noting that (A3) implies

$$
W_{i h t}^{m}=\widetilde{W}_{i t}^{m}\left(\frac{\widetilde{M}_{i t}}{M_{i h t}}\right)^{\frac{1}{\sigma_{i}^{n}}} \alpha_{i h t}^{\frac{\sigma_{m}^{m}-1}{\sigma_{n}^{m}}}
$$

Hence:

$$
\ln \left(\frac{W_{i h t}^{m}}{W_{i h t-1}^{m}}\right)=\ln \left(\frac{\widetilde{W}_{i t}^{m}}{\widetilde{W}_{i t-1}^{m}}\right)+\frac{1}{\sigma_{i}^{y}} \ln \left(\frac{\widetilde{M}_{i t}}{\widetilde{M}_{i t-1}}\right)-\frac{1}{\sigma_{i}^{y}} \ln \left(\frac{W_{i h t}^{m}}{W_{i h t-1}^{m}}\right)+\frac{\sigma_{i}^{y}}{\sigma_{i}^{y}-1} \ln \left(\frac{\alpha_{i h t}}{\alpha_{i h t-1}}\right)
$$

Plugging this into (A9), re-arranging, and using the fact that $\sum_{h \in \Omega_{i t, t-1}^{m *}} \psi_{i h t}^{m}=1$ gives the log change in $\widetilde{M}_{i t}$ :

$$
\ln \left(\frac{\widetilde{M}_{i t}}{\widetilde{M}_{i t-1}}\right)=\sum_{h \in \Omega_{i t, t-1}^{m *}} \psi_{i h t}^{m} \ln \left(\frac{M_{i h t}}{M_{i h t-1}}\right)+\sum_{h \in \Omega_{i t, t-1}^{m *}} \psi_{i h t}^{m} \ln \frac{\alpha_{i h t}}{\alpha_{i h t-1}}+\frac{\sigma_{i}^{m}}{\sigma_{i}^{m}-1} \ln \left(\frac{\chi_{i t-1, t}^{m}}{\chi_{i t, t-1}^{m}}\right)
$$

which is (11) in the main text. The fact that $\widetilde{W}_{i t}^{m *} \widetilde{M}_{i t}^{*}=E_{i t}^{*}$ can be shown as in (A4), using just common goods.

## S1.3 Construction of Alternative Quantity Indexes

On the input side, following standard formulations (see e.g. Dodge (2008)), we define the Laspeyres input quantity index for $t-1$ and $t$ as:

$$
\begin{equation*}
\widetilde{M}_{i t, t-1}^{\text {Lasp }}=\frac{\sum_{h \in \Omega_{i t, t-1}^{m *}} W_{i h t-1}^{m} M_{i h t}}{\sum_{h \in \Omega_{i t, t-1}^{m *}} W_{i h t-1}^{m} M_{i h t-1}} \tag{A10}
\end{equation*}
$$

and the Paasche input quantity index as:

$$
\begin{equation*}
\widetilde{M}_{i t, t-1}^{P a a s}=\frac{\sum_{h \in \Omega_{i t, t-1}^{m *}} W_{i h t}^{m} M_{i h t}}{\sum_{h \in \Omega_{i t, t-1}^{m, t}} W_{i h t}^{m} M_{i h t-1}}, \tag{A11}
\end{equation*}
$$

The Tornqvist quantity index is defined as:

$$
\begin{equation*}
\widetilde{M}_{i t, t-1}^{\text {Torn }}=\prod_{h \in \Omega_{i t, t-1}^{m *}}\left(\frac{M_{i h t}}{M_{i h t-1}}\right)^{\frac{1}{2}\left(S_{i h t}^{m *}+S_{i h t-1}^{m *}\right)} \tag{A12}
\end{equation*}
$$

where $S_{i h t}^{m *}$ and $S_{i h t-1}^{m *}$ are as defined in footnote 22 of the main text.
Note that the Laspeyres quantity index is related to the Paasche price index, and vice-versa. If
we define the Laspeyres price index as:

$$
\begin{equation*}
\widetilde{W}_{i t, t-1}^{m, L a s p}=\frac{\sum_{h \in \Omega_{i t, t-1}^{m *}} W_{i h t}^{m} M_{i h t-1}}{\sum_{h \in \Omega_{i t, t-1}^{m *}} W_{i h t-1}^{m} M_{i h t-1}} . \tag{A13}
\end{equation*}
$$

and the Paasche price index as:

$$
\begin{equation*}
\widetilde{W}_{i t, t-1}^{m, P a a s}=\frac{\sum_{h \in \Omega_{i t, t-1}^{m *}} W_{i h t}^{m} M_{i h t}}{\sum_{h \in \Omega_{i t, t-1}^{m *}} W_{i h t-1}^{m} M_{i h t}} \tag{A14}
\end{equation*}
$$

then the common-input expenditure ratio between $t$ and $t-1$ is the product of the Laspeyres price index and the Paasche quantity index and also the product of the Laspeyres quantity index and the Paasche price index:

$$
\frac{E_{i t}^{*}}{E_{i t-1}^{*}}=\frac{\sum_{h \in \Omega_{i t t-1}^{m *}} W_{i h t}^{m} M_{i h t}}{\sum_{h \in \Omega_{i t, t-1}^{m *}} W_{i h t-1}^{m} M_{i h t-1}}=\widetilde{M}_{i t, t-1}^{\text {Lasp }} \times \widetilde{W}_{i t, t-1}^{m, \text { Paas }}=\widetilde{M}_{i t, t-1}^{\text {Paas }} \times \widetilde{W}_{i t, t-1}^{m, \text { Lasp }}
$$

The definition of the alternative output quantity indexes is analogous to the definition of the input quantity indexes (A10), (A11) and (A12).

## S1.4 Variance Correction for $\beta_{k}$ in Levels-Equation Estimation

Our sequential production function estimation belongs to a general class of two-step M-Estimators discussed for instance in Wooldridge (2002, Section 12.4) and previously in Newey (1984). The results there can be applied directly in our setting. Under our assumptions, our first-step estimates $\widehat{\beta}_{m}$ and $\widehat{\beta}_{l}$ and their standard errors are consistently estimated. The levels-equation estimate of $\beta_{k}$, call it $\widehat{\widehat{\beta}}_{k}$, can be calculated by solving:

$$
\begin{equation*}
\sum_{t=1}^{T} \sum_{i=1}^{N} \Delta k_{i t-1}\left(\left(\widetilde{y}_{i t}^{S V}-\widehat{\beta}_{m} \widetilde{m}_{i t}^{S V}-\widehat{\beta}_{l} l_{i t}\right)-\widehat{\widehat{\beta}}_{k} k_{i t}\right)=0 . \tag{A15}
\end{equation*}
$$

As noted in the main text, the consistency of $\widehat{\beta}_{m}$ and $\widehat{\beta}_{l}$ is sufficient to guarantee the consistency of $\widehat{\widehat{\beta}}_{k}$. In the special case when $\mathbb{E}\left(\Delta k_{i t-1} \widetilde{m}_{i t}^{S V}\right)=0$ and $\mathbb{E}\left(\Delta k_{i t-1} \ell_{i t}\right)=0$, the first step estimation can be ignored when computing the asymptotic variance of $\widehat{\widehat{\beta}}_{k} .{ }^{1}$ If those conditions do not hold, then we need to use a corrected expression for the asymptotic variance of $\widehat{\beta}_{k}$, which takes into account that $\widehat{\beta}_{m}$ and $\widehat{\beta}_{l}$ were estimated in a previous step. A consistent estimate of the corrected asymptotic

[^0]variance for $\widehat{\widehat{\beta}}_{k}$, call it $\widehat{V}_{\beta_{k}}$, is given by Newey and McFadden (1994): ${ }^{2}$
\[

$$
\begin{equation*}
\widehat{V}_{\beta_{k}}=\frac{(T \times N)^{-1}\left(\sum_{t=1}^{T} \sum_{i=1}^{N}\left(\widehat{s}_{i t}+\widehat{F} \widehat{\psi}_{i t}\right)^{2}\right)}{\widehat{G}^{2}} \tag{A16}
\end{equation*}
$$

\]

where

$$
\begin{aligned}
\widehat{G} & =-\frac{1}{N T} \sum_{t=1}^{T} \sum_{i=1}^{N} \Delta k_{i t-1} k_{i t} \\
\widehat{F} & =-\frac{1}{N T} \sum_{t=1}^{T} \sum_{i=1}^{N} \Delta k_{i t-1}\left[\widetilde{m}_{i t}^{S V}, l_{i t}, 0\right] \\
\widehat{s}_{i t} & =\Delta k_{i t-1}\left(\widetilde{y}_{i t}^{S V}-\widehat{\beta}_{m} \widetilde{m}_{i t}^{S V}-\widehat{\beta}_{l} l_{i t}-\widehat{\widehat{\beta}}_{k} k_{i t}\right) \\
\widehat{\psi}_{i t} & =-\left(\widehat{H}^{\prime} \widehat{W} \widehat{H}\right)^{-1} \widehat{H}^{\prime} \widehat{W} \widehat{m}_{i t}
\end{aligned}
$$

and the terms in $\widehat{\psi}_{i t}$ are defined as:

$$
\begin{aligned}
\widehat{H} & =\frac{1}{N T} \sum_{t=1}^{T} \sum_{i=1}^{N}\left[\Delta \widehat{\bar{w}}_{i t}^{i m p}, \Delta z_{i t}, k_{i t-2}, \widetilde{m}_{i t-2}^{S V}, l_{i t-2}\right]\left[\Delta \widetilde{m}_{i t}^{S V}, \Delta l_{i t}, \Delta k_{i t}\right]^{\prime} \\
\widehat{W} & =\frac{1}{N T} \sum_{t=1}^{T} \sum_{i=1}^{N}\left[\Delta \widehat{\bar{w}}_{i t}^{i m p}, \Delta z_{i t}, k_{i t-2}, \widetilde{m}_{i t-2}^{S V}, l_{i t-2}\right]\left[\Delta \widehat{\bar{g}}_{i t}, \Delta z_{i t}, k_{i t-2}, \widetilde{m}_{i t-2}^{S V}, l_{i t-2}\right]^{\prime} \\
\widehat{m}_{i t} & =\left[\Delta \widehat{g}_{i t}, \Delta z_{i t}, k_{i t-2}, \widetilde{m}_{i t-2}^{S V}, l_{i t-2}\right]^{\prime}\left(\Delta \widetilde{y}_{i t}^{S V}-\widehat{\beta}_{m} \Delta \widetilde{m}_{i t}^{S V}-\widehat{\beta}_{l} \Delta l_{i t}-\widehat{\beta}_{k} \Delta k_{i t}\right)
\end{aligned}
$$

We report the corresponding corrected standard errors when we report $\widehat{\widehat{\beta}_{k}}$.

[^1]
## S2 Additional Empirical Details

## S2.1 Real Exchange Rate Fluctuations

Figure S1 depicts the movements in real exchange rates (where increases reflect real appreciations in the trading partner) for the 12 countries from which rubber and plastics producers purchased the most imports during the period of our analysis. We see that several of the most important import origins had significant RER fluctuations. Venezuela and Mexico, both major oil producers, had large real appreciations in 1995-2000 and large real depreciations subsequently. Indonesia suffered a major crisis accompanied by sharp real devaluation in 1997 (as did Argentina (not pictured) in 2001). Even the US and Eurozone countries, which were less volatile overall, experienced non-trivial variation in the RER relative to Colombia.

## S2.2 Real Minimum Wage, 1994-2009

Figure S2 shows the steady increase of the real minimum wage from 1996 to 2009.

## S2.3 Comparison to System GMM

For purposes of comparison to the System GMM results using our quantity indexes in Table 5, Table S3 below presents the results from applying standard System GMM, using deflated sales and expenditures for output and inputs as in typical applications. Table S4 presents the corresponding weak-instrument diagnostics. The message is broadly similar to Table 5, except for the capital coefficient, which seems implausibly small, pointing to a potential advantage of using the quantity indexes in System GMM, even when external instruments are not available.

## References

Blundell, Richard and Stephen Bond, "GMM Estimation with Persistent Panel Data: An Application to Production Functions," Econometric Reviews, 2000, 19 (3), 321-340.
Chen, Bang-Chen, "An Explicit Formula of Hessian Determinants of Composite Functions and Its Applications," Kragujevac Journal of Mathematics, 2012, 17, 27-39.
Dodge, Yadolah, The Concise Encyclopedia of Statistics, New York: Springer, 2008.
Doornik, Jurgen A, Manuel Arellano, and Stephen Bond, "Panel Data Estimation Using DPD for Ox," 2012. Available at https://www.doornik.com/download/oxmetrics7/Ox_Packages/dpd.pdf.
Kleibergen, Frank and Richard Paap, "Generalized Reduced Rank Tests using the Singular Value Decomposition," Journal of Econometrics, 2006, 133 (1), 97 - 126.
Kripfganz, Sebastian and Claudia Schwarz, "Estimation of Linear Dynamic Panel Data Models with Time-Invariant Regressors," Journal of Applied Econometrics, 2019, 34 (4), 526-546.
Newey, Whitney K, "A Method of Moments Interpretation of Sequential Estimators," Economics Letters, 1984, 14 (2-3), 201-206.
Newey, Whitney K. and Daniel L. McFadden, "Large Sample Estimation and Hypothesis Testing," in Robert F. Engle and Daniel L. McFadden, eds., Handbook of Econometrics, Vol. 4, Amsterdam: Elsevier Science, 1994, chapter 36, pp. 2111-2245.

Roodman, David, "How To Do xtabond2: An Introduction to Difference and System GMM in Stata," Stata Journal, 2009, 9 (1), 86-136.
Sanderson, Eleanor and Frank Windmeijer, "A Weak Instrument F-test in Linear IV models with Multiple Endogenous Variables," Journal of Econometrics, 2016, 190 (2), 212-221.
Sun, Ning and Zaifu Yang, "Equilibria and Indivisibilities: Gross Substitutes and Complements," Econometrica, 2006, 74 (5), 1385-1402.
Sydsaeter, Knut, Peter Hammond, Atle Seierstad, and Arne Strom, Further Mathematics for Economic Analysis, Prentice Hall, 2005.
Windmeijer, Frank, "A Finite Sample Correction for the Variance of Linear Efficient Two-Step GMM Estimators," Journal of Econometrics, 2005, 126 (1), 25-51.
Wooldridge, Jeffrey M., Econometric Analysis of Cross Section and Panel Data, Cambridge, MA: MIT Press, 2002.

Figure S1. Real Exchange Rate Variation, 1994-2009


Notes: Figure plots real exchange rate (RER), normalized to 100 in 1994, calculated as in equation (21) in text, for top six import origins for rubber and plastics sectors. An RER increase reflects a real appreciation in the trading partner.

Figure S1. Real Exchange Rate Variation, 1994-2009 (cont.)


Notes: Figure plots real exchange rate (RER), normalized to 100 in 1994, calculated as in equation (21) in text, for import origins ranked $7-12$ for rubber and plastics sectors. (See Fig. S1 for ranks 1-6.) An RER increase reflects a real appreciation in the trading partner.

Figure S2. Real Minimum Monthly Wage, 1994-2009


Notes: Figure plots Colombian national real monthly minimum wage, in thousands of 2000 pesos, for 1994-2009. Average 2000 exchange rate is approximately 2,000 pesos/USD.

Figure S3. Coefficients from Import-Price Regressions

Wading, felt and nonwovens, [.0046] , 2.67(-.81,6.14) Other made-up textile articles, [.0012] , 1.43(-.06,2.93) Dairy produce, [.0022] , 1.43(-1.27,4.13)
Optical, photographic, cinematographic, measuring, checking,, [.0184], 1.36(-.53,3.24) Impregnated, coated, covered or laminated textile fabrics, [.0028] , 1.34(-.12,2.81) Miscellaneous edible preparations, $[.009], 1.29(.03,2.55)$

Articles of iron or steel, [.016] , 1.25(.24,2.26) Organic chemicals, [.1519], 1.19(.32,2.06) Preparations of cereals, flour, starch or milk, [.0054] , .99(.04,1.93) Animal or vegetable fats and oils and their cleavage product, [.0213], . $94(-.47,2.35)$ Headgear and parts thereof, [.0002], , 8(-4.08,5.68) Articles of stone, plaster, cement, asbestos, mica or simila, [.0021], . $8(-.54,2.14)$ Oil seeds and oleaginous fruits, $[.0141], .78(-1.67,3.23)$

Cotton, [.0245] , .74(-2.42,3.89) Ships, boats and floating structures, $[.0001], .73(.13,1.33)$ Ceramic products, [.003], .65(-.09,1.39) Pharmaceutical products, $[.0144], .6(-.78,1.98)$ Sugars and sugar confectionery, [.0016] , .57(-1.23,2.37) Beverages, spirits and vinegar, [.0034], .57(-1.08,2.21) Explosives, $[.0008], .5(-.58,1.58)$
Preparations of meat, of fish or of crustaceans, molluscs or, [.001], .5(-1,1.99)
Live trees and other plants, [.0001] , .49(-1.93,2.91) Ores, slag and ash, [.0018] , .48(-.01,.96) Fish and crustaceans, molluscs and other aquatic invertebrat, [.0004] , .46(-1.18,2.11) Furskins and artificial fur; manufactures thereof, [0] , .41(-.59,1.42) Cocoa and cocoa preparations, $[.001], .41(-.74,1.55)$ Paper and paperboard, [.0377] , .39(-.93,1.72) Rubber and articles thereof, [.0244], .31(-1.29,1.91) Man-made staple fibres, [.0121], . $31(-.77,1.39)$ Edible vegetables and certain roots and tubers, [.0008] , .3(-.99,1.59) Printed books, newspapers, pictures and other products of th, [.0023], .29(-3.21,3.8) Toys, games and sports requisites, $[.0015], .27(-.58,1.13)$ Carpets and other textile floor coverings, $[.0004], .25(-1.18,1.68)$ Edible fruit and nuts; peel of citrus fruit or melons, [.0009] , .24(-1.11,1.59) Fertilisers, [.0247] , .23(-.64,1.09) Pulp of wood or of other fibrous cellulosic material, [.0137], .21(-.37,.79) Vegetable plaiting materials, [0] , .21(-.7,1.12) Iron and steel, [.0724] , .18(-.38,.74) Preparations of vegetables, fruit, nuts or other parts of pl, [.0028] , .17(-.92,1.26) Aluminium and articles thereof, [.0256] , .17(-2.93,3.27) Photographic or cinematographic goods, [.0013] , .16(-.42,.73) Knitted or crocheted fabrics, [.0024] , .15(-1.01,1.3)
Natural or cultured pearls, precious or semi-precious stones, [.0009] , .13(-.26,.52)


Notes: Average sector-specific coefficients from estimating equation (23) in main text. We generate 362 sets of leave-one-out coefficient estimates, then average estimates and standard errors across firms and years. All Harmonized System 2-digit categories except except petroleum products, machinery and equipment (HS2 categories 27, 84 and 85) included; see Section 2.5 .1 for details. Import share calculated as imports in HS2 category over total imports for 1994-2009 period. After sector names at left, we list share of total imports (square brackets), the regression coefficient, and $95 \%$ confidence interval (parentheses).

## Figure S3. Coefficients from Import-Price Regressions (cont.)

Coffee, tea, maté and spices, [.0008] , .12(-1.84,2.08)
Railway or tramway locomotives, rolling stock and parts ther, [.0001] , .11(-.56,.78) Soap, organic surface-active agents, washing preparations, $I,[.0062], .06(-1.16,1.29)$ Miscellaneous chemical products, $[.0327], .06(-.11, .23)$ Plastics and articles thereof, [.085] , .04(-.54,.63) Miscellaneous articles of base metal, [.0041] , .02(-1.74,1.77) Articles of apparel and clothing accessories, not knitted or, [.0034] , 0(-.18,.18) Silk, [.0002], 0(-1.29,1.29) Wood and articles of wood; wood charcoal, [.0015] , -.02(-2.75,2.72) Cereals, [.0835], -.03(-1.61,1.55) Inorganic chemicals, $[.0209],-.03(-.35, .29)$
ntial oils and resinoids [0159] $-.07(-42,28)$ Other base metals; cermets; articles thereof, [.0003] , -.07(-.86,.72)

Furniture, $[.0024],-.12(-1.99,1.76)$ Tools, implements, cutlery, spoons and forks, of base metal, [.0031] , -.12(-1.42,1.18) Products of the milling industry, [.0011] , -.13(-1.42,1.16) Glass and glassware, $[.0072],-.13(-1.38,1.12)$ Wool, fine or coarse animal hair, [.0042], -.14(-.84,.57) Manufactures of straw, of esparto or of other plaiting mater, $[0],-.15(-2,1.69)$ Clocks and watches and parts thereof, $[.0008],-.15(-1.88,1.57)$ Nickel and articles thereof, $[.0007],-.16(-.97, .65)$ Special woven fabrics, $[.002],-.17(-1.48,1.14)$ Speciar woven fabrics, $[.002],-.17(-1.40,1.44)$
Man-made filaments, $[.0119],-.19(-.82, .43)$ Other vegetable textile fibres, [.0003] , -.22(-.86,.41) Works of art, collectors' pieces and antiques, [0] , -.28(-3.17,2.61) Tobacco and manufactured tobacco substitutes, $[.0016],-.29(-1.16, .58)$ Zinc and articles thereof, $[.0055],-.29(-3.49,2.9)$ Articles of apparel and clothing accessories, knitted or cro, [.0033], ,-32(-1.36,.71) $\begin{array}{r}\text { Salt, }[.0065],-.38(-1.95,1.19) \\ \text { xtracts, }[.0013],-44(-3.16,2.28) \\ \hline\end{array}$ Lac; gums, resins and other vegetable saps and extracts, $[.0013],-.44(-3.16,2.28)$ Arms and ammunition; parts and accessories thereof, $[.005],-.5(-1.76, .77)$ Lead and articles thereof, [0015] , -8(-3.08,1.49) Raw hides and skins (other than furskins) and leather, [.0017] , -.81(-2.31,.7) Prepared feathers and down and articles made of feathers or , [0] , -.85(-1.87,.16) Copper and articles thereof, $[.0165],-.93(-3.32,1.46)$ Footwear, gaiters and the like; parts of such articles, $[.0021],-1.07(-3.49,1.34)$ Tin and articles thereof, $[.0003],-1.13(-3.02, .75)$ Musical instruments; parts and accessories of such articles, [.0004] , -1.36(-2.72,0) Live animals, $[.0002],-1.38(-4.67,1.91)$ Umbrellas, sun umbrellas, walking sticks, seat-sticks, whips, [.0001], ,-1.51(-7.03,4.01) Albuminoidal substances; modified starches; glues; enzymes, [.0037], -1.71(-3.81,.4) Residues and waste from the food industries, [.028] , -1.79(-5.73,2.15)


Notes: See notes on previous page.

Table S1. Primary Outputs and Inputs, Glass Products Producers

| CPC code | Share of total revenues or expenditures | Export/Import share | CPC description |
| :---: | :---: | :---: | :---: |
| A. Outputs |  |  |  |
| 3719102 | 0.21 | 0.16 | Glass bottles for soft drinks |
| 3719103 | 0.18 | 0.19 | Glass bottles of a capacity not exceeding 1 liter |
| 3711502 | 0.18 | 0.53 | Safety glass |
| 3711201 | 0.12 | 0.17 | Unworked flat glass |
| 3719104 | 0.11 | 0.18 | Glass bottles of a capacity exceeding 1 liter |
| 3711503 | 0.06 | 0.28 | Safety glass for motor cars, windshields, similar |
| 3719101 | 0.03 | 0.22 | Small glass jars for perfumery, pharmacy, laboratory |
| 3712204 | 0.01 | 0.49 | Glass wool sheet |
| 3719309 | 0.01 | 0.58 | Glass vases |
| 2799704 | 0.01 | 0.01 | Asphalt fabrics |
| 4299942 | 0.01 | 0.11 | Wire rods and rings, for brassieres |
| 3719302 | 0.01 | 0.24 | Glasswares of a kind used for table and kitchen |
| 3712203 | 0.01 | 0.27 | Fiberglass ducts |
| 3719503 | 0.01 | 0.04 | Glass ampoules |
| 3712101 | 0.01 | 0.46 | Fiberglass |
| 3711601 | 0.01 | 0.06 | Unframed mirror |
| 3712907 | 0.01 | 0.38 | Fiberglass bathtubs |
| 3712908 | 0.01 | 0.09 | Fiberglass tanks |
| 3711501 | 0.00 | 0.00 | Tempered glass |
| 3719903 | 0.00 | 0.10 | Glass screens |
| B. Inputs |  |  |  |
| 3711201 | 0.30 | 0.79 | Unworked flat glass |
| 3424501 | 0.22 | 0.44 | Sodium carbonate |
| 3711103 | 0.10 | 0.07 | Waste and scrap of glass |
| 3633019 | 0.07 | 0.93 | Plastic fabric |
| 3633007 | 0.05 | 0.98 | Polyvinyl film |
| 1531201 | 0.05 | 0.00 | Siliceous sands and gravels |
| 1639902 | 0.03 | 0.00 | Feldspar |
| 3219702 | 0.03 | 0.00 | Printed labels |
| 3474002 | 0.02 | 0.37 | Polyester resins |
| 1512004 | 0.02 | 0.00 | Crushed or ground limestone |
| 3215308 | 0.02 | 0.00 | Partitions and dividers of cardboard for boxes |
| 3215302 | 0.01 | 0.01 | Corrugated cardboard boxes |
| 4151203 | 0.01 | 0.36 | Angles, shapes and sections of copper |
| 3511104 | 0.01 | 0.37 | Anticorrosive bases and paints |
| 3712101 | 0.01 | 0.45 | Fiberglass |
| 3170101 | 0.01 | 0.00 | Wooden packaging box |
| 4299942 | 0.01 | 0.72 | Wire rods and rings, for brassieres |
| 3170105 | 0.01 | 0.00 | Pallets |
| 3424202 | 0.01 | 0.07 | Sodium sulfate |
| 3641002 | 0.01 | 0.00 | Unprinted plastic film in tubular form |

Notes: Table similar to Tables A1-A2 for producers of glass products (ISIC rev. 2 category 362). See notes for those tables.

Table S2. Summary Statistics, Glass Products
A. Period: 1996-2009
Number of observations ..... 410
Number of firms ..... 34
Number of workers per firm ..... 122.97
Share of firms that are single-product ..... 0.15
Production value (billions 2000 pesos) per firm ..... 16.41
Earnings per year per firm, permanent workers (millions 2000 pesos) ..... 7.06
B. Period: 2000-2009
Input variables
No. inputs per firm ..... 9.43
Share of firms that import ..... 0.65
No. inputs per firm, conditional on importing ..... 9.50
Share of expenditure on imported inputs ..... 0.30
No. imported HS8 categories, conditional on importing ..... 24.74
Output variables
No. outputs per firm ..... 2.89
Share of firms that export ..... 0.53
No. outputs per firm, conditional on exporting ..... 3.44
Fraction of revenues from exported outputs ..... 0.11
No. exported HS8 categories, conditional on exporting ..... 5.31

Table S3. System GMM, Using Sales and Expenditures

|  | $\log$ output index $\left(\triangle \widetilde{y}_{i t}^{S V}\right)$ |  |  |
| :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) |
| $\log \operatorname{sales}_{i t-1}$ | $\begin{gathered} 0.626^{* * *} \\ (0.080) \end{gathered}$ | $\begin{gathered} 0.549^{* * *} \\ (0.080) \end{gathered}$ | $\begin{gathered} 0.515^{* * *} \\ (0.060) \end{gathered}$ |
| log expenditures ${ }_{i t}$ | $\begin{gathered} 0.548^{* * *} \\ (0.088) \end{gathered}$ | $\begin{gathered} 0.573^{* * *} \\ (0.073) \end{gathered}$ | $\begin{gathered} 0.490^{* * *} \\ (0.057) \end{gathered}$ |
| $\log$ expenditures ${ }_{i t-1}$ | $\begin{gathered} -0.211^{* *} \\ (0.086) \end{gathered}$ | $\begin{gathered} -0.202^{* * *} \\ (0.067) \end{gathered}$ | $\begin{gathered} -0.127^{* *} \\ (0.049) \end{gathered}$ |
| $\log \operatorname{labor}_{i t}\left(\ell_{i t}\right)$ | $\begin{gathered} 0.347^{* *} \\ (0.140) \end{gathered}$ | $\begin{gathered} 0.339 * * \\ (0.148) \end{gathered}$ | $\begin{gathered} 0.353^{* * *} \\ (0.075) \end{gathered}$ |
| $\log \operatorname{labor}_{i t-1}\left(\ell_{i t-1}\right)$ | $\begin{gathered} -0.326^{* *} \\ (0.145) \end{gathered}$ | $\begin{aligned} & -0.269^{*} \\ & (0.141) \end{aligned}$ | $\begin{gathered} -0.236^{* * *} \\ (0.077) \end{gathered}$ |
| $\log \operatorname{capital}_{i t}\left(k_{i t}\right)$ | $\begin{gathered} 0.042 \\ (0.071) \end{gathered}$ | $\begin{gathered} 0.061 \\ (0.074) \end{gathered}$ | $\begin{gathered} 0.014 \\ (0.063) \end{gathered}$ |
| $\log$ capital $_{i t-1}\left(k_{i t-1}\right)$ | $\begin{aligned} & -0.025 \\ & (0.061) \end{aligned}$ | $\begin{aligned} & -0.030 \\ & (0.063) \end{aligned}$ | $\begin{gathered} 0.022 \\ (0.055) \end{gathered}$ |
| Observations | 4,247 | 4,247 | 4,247 |
| Lag limit | 3 | 4 | All |
| Hansen test | 120.700 | 171.000 | 347.600 |
| Hansen p-value | 0.141 | 0.279 | 1.000 |

Notes: Table presents estimates of standard System GMM model (Blundell and Bond, 2000), using sales and expenditures for output and inputs, and using the "two-step" procedure described in Roodman (2009), with initial weighting matrix defined in Doornik et al. (2012) and finite-sample correction from Windmeijer (2005). The Stata command is xtabond2 (Roodman, 2009), with options h(2), twostep, and robust. The difference equation includes lags to $t-3$ in Column 1, lags to $t-4$ in Column 2, and all available lags in Column 3. The numbers of instruments are as indicated in Appendix Table S4. The Hansen test of overidentifying restrictions is appropriate in the non-homoskedastic case, but should be interpreted with caution, as it is weakened by the presence of many instruments. See Section 5 for further details. Robust standard errors in parentheses. ${ }^{*} 10 \%$ level, ${ }^{* *} 5 \%$ level, ${ }^{* * *} 1 \%$ level.

Table S4. Weak IV Diagnostics for System GMM, Using Revenues and Expenditures

| Differences |  |  |  | Levels |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Dep. var.: $\Delta \log$ sales $_{i t}$ <br> (1) <br> (2) <br> (3) |  |  | Dep. var.: $\log$ sales $_{i t}$ <br> (4) |  |
| $\Delta \log \operatorname{sales}_{i t-1}$ | $\begin{gathered} 0.277^{* * *} \\ (0.091) \end{gathered}$ | $\begin{gathered} 0.255^{* * *} \\ (0.078) \end{gathered}$ | $\begin{gathered} 0.183^{* * *} \\ (0.060) \end{gathered}$ | $\log$ sales $_{\text {it-1 }}$ | $\begin{gathered} 0.680^{* * *} \\ (0.108) \end{gathered}$ |
| $\Delta \log$ expenditure ${ }_{i t}$ | $\begin{gathered} 0.387^{* * *} \\ (0.092) \end{gathered}$ | $\begin{gathered} 0.401 * * * \\ (0.081) \end{gathered}$ | $\begin{gathered} 0.397^{* * *} \\ (0.053) \end{gathered}$ | $\log$ expenditure ${ }_{i t}$ | $\begin{gathered} 0.556^{* * *} \\ (0.172) \end{gathered}$ |
| $\Delta \log$ expenditure ${ }_{i t-1}$ | $\begin{gathered} -0.115^{*} \\ (0.065) \end{gathered}$ | $\begin{gathered} -0.096^{*} \\ (0.058) \end{gathered}$ | $\begin{aligned} & -0.073 \\ & (0.045) \end{aligned}$ | log expenditure ${ }_{i t-1}$ | $\begin{gathered} -0.274^{* * *} \\ (0.103) \end{gathered}$ |
| $\Delta \log$ labor $\left(\Delta \ell_{i t}\right)$ | $\begin{aligned} & 0.339^{* *} \\ & (0.144) \end{aligned}$ | $\begin{gathered} 0.453^{* * *} \\ (0.123) \end{gathered}$ | $\begin{gathered} 0.341^{* * *} \\ (0.070) \end{gathered}$ | $\log \operatorname{labor}\left(\ell_{i t}\right)$ | $\begin{aligned} & -0.432^{*} \\ & (0.239) \end{aligned}$ |
| $\Delta \log$ labor $\left(\Delta \ell_{i t-1}\right)$ | $\begin{gathered} 0.069 \\ (0.116) \end{gathered}$ | $\begin{gathered} 0.039 \\ (0.102) \end{gathered}$ | $\begin{gathered} 0.046 \\ (0.062) \end{gathered}$ | $\log \operatorname{labor}\left(\ell_{i t-1}\right)$ | $\begin{aligned} & 0.481^{* *} \\ & (0.209) \end{aligned}$ |
| $\triangle \log$ capital $\left(\triangle k_{i t}\right)$ | $\begin{gathered} -0.003 \\ (0.081) \end{gathered}$ | $\begin{gathered} -0.017 \\ (0.072) \end{gathered}$ | $\begin{aligned} & -0.004 \\ & (0.053) \end{aligned}$ | $\log \operatorname{capital}\left(k_{i t}\right)$ | $\begin{gathered} 0.016 \\ (0.127) \end{gathered}$ |
| $\Delta \log$ capital $\left(\Delta k_{i t-1}\right)$ | $\begin{aligned} & -0.146^{*} \\ & (0.084) \end{aligned}$ | $\begin{aligned} & -0.103 \\ & (0.070) \end{aligned}$ | $\begin{aligned} & -0.084^{*} \\ & (0.046) \end{aligned}$ | $\log \operatorname{capital}\left(k_{i t-1}\right)$ | $\begin{gathered} 0.010 \\ (0.125) \end{gathered}$ |
| N | 4,247 | 4,247 | 4,247 |  | 4,247 |
| R-squared | 0.203 | 0.217 | 0.264 |  | 0.961 |
| Lag Limit | 3 | 4 | All |  | NA |
| Number of excluded instruments | 108 | 156 | 420 |  | 56 |
| SW F-stat log sales ${ }_{i t}$ | 2.070 | 2.060 | 2.233 |  | 3.970 |
| SW F-stat log expenditure ${ }_{i t}$ | 2.034 | 2.161 | 2.473 |  | 1.845 |
| SW F-stat log expenditure ${ }_{i t-1}$ | 2.334 | 2.499 | 3.869 |  | 2.094 |
| SW F-stat log labor ( $\ell_{i t}$ ) | 1.643 | 1.504 | 1.985 |  | 1.238 |
| SW F-stat log labor ( $\ell_{i t-1}$ ) | 2.334 | 2.499 | 3.869 |  | 1.392 |
| SW F-stat log capital ( $k_{i t}$ ) | 2.120 | 2.227 | 1.970 |  | 1.339 |
| SW F-stat log capital ( $k_{i t-1}$ ) | 2.208 | 2.022 | 1.855 |  | 1.400 |
| KP LM test (underidentification) | 124.000 | 160.100 | 444.000 |  | 51.840 |
| KP LM p-value | 0.069 | 0.271 | 0.149 |  | 0.402 |
| KP Wald test (weak instruments) | 1.462 | 1.447 | 1.835 |  | 0.968 |

[^2] expenditures (Table S3), with weak-instrument diagnostic statistics. Robust standard errors in parentheses. ${ }^{*} 10 \%$ level, ${ }^{* * 5} \%$ level, ${ }^{* * *} 1 \%$ level.

Table S5. Monte Carlo Simulation: TSIV, Differences (Step 1), First Stage

|  | DGP1 |  | DGP2 |  | DGP3 |  | DGP4 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\Delta \widetilde{m}_{i t}^{S V}$ <br> (1) | $\triangle k_{i t}$ <br> (2) | $\triangle \widetilde{m}_{i t}^{S V}$ <br> (3) | $\triangle k_{i t}$ <br> (4) | $\triangle \widetilde{m}_{i t}^{S V}$ <br> (5) | $\triangle k_{i t}$ <br> (6) | $\triangle \widetilde{m}_{i t}^{S V}$ <br> (7) | $\triangle k_{i t}$ <br> (8) |
| Mean coefficient $\widetilde{m}_{i t-2}^{S V}$ | $\begin{gathered} -0.107 \\ (0.004) \end{gathered}$ | $\begin{gathered} \hline 0.020 \\ (0.005) \end{gathered}$ | $\begin{aligned} & -0.122 \\ & (0.002) \end{aligned}$ | $\begin{gathered} 0.013 \\ (0.004) \end{gathered}$ | $\begin{aligned} & -0.091 \\ & (0.002) \end{aligned}$ | $\begin{gathered} 0.028 \\ (0.004) \end{gathered}$ | $\begin{aligned} & -0.086 \\ & (0.001) \end{aligned}$ | $\begin{gathered} 0.019 \\ (0.003) \end{gathered}$ |
| Mean std. error $\widetilde{m}_{i t-2}^{S V}$ | $\begin{gathered} 0.039 \\ (0.004) \end{gathered}$ | $\begin{gathered} 0.054 \\ (0.005) \end{gathered}$ | $\begin{gathered} 0.029 \\ (0.002) \end{gathered}$ | $\begin{gathered} 0.060 \\ (0.004) \end{gathered}$ | $\begin{gathered} 0.026 \\ (0.002) \end{gathered}$ | $\begin{gathered} 0.054 \\ (0.004) \end{gathered}$ | $\begin{gathered} 0.025 \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.051 \\ (0.003) \end{gathered}$ |
| Mean coefficient $k_{i t-2}$ | $\begin{gathered} 0.065 \\ (0.003) \end{gathered}$ | $\begin{gathered} -0.031 \\ (0.004) \end{gathered}$ | $\begin{gathered} 0.038 \\ (0.001) \end{gathered}$ | $\begin{aligned} & -0.047 \\ & (0.002) \end{aligned}$ | $\begin{gathered} 0.023 \\ (0.001) \end{gathered}$ | $\begin{gathered} -0.054 \\ (0.002) \end{gathered}$ | $\begin{gathered} 0.021 \\ (0.001) \end{gathered}$ | $\begin{gathered} -0.050 \\ (0.002) \end{gathered}$ |
| Mean std. error $k_{i t-2}$ | $\begin{gathered} 0.028 \\ (0.003) \end{gathered}$ | $\begin{gathered} 0.039 \\ (0.004) \end{gathered}$ | $\begin{gathered} 0.014 \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.029 \\ (0.002) \end{gathered}$ | $\begin{gathered} 0.012 \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.026 \\ (0.002) \end{gathered}$ | $\begin{gathered} 0.012 \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.024 \\ (0.002) \end{gathered}$ |
| Mean coefficient $\triangle \widehat{\bar{w}}_{i t}^{i m p}$ | $\begin{gathered} -0.537 \\ (0.050) \end{gathered}$ | $\begin{gathered} 0.016 \\ (0.068) \end{gathered}$ | $\begin{gathered} -0.425 \\ (0.027) \end{gathered}$ | $\begin{gathered} 0.008 \\ (0.054) \end{gathered}$ | $\begin{gathered} -0.434 \\ (0.028) \end{gathered}$ | $\begin{gathered} -0.008 \\ (0.058) \end{gathered}$ | $\begin{aligned} & -0.429 \\ & (0.026) \end{aligned}$ | $\begin{gathered} 0.007 \\ (0.052) \end{gathered}$ |
| Mean std. error $\triangle \widehat{\bar{w}}_{i t}^{i m p}$ | $\begin{gathered} 0.046 \\ (0.005) \end{gathered}$ | $\begin{gathered} 0.063 \\ (0.007) \end{gathered}$ | $\begin{gathered} 0.026 \\ (0.002) \end{gathered}$ | $\begin{gathered} 0.053 \\ (0.003) \end{gathered}$ | $\begin{gathered} 0.026 \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.053 \\ (0.003) \end{gathered}$ | $\begin{gathered} 0.026 \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.053 \\ (0.003) \end{gathered}$ |
| Mean R-squared | $\begin{gathered} 0.020 \\ (0.003) \end{gathered}$ | $\begin{gathered} 0.010 \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.041 \\ (0.003) \end{gathered}$ | $\begin{gathered} 0.022 \\ (0.002) \end{gathered}$ | $\begin{gathered} 0.042 \\ (0.003) \end{gathered}$ | $\begin{gathered} 0.023 \\ (0.002) \end{gathered}$ | $\begin{gathered} 0.040 \\ (0.003) \end{gathered}$ | $\begin{gathered} 0.022 \\ (0.002) \end{gathered}$ |
| Mean F - statistic | $\begin{gathered} 90.819 \\ (16.139) \end{gathered}$ | $\begin{aligned} & 34.510 \\ & (5.882) \end{aligned}$ | $\begin{aligned} & 196.166 \\ & (29.523) \end{aligned}$ | $\begin{gathered} 85.574 \\ (19.588) \end{gathered}$ | $\begin{aligned} & 195.149 \\ & (29.442) \end{aligned}$ | $\begin{gathered} 86.546 \\ (23.672) \end{gathered}$ | $\begin{aligned} & 192.040 \\ & (28.385) \end{aligned}$ | $\begin{gathered} 89.260 \\ (19.951) \end{gathered}$ |
| Mean F - SW | $\begin{aligned} & 2038.723 \\ & (583.469) \end{aligned}$ | $\begin{gathered} 97.334 \\ (19.661) \end{gathered}$ | $\begin{aligned} & 3052.729 \\ & (340.280) \end{aligned}$ | $\begin{aligned} & 263.593 \\ & (53.826) \end{aligned}$ | $\begin{aligned} & 2904.559 \\ & (316.686) \end{aligned}$ | $\begin{aligned} & 266.689 \\ & (59.610) \end{aligned}$ | $\begin{aligned} & 2147.679 \\ & (225.227) \end{aligned}$ | $\begin{aligned} & 266.264 \\ & (49.561) \end{aligned}$ |
| Mean KP LM statistic |  |  |  |  |  |  |  |  |
| Mean KP Wald F-statistic |  |  |  |  |  |  |  |  |

Notes: Table presents the first stage of step 1 of TSIV procedure for the four DGPs in our Monte Carlo simulation. See Section 5.2 and Appendix D for details. Table reports means of statistics across 100 simulated samples for each DGP. In parentheses are standard deviations of statistics across the 100 samples. $\mathrm{N}=15,000$ for each sample. Dependent variables are indicated at the tops of columns. SW refers to Sanderson and Windmeijer (2016) and KP to Kleibergen and Paap (2006). The F-statistic is the standard F for a test that the coefficients on the excluded instruments (indicated at left) are zero. The KP statistics (LM test for under-identification and Wald F test for weak instruments) are not specific to a particular dependent variable.

Table S6. Monte Carlo Simulation: TSIV, Levels (Step 2), First Stage

|  | Dep. var.: log capital $\left(k_{i t}\right)$ |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | DGP 1 | DGP 2 | DGP 3 | DGP 4 |
| Mean coefficient $\triangle k_{i t-2}$ | $(1)$ | $(2)$ | $(3)$ | $(4)$ |
|  | 0.496 | 0.490 | 0.486 | 0.486 |
| Mean standard error $\triangle k_{i t-2}$ | $(0.046)$ | $(0.023)$ | $(0.021)$ | $(0.020)$ |
|  | 0.039 | 0.022 | 0.022 | 0.022 |
| Mean R-squared | $(0.007)$ | $(0.003)$ | $(0.002)$ | $(0.003)$ |
|  |  |  |  |  |
| Mean KP LM test | 0.009 | 0.038 | 0.037 | 0.037 |
|  | $(0.002)$ | $(0.006)$ | $(0.007)$ | $(0.006)$ |
| Mean KP Wald - F test | 37.560 | 93.910 | 94.114 | 92.324 |
|  | $(9.274)$ | $(16.577)$ | $(16.518)$ | $(15.594)$ |

Notes: Table presents the first-stage of step 2 of TSIV procedure for the four DGPs in our Monte Carlo simulation. See Section 5.2 and Appendix D for details. Table reports means of statistics across 100 simulated samples for each DGP. In parentheses are standard deviations of statistics across the 100 samples. $\mathrm{N}=15,000$ for each sample.

Table S7. Monte Carlo Simulation: TSIV, Steps $1 \& 2$, Second Stages

|  | Dep. var.: $\triangle \log$ output quantity $\left(\triangle y_{i t}\right)$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | DGP1 |  | DGP2 |  | DGP3 |  | DGP4 |  |
|  | Step 1 <br> (1) | Step 2 <br> (2) | Step 1 <br> (3) | Step 2 (4) | Step 1 (5) | Step 2 (6) | Step 1 <br> (7) | Step 2 <br> (8) |
| Materials: |  |  |  |  |  |  |  |  |
| Mean elasticity | $\begin{gathered} 0.646 \\ (0.048) \end{gathered}$ |  | $\begin{gathered} 0.655 \\ (0.066) \end{gathered}$ |  | $\begin{gathered} 0.651 \\ (0.070) \end{gathered}$ |  | $\begin{gathered} 0.657 \\ (0.059) \end{gathered}$ |  |
| Mean standard error | $\begin{gathered} 0.048 \\ (0.002) \end{gathered}$ |  | $\begin{gathered} 0.062 \\ (0.002) \end{gathered}$ |  | $\begin{gathered} 0.062 \\ (0.002) \end{gathered}$ |  | $\begin{gathered} 0.063 \\ (0.002) \end{gathered}$ |  |
| Capital: |  |  |  |  |  |  |  |  |
| Mean elasticity | $\begin{gathered} 0.253 \\ (0.037) \end{gathered}$ | $\begin{gathered} 0.253 \\ (0.036) \end{gathered}$ | $\begin{gathered} 0.248 \\ (0.033) \end{gathered}$ | $\begin{gathered} 0.248 \\ (0.033) \end{gathered}$ | $\begin{gathered} 0.249 \\ (0.033) \end{gathered}$ | $\begin{gathered} 0.250 \\ (0.034) \end{gathered}$ | $\begin{gathered} 0.246 \\ (0.030) \end{gathered}$ | $\begin{gathered} 0.246 \\ (0.030) \end{gathered}$ |
| Mean standard error | $\begin{gathered} 0.035 \\ (0.002) \end{gathered}$ | $\begin{gathered} 0.036 \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.030 \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.030 \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.030 \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.030 \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.031 \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.031 \\ (0.001) \end{gathered}$ |
| Mean R-squared | $\begin{gathered} 0.729 \\ (0.022) \end{gathered}$ | $\begin{gathered} 0.950 \\ (0.014) \end{gathered}$ | $\begin{gathered} 0.542 \\ (0.017) \end{gathered}$ | $\begin{gathered} 0.839 \\ (0.035) \end{gathered}$ | $\begin{gathered} 0.542 \\ (0.016) \end{gathered}$ | $\begin{gathered} 0.838 \\ (0.040) \end{gathered}$ | $\begin{gathered} 0.545 \\ (0.015) \end{gathered}$ | $\begin{gathered} 0.836 \\ (0.041) \end{gathered}$ |

Notes: Table presents the second stages of steps 1 and 2 of TSIV procedure for the four DGPs in our Monte Carlo simulation. See Section 5.2 and Appendix D for details. Corresponding Monte Carlo first-stage estimates are in Tables S5 and S6. Table reports means of the statistics across 100 simulated samples for each DGP. In parentheses are standard deviations of statistics across the 100 samples. $\mathrm{N}=15,000$ for each sample. The true values for the elasticities are 0.65 for materials and 0.25 for capital.


[^0]:    ${ }^{1}$ The score function corresponding to the levels-equation IV estimation is $s\left(a_{i t}, \beta_{k} ; \beta_{m}, \beta_{l}\right)=\Delta k_{i t-1}\left(\widetilde{y}_{i t}^{S V}-\beta_{m} \widetilde{m}_{i t}^{S V}-\right.$ $\left.\beta_{l} l_{i t}-\beta_{k} k_{i t}\right)$, where $a_{i t}=\left(\widetilde{y}_{i t}^{S V}, \widetilde{m}_{i t}^{S V}, l_{i t}, k_{i t}, \Delta k_{i t-1}\right)$. If $\mathbb{E}\left(\Delta k_{i t-1} \widetilde{m}_{i t}^{S V}\right)=0$ and $\mathbb{E}\left(\Delta k_{i t-1} \ell_{i t}\right)=0$ then the gradient of the score function with respect to $\beta_{m}$ and $\beta_{\ell}$ is zero and equation 12.37 of Wooldridge (2002) holds, implying that we can ignore the first step in calculating the asymptotic variance of $\widehat{\widehat{\beta}_{k}}$.

[^1]:    ${ }^{2}$ See also Proposition 2 of Kripfganz and Schwarz (2019).

[^2]:    Notes: Table reports IV estimates corresponding to differences (Columns 1-3) and levels (Column 4) equations of System GMM using sales and

