# Strict Finitism

## Nominalism

- We have seen that formalists appeal to "concrete object[s]...finite configuration[s]...of recognizable symbols [Huber-Dyson 1991, 16]." But the symbols appealed to in proof theory, like **(**, $\forall$, **x**, **)**, and ~, are *symbol types*, knowledge of which seems hardly more tractable than knowledge of ordinary mathematical entities, like numbers and tensors.

  - *Recall*: Tokens of types do not generally resemble the types they are tokens of. On the one hand, it is hard to even understand what it could mean to say that a non-spatiotemporal entity, like *The Letter A*, has a shape. On the other, tokens of that letter can come in all manner of fonts, can be incorrectly written, and so forth.

  - There is also the fact that types have occurrences, as distinct from tokens. We say that each occurrence of a variable, x, is bound by a quantifier, $\forall$x, in the formula **(**$\forall$**x)(Fx v ~Fx)**. These occurrences cannot be tokens, because they 'ihere' in a formula type! What *are* occurrences, and how do they relate to types and tokens?

- Quine and Goodman are among the very few who recognize the problem, and respond to it. They concede that it is hopeless to translate all of mathematics into claims about concrete objects. They propose only to develop a *proof theory* that is nominalist — i.e., that replaces talk of mathematical (or otherwise abstract) entities with talk of concrete ones. The claim that, e.g., Fermat's Last Theorem is true turns into the claim that it is *provable*. But *unlike* Hilbert and even his 'ultrafinitist' followers, they go on to argue that talk of provability, which looks tantamount to arithmetic, can be made nominalistic.

  - *Recall*: One can 'cheaply' avoid reference to mathematical entities by simply trading ontology for ideology. Take a mathematical theorem, *S*, to be shorthand for [M]*S*, to be read *it is mathematically necessary that S*, where [M] is a logical primitive, like negation. But whatever puzzles plague knowledge of mathematics would just follow this trade. Now the question is: how do we know [M]$AX_{ZFC}$? Similar problems plague appeal to primitive *logical* modalities, as in Field [1989].

- Besides avoiding the epistemological problems that plague 'platonism' about mathematics (or proof theory), nominalism is attractive from a metaphysical, indeed physical, point of view. Is it not an open empirical question whether the universe is finite? If it is, then we should not be *a priori* committed to theories, like ordinary arithmetic, that have only infinite models! For example, quantum theories of gravity, incorporating discrete spacetime, should at least be *consistent* with the claim that the universe is finite. But even if a theory itself admits of finite models, its metatheory, *as ordinarily understood*, will not. It will be bi-interpretable with arithmetic. So, an authentic physical theory according to which the universe might be finite *needs* a surrogate for the theory of syntax, as well as set theory, like Quine and Goodman's.

Hermeneutic vs. Revolutionary

- Burgess [1983] distinguishes nominalists that purport to conceptually analyze what practicing mathematicians actually assert, from nominalists who purport to reveal what we (philosophers) should believe – whatever mathematicians happen to assert. Burgess calls the former hermeneutic nominalists, and the latter revolutionary nominalists.
    - *Note*: Sometimes Burgess makes it sound as if the choice is between analyzing what mathematicians assert and telling them what they should assert. But the latter alternative is silly (as he notes). *Insofar as mathematicians wish to take a philosophical position on ontology*, nominalists *might* tell them what to *believe*. However, this has no obvious relevance to what they should *assert*, much less *do*.

- Alston [1958] points out that hermeneutic nominalists face a dilemma. Suppose that they claim that, e.g., 'the number of apples is 3' *really means* that $(\exists x)(\exists y)(\exists z)(x \neq y \;\&\; y \neq z \;\&\; x \neq z \;\&\; (w)(Aw \longleftrightarrow [w = x \vee w = y \vee w = z])$. Then, equally, $(\exists x)(\exists y)(\exists z)(x \neq y \;\&\; y \neq z \;\&\; x \neq z \;\&\; (w)(Aw \longleftrightarrow [w = x \vee w = y \vee w = z])$ *really means* that the number of apples is 3! We have not made any progress. The latter appears to quantify over numbers, while the former appears not to. But why should we take the former as ontologically perspicuous, and the latter as misleading, rather than the other way around?

- Even if there were a principled answer to this question, why defer to what mathematicians happen to assert? Mathematicians do not, *qua* mathematicians, purport to speak to ontology. They may be ignorant of relevant arguments, and, as Kreisel [1967] notes, may not even know the axioms of standard foundational systems, like *ZFC*. (This is no indictment of mathematicians (contra Frege [1884])! It is the banality that one can be an expert engineer while knowing little of the philosophical foundations of physics.)

- Quine and Goodman are revolutionary nominalists in the above sense. They make no claims about natural language semantics, much less about the practice of mathematicians. Their claim is that we need not believe in abstract objects in order to explain the fact that, e.g., Euclid's Theorem is – in some relevant sense – 'right' while its negation 'wrong'.

- *Quine & Goodman*: "The gains which seem to have accrued to natural science from the use of mathematical formulas do not imply that those formulas are true statements. No one, not even the hardiest pragmatist, is likely to regard the beads of an abacus as true; and our position is that the formulas of platonistic mathematics are, like the beads of an abacus, convenient computational aids which need involve no question of truth (122)."

- "[T]he formula which is the full expansion in our object language of '(n)(n + n = 2n)' will contain variables calling for abstract entities as values….But, taking that formula as a string of marks, we can determine whether it is indeed a proper formula of our object language, and what consequence-relations it has to other formulas. We can thus handle much of classical logic and mathematics without…granting the truth of…the formulas."

<u>Getting Started</u>

- Various locutions of basic set theory are nominalistically paraphrasable in obvious ways.
  - <u>Example 1</u>: $A \subseteq B$ is paraphrasable as $(x)(A \rightarrow B)$.
  - <u>Example 2</u>: $A \subseteq C \neq A$ is paraphrasable as $(\exists x)(\sim Ax)$.
  - <u>Example 3</u>: As above, Card$(A) = 3$ is paraphrasable as $(\exists x)(\exists y)(\exists z)(x \neq y \ \& \ y \neq z \ \& \ x \neq z \ \& \ (w)(Aw \longleftrightarrow [w = x \vee w = y \vee w = z])$. (In English: there are distinct objects x, y, and z such that anything is an *A* if and only if it is x or y or z.)
    - In general, the claim that *there are,* or *there are not, exactly n*, *at least n*, or *at most n*, concrete *A*s (for finite *n*) is nominalistically paraphrasable using this technique *just in case the resulting expression is not too long*.

- Other locutions are more recalcitrant.
  - <u>Problem 1</u>: How could we paraphrase *b is ancestor of c* without quantifying over *sets*, as in $b \neq c \ \& \ (x)\{c \in x \ \& \ (y)(z)(z \in x \ \& \ \mathit{Parent}\, yz \rightarrow y \in x) \rightarrow b \in x\}$? (In English: *b* is distinct from *c* and, for any *set* x, if *c* is a member of x and all parents of members of x are members of x then b is a member of x as well.)
    - Quine and Goodman show that *in this case* a paraphrase is actually available, in terms of *mereological* predicates. In general, such a paraphrase is available when 'every individual in the field of the predicate

has some part that has no part in common with any other individual in that field'. But it is not available in other cases. This is a significant problem!

- Problem 2: How could we paraphrase *There are more human cells than humans*? A long disjunction using the technique above would presuppose the availability of too many symbols. So, again, Quine and Goodman resort to mereology. They say that *Every individual that contains a 'bit' of each human cell is bigger than some individual that contains a bit of each human*. (Details on this idea below.)

  - *Question*: Would this work for *There are more quarks than electrons*?

- Problem 3: How could we paraphrase *There are $10^{1000}$ objects in the universe* (assuming that there are not so many concrete ones)? Quine and Goodman suggest (in a footnote) that we could take 'has $10^{1000}$ objects as parts' as a *primitive predicate* and apply it to the universe as a whole concrete individual. But how is the nominalist supposed to *learn the meaning* of such a predicate?

- *Objective (Nominalist Syntax):* "Since…we have not as yet discovered how to translate all statements that we are unwilling to discard as meaningless, we describe in the following sections a course that enables us…without any retreat from our position…to talk *about* certain statements without being able to *translate* them [my emphases]."

Vocabulary

- Again, unlike Hilbert, Quine and Goodman are sensitive to the fact that *syntax, ordinarily construed, is no more concrete than mathematics.* So, an authentic formalist, who countenances only 'concrete marks', must invent a *nominalistic surrogate for syntax*.

- Quine and Goodman help themselves to the following primitive (undefined) predicates:

  - ***Vee* x** =: x is a *v* (a *v*-shaped inscription, serving as a variable)
  - ***Ac* x** =: x is a *'* (an accent, following a variable, as in *v'* or, as we will see, *v'''*)
  - ***LPar* x** := x is a left parenthesis
  - ***RPar* x** =: x is a right parenthesis
  - ***Str* x** := x is a stroke (a | shaped inscription, with the meaning of the Sheffer stroke, i.e. *not both*)
  - ***Ep* x** =: x is a ∈ (x is an epsilon symbol)
  - ***C* xyz** =: the instription, x, consists of y followed by z (x is the concatenation of y and z)

- ***Part* xy** =: x is a part of y (x is contained entirely within y in a sense according to which x is contained entirely within x itself)
- ***Bgr* xy** =: x is spatially larger than y

    - *Note*: Strictly, ***LPar* x** and ***RPar* x** should be two-place predicates, since the symbol tokens **(** and **)** are intrinsically alike. But this could be avoided by simply using nonstandard symbols for parentheses. So, we ignore this.

    - *Note*: The concatenation relation is not straightforward! ***C* xyz** means that 'y and z are composed of whole characters of the language, in normal orientation…and contain neither split-off fragments of characters nor anything extraneous….The characters comprising y and z may be irregularly spaced; furthermore the inscription x will be considered to consist of y followed by z no matter what the spatial interval between y and z, provided that x contains no characters that occurs in that interval.'

    - *Question*: What can 'normal orientation' mean for Quine and Goodman?

Definitions

- Using the above primitive predicates, Quine and Goodman define the following.

    - **[D1] *C* xyzw** =: (∃t)(*C* xyt & *C* tzw)
    - **[D2] *C* xyzwu** =: (∃t)(*C* xyt & *C* tzwu)
    - **[D3] *C* xyzwus** =: (∃t)(*C* xyt & *C* tzwus)

- **[D4] *Char* x** =: *Vee* x v *Ac* x v *LPar* x v *RPar* x v *Str* x v *Ep* x (x is a character)
- **[D5] *Insc* x** =: *Char* x v (∃y)(∃z)*C* zyz (x is an inscription, i.e. a character or the concatenation of some characters)
- **[D6] *InitSeg* xy** =: *Insc* x & x = y v (∃z)*C* yxz (x is the initial segment of y)
- **[D7] *FinSeg* xy** =: *Insc* x & x = y v (∃z)*C* yzx (x is the final segment of y)
- **[D8] *Seg* xy** =: (∃z)(*InitSeg* xz & *FinSeg* zy) (x is a segment of y)
- **[D9] *Bit*** x =: (y)(*Char* y → ~*Bgr* xy) & (∃z)(*Char* z & ~*Bgr* zx) (x is exactly as big as every smallest character, including itself)
- **[D10] *Lngr* xy** =: *Insc* x & *Insc* y & (z)((w)[*Char* w & *Seg* wx → (∃u)(*Bit* u & *Part* uw & *Part* uz)] → (∃t)[(r)(*Char* r & *Seg* ry → (∃s)(*Bit* s & *Part* sr & *Part* st)) & *Bgr* zt]) (x contains more characters than y)
- **[D11] *EqLng* xy** =: *Insc* x & *Insc* y & ~*Lngr* xy & ~*Lngr* yx (x is equally as long as y)

- **[D12]** ***Like*** **xy** =: *EqLng* xy & (z)(w)[*EqLng* zw & *InitSeg* zx & *InitSeg* wy → (∃s)(∃t)(*FinSeg* sz & *FinSeg* tw : *Vee* s & *Vee* t v *Ac* s & *Ac* t v *LPar* s & *LPar* t v *RPar* t v *Str* s & *Str* t v *Ep* s & *Ep* t)] ( x and y are alike inscriptions – i.e., if they are characters, then they are both vees, accents, etc., and if they are inscriptions, then they are equally long and any of their initial segments end in like characters)
- **[D13]** ***AcString*** **x** =: *Insc* x & (z)(*Seg* zx & *Char* z → *Ac* z) (x is a string of accents ')
- **[D14]** ***Vbl*** **x** =: *Vee* x v (∃y)(∃z)(*Vee* y & *AcString* z & *C* xyz) (x is a variable)
- **[D15]** ***QfrString*** **x** =: (∃y)(∃z)(*LPar* y & *RPar* z & (∃w)*C* xywz & (s)(t)(u)(k)[*LPar* t & *RPar* k & *Seg* sx → ~*Cstk* : *Cstuk* → *Vbl* u v (∃p)(∃q)(∃r)(*RPar* q & *LPar* r & *C* pqr & *Seg* pu)]) (x is a string of quantifiers)
- **[D16]** ***Qfn*** **xy** =: (∃z)(*QfrString* z & *C* xzy) (x is a quantification of y)
- **[D17]** ***AtFmla*** **x** =: (∃w)(∃y)(∃z)(*Vbl* w & *Ep* y & *Vbl* z & *C* xwyz) (x is an atomic formula if it consists of two variables with an epsilon symbol in between)
- **[D18]** ***EqPar*** **x** =: (u)(*LPar* u v *RPar* u → ~*Seg* ux) v (∃y)(∃z)[*EqLng* yz & (w)(*Char* w → : *LPar* w & *Seg* wz ←→ *Seg* wy : *RPar* w & *Seg* wx ←→ *Seg* wz)] (x contains exactly as many left as right parentheses)
- **[D19]** ***AD*** **xyz** =: *EqPar* y & *EqPar* z & (r)(s)(*C* xrs → ~*EqPar* r) & (∃t)(∃u)(∃w)(*LPar* t & *Str* u & *RPar* w & *C* xtyuzw) (x is the alternative denial, or Sheffer stroke, of y and z, so that x is y followed by a '|' followed by z).
    - *Recall*: (P | Q) =: ~(P & Q)
- **[D20]** ***QuasiFmla*** **x** =: (∃y)(x = y v *Qfn* xy : *AtFmla* y v (∃w)(∃z)*AD* ywz) (x is a quasi-formula, i.e., an atomic formula, an alternative denial [not necessarily of formulas!], or a quantification of an atomic formula or alternative denial)
- **[D21]** ***Fmla x*** =: *QuasiFmla* x & (w)(y)(z)(*AD* wyz & *Seg* wx → *QuasiFmla* y & *QuasiFmla* z) (a formula is a quasi-formula such that each of its alternative denials is an alternative denial of a quasi-formula)

    - *Remember*: Nominalists like Quine and Goodman *cannot* define ***Fmla x*** *a la* Frege as the formula that x belongs to every set that contains all atomic formulas and quantifications and alternative denials of its members!

Axioms and Rules

- Quine's and Goodman's (incomplete) proof system may be formulated in terms of two groups of logical axioms, and a group of axioms governing the epsilon (membership) sign, corresponding to mathematics proper. (It helps to note that **| ~** is equivalent to **→.** In other words, $(P \mid {\sim}Q) =: {\sim}(P \;\&\; {\sim}Q) \Longleftrightarrow ({\sim}P \vee {\sim\sim}Q) \Longleftrightarrow (P \rightarrow {\sim\sim}Q) \Longleftrightarrow (P \rightarrow Q)$.

- **Alternative Denial**: Every axiom of the form (like letters replaced by like formulas):
    - $((P \mid (Q \mid R)) \mid ((S \mid {\sim}S) \mid ((S \mid Q) \mid {\sim}(P \mid S))))$
- **Quantification**: Every axiom of the form:
    - **(1)** $((v)(P \mid {\sim}Q) \mid {\sim}((v)P \mid {\sim}(v)Q))$
    - **(2)** $(R \mid {\sim}(v)R)$ (where '$v$' is not free in '$R$')
        - *Remember*: What **(2)** officially means is that the formulas replacing the 'R's contain no free variables <u>like</u> the variables replacing the vees.
    - **(3)** $((v)P \mid {\sim}S)$ ('$S$' is the result of substituting a variable for '$v$' in '$P$')
- **Set Theory**: All axioms *like* those on Hailperin's list ['A Set of Axioms for Logic', *Journal of Symbolic Logic*, Vol. 9 (1944)], once transcribed into primitive notation.
- The following two rules of inference:
    - **(1)** From a formula, and the result of putting a formula *like* it for '$P$' and any formula for '$Q$' and '$R$' in '$(P \mid (Q \mid R))$', infer any formula like the one which was put for '$Q$'.
    - **(2)** Infer any quantification of a formula from that formula.

<u>Final Definitions</u>

- The final string of definitions lead up to that of theoremhood.
- **[D22]** ***D*** **xy** =: (∃z)(*Like* yz & *AD* xyz) (x is a <u>denial</u> of y, i.e., x is the <u>alternative denial</u> of y and some other inscription just like y, since $\sim(P \;\&\; P) \Longleftrightarrow (\sim P \vee \sim P) \Longleftrightarrow \sim P$).
- **[D23]** ***AAD*** **x** = (∃f)(∃g)(∃h)(∃i)(∃j)(∃k) (∃l)(∃m)(∃n)(∃p)(∃q)(∃r) (∃s)(∃t)(∃u)(∃w)(∃y)(∃z)(*Fmla* f & *Fmla* g & *Fmla* h & *Fmla* i & *Like* ki & *Like* lg & *Like* mf & *Like* ni & *AD* pgh & *AD* qfp & *D* ri & *AD* sir & *AD* tkl & *AD* umn & *D* wu & *AD* ytw & *AD* zsy & *AD* xqz) (x is an <u>axiom of alternative denial</u>, i.e., 'every

axiom of alternative denial is an alternative denial of two formulas; one of these…is an alternative denial of formulas of which one is an alternative denial of formulas; the other of the two main components is an alternative denial of formulas of which one is an alternative denial of a formula with a formula like the denial of that formula, while the other is…etc., etc.').

- **Note:** ***AQ1 x***, i.e., x is an axiom of quantification of kind (1), can be formulated similarly.
- **[D24]** *Free* xy =: *Vbl* x & *Seg* xy & (z)(w)(*Ac* w & *C* zxw & → ~*Seg* zy) & (q)(r)(s)(t)(u)(*LPar* q & *Like* rx & *RPar* s & *Fmla* t & *C* uqrst & *Seg* uy & → ~*Seg* xu) (x is a free variable in an inscription y if 'x is a segment of y not followed by any additional accents in y, and…x is not a segment of any segment of y that consists of a formula preceded by a quantifier consisting of a variable *like* x…in parentheses [my emphasis])
- **Note**: As with ***AQ1*** **x**, ***AQ2*** **x** is now tedious, but unproblematic.
- **[D25]** ***Subst*** **wxyz** =: *Fmla* w & *Fmla* z & (∃t)(∃u)[*Like* tu & (s)[*Char* s → : (r)(*Like* ry & *Free* rz → ~*Seg* sr) & *Seg* sz ←→ *Seg* su : (r)(*Like* rx & *Free* rw → ~*Seg* sr) → *Seg* sw ←→ *Seg* st] & (s)(r)(*Like* rz & *Free* rw & *Seg* sr & *Seg* st → *Seg* rt)] (w is the substitution of variable x for variable y in formula z – that is, the formula w is *like* the formula z but has free variables *like* x wherever z contains free variables *like* y. As they put it: 'what remains when all free variables like y are omitted from the formula z is like what remains when some free variables like x are omitted from the formula w'.)
- **Note**: As with ***AQ1*** **x** and ***AQ2*** **x**, ***AQ3*** **x** is now tractable, as are the axioms of **Set Theory**, **AM**.

- **[D26]** ***Axiom x*** =: *AAD* x v *AQ1* x v *AQ2* x v *AQ3* x v *AM* x (x is an axiom when x is an Alternative Denial, Quantification or Set Theory axiom)

- **[D27]** ***IC*** **xyz** =: (∃r)(∃s)(∃t)(∃u)(∃w)(*Like* rx & *Like* sy & *Like* tz : *AD* urw & *AD* stu v *AD* tsu v *Qfn* rs) (an inscription x is an immediate consequence of inscriptions y and z when x follows from y and z by *one* application of rule of inference (1), or from y by rule of inference (2))

- **[D28]** ***Line*** **xy** =: (z)(*Fmla* z & *Part* xz & *Part* zy ←→ z = x) (x is a line of y when x is a formula that is part of y but not part of a subformula of y)

- **[D29]** ***Proof*** **x** =: (y)[(∃z)(*Line* zx & Part zy) & (w)(*Axiom* w & *Line* wx → ~*Part* wy) → (∃s)(∃t)(∃u)(*Line* sx & *Part* sy & *Line* tz & ~*Part* ty & *Line* ux & ~*Part* uy & *IC* stu)] (x is a proof when it is a line with the property that 'if any individual y contains as parts some lines of a proof x but none of which are axioms, then some line of x which lies in y must be an immediate consequence of lines of x which lie outside of y (120)')

- *Note:* This allows that proofs contain 'debris' besides formulas or even inscriptions. It also stipulates no order among lines, allowing that they may be spread across spacetime.

- **[D30]** ***Thm*** **x** =: (∃y)(∃z)(*Proof* y & *Line* zy & *Like* xz) (x is a theorem)

Problems with Nominalism

- *Problem 1*: How can nominalists deal with *unbounded* claims – including, crucially, consistency claims? Such claims say that for *all* formulas, x, such that ***Thm*** **x**, x is not a contradiction. Couldn't there be such an x, but only one with a few more lines than there are in the universe? Quine and Goodman suggest not. Any 'proof' that is too long to fit in the universe does not exist. They, thus, equate *concrete* provability with provability.

    - *Quine and Goodman*: "[S]ome formulas may still fail to qualify as theorems solely because no inscription exists anywhere at any time to stand as a needed intermediate line in an otherwise valid proof….But [if] we…construe inscriptions as all appropriately shaped portions of matter…[t]hen the only syntactical descriptions that will fail to have actual inscriptions answering to them will be those that describe inscriptions too long to fit into the whole spatio-temporally extended universe (121)."

    - *Note*: Contra Weir [1998], the same problem, among others, plagues his *non-cognitivist formalism* insofar as this involves cognitivism about metamathematics. According to this view, a mathematical sentence is true just in case it 'is derivable using meaning-constitutive rules implicit in the utterer's practice.' But the claim that, say, **0 = 1** is *not* so derivable is equivalent to a $\Pi_1$ arithmetic claim that is consistent to deny if the theory is consistent – and also, what seems necessary, recursively axiomatizable.

- *Problem 2*: 'There are countless sentences with this property: concrete tokens of them exist but no concrete proof or refutation actually exists, none that a human could manipulate as a meaningful utterance anyway. (Cf. Boolos, 1987.) Formalists of Goodman and Quine's persuasion seem forced to the conclusion that sentences like [*Is 2^2^2^2^2^2^2^2 + 1 prime*?], sentences which are decidable in the usual formal sense, are neither true nor false, since neither (concretely) provable nor refutable [Weir 2011].'

    - *Upshot*: Even if the idea that there are 'theorems' that lack truth-values because their only proofs are too long is tolerable, the idea that there are simple written down and conjectured theorems with this quality seems too much to swallow!

- *Problem 3*: Quine and Goodman cannot assume induction, or recursion (107). They accordingly deny understanding of the ancestral relation of, , e.g., a formula-building operation or of the successor operation. However, it is not obvious how to understand their own arguments without it. Consider their discussion of *formulas* of their language.

    - *Quine & Goodman*: 'By requiring also the next more complex alternative denials in x to be alternative denials of quasi-formulas, the definition guarantees these also will be formulas in the intuitively intended sense; and so on, to x itself (116)'.

- *Weir*: 'Goodman and Quine are trying to work their way up through an arbitrary formula showing that their definition will ensure that each larger component is a formula. It is not clear how we can have a guarantee of this for arbitrary x, without something like <u>induction over formula complexity</u>; but this is not available as formulas are not generated in the usual inductive set-theoretic fashion [2011].' So, what do Quine and Goodman, who lack recourse to the set-theoretic notion of <u>transitive closure</u>, mean by 'and so on'?

- *Problem 4*: The two final problems concern the presuppositions of Quine's and Goodman's project. The first is that there is a useful <u>abstract/concrete distinction</u>. But what does this distinction really come to? It cannot be that an object is abstract if it lacks spacetime location, since the whole point of prominent theories of quantum gravity is that spacetime emerges from non-spatiotemporal building blocks.[1] Nor can it be that abstract entities are causally inert, since, e.g., had arithmetic been (feasibly) inconsistent, a computer checking for would have said so. Maybe the worry is that if physical facts depended on mathematical ones, then this would involve objects "acting at a distance" in Newton's sense? The problem with this is that <u>Bell's Theorem</u> is widely taken to establish that *any* formulation of quantum mechanics -- and, hence, of physics generally -- must be non-local (as the Copenhagen, de Broglie-Bohm, and GRW formulations are).[2]

- *Problem 5*: The second presupposition is that <u>there may be only finitely-many things</u>. They begin, "Not only is our own experience finite, but there is no general agreement among physicists that there are more than finitely many physical objects in all of space-time (106)." So, "If in fact the concrete world is finite, acceptance of any theory that presupposes infinity would require us to assume that in addition to the concrete objects…there are also abstract entities (106)." The problem is to say what this means <u>in terms that Quine and Goodman accept</u>. Depending on how many (concrete) lines there

[1] Also, as I said last time, particles cannot be assigned trajectories through spacetime even in non-relativistic quantum mechanics. Moreover, in quantum field theory, there is not even a position operator for photons!
[2] This is not beyond dispute because *Bell's Theorem* assumes that measurements have a unique outcome (contra Everett's interpretation), that there is not a global experimental conspiracy (contra 'superdeterminism'), and that that measurements do not affect the prior states of the particles that are measured (contra 'retrocausality').

are in, they could say that there are exactly *1* or *2* or *3* or… *n* things, for a fixed finite *n*. But that is not to say that there are exactly *1* or *2* or *3* or… *n* things for *some* n $\in$ *N*. If they said that the universe was finite using the Dedekind definition, then they would end up saying that there is no bijective class function between it and one of its proper subsets.

- *Question*: Could they express what they need to using the idea of a *class* function, the kind at issue when we say that there is a bijective function (formula) mapping *On* onto *V*?