Quantum Logic

- That there are alternatives to so-called **classical logic** is, in one sense, entirely uncontroversial. In particular, there are mathematical constructions called 'logics' that at least appear to disagree about the what follows from what, what is a tautology, and so on.
  - *Example 1 (Strong Kleene, K_3):* In this logic, one lets statements take on a third truth-value, commonly called **indeterminate**, i. For example, if P is i, then so is ~P, and (P→Q) is i if P or Q or both is. There are no tautologies in this logic, but there are valid inference rules, like *modus ponens*. The Deduction Theorem fails.
  - *Example 2 (Łukasiewicz, Ł3)*: This logic is just like Strong Kleene, except that (P→Q) only gets truth-value, i if P is i and Q is F, or if P is T and Q is i. This has tautologies, like (P→P) or (P & Q)→P, and validates *modus ponens*. But the Law of the Excluded Middle fails ((P v ~P) is not a tautology). This is true of *K_3* too.
  - *Example 3 (Logic of Paradox, LP)*: This is just *K_3* but with i as a **designated value** -- i.e., a value that valid inferences preserve. Value i is now taken to mean **both true and false** (if you are both true and false then at least you are true!). This logic again has tautologies and the Law of the Excluded Middle is valid. However, both *reductio ad absurdum* and *modus ponens* are invalid inferences.
  - *Example 4 (First-Degree Entailment→, FDE→)*: *K_3* is an example of a **paracomplete** logic (i.e., the Law of the Excluded Middle is invalid), while LP is an example of a **paraconsistent** one (i.e., the *reductio ad absurdum* is invalid). FDE is both paracomplete and paraconsistent. However, when supplemented with an appropriate conditional, it can still manage to validate *modus ponens*.
- Logics only disagree if they are all-purpose logics, i.e. 'true logics of the world'. (What in the world this could mean is something to which we return!) Dummett understood intuitionistic logic this way, and Putnam [1968] seems to so understand **quantum logic**.
  - *Putnam*: "It makes as much sense to speak of 'physical logic' as of 'physical geometry'. We live in a world with a non-classical logic….Quantum mechanics itself explains the *approximate* validity of *classical* logic...just as non-Euclidean geometry explains the *approximate* validity of *Euclidean* geometry..."

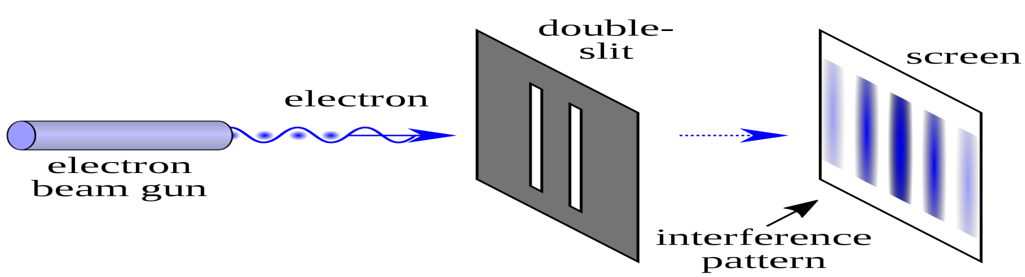
- *Note*: Disagreements between logics are technically **meta-logical**. They concern what is **valid**, **what follows from what**, and so on, not what (non-meta-logical) claims are **true**. But such disagreements translate into disagreements over truths insofar as they **license different inferences**. An LP advocate will not infer in accord with *disjunctive syllogism*.
    - *Question*: Is this actually a reasonable policy? Scenarios in which the premises of an argument have a designated value but the conclusion fails to may be 'remote'. (We already violate the letter of this policy when inferring that something is colored from the premise that it is red.) Why not infer Q from T whenever Q is true in all physically possible, or physically reasonable, worlds where P is true?
- Arguments for one logic over another often appeal to whether "the implication relation so defined agrees with the pre-theoretic notion of implication between statements" [Zach Forthcoming, 1]. But assuming that the question of what logic is the 'true logic of the world' makes sense, it is hard to see how this could be a reliable method of inquiry. Presumably Euclidean geometry more closely 'agrees with the pre-theoretic' notion of line. But whether General Relativity is true is not a matter of natural language semantics!
- More serious arguments for new logics are analogous to arguments for new physical laws. Just as we might adopt new physical laws because they systematize and explain diverse and recalcitrant data, we might adopt new logical principles for this reason.
    - *Quine*: "[N]o statement is immune to revision. Revision even of the logical law[s]...has been proposed as a means of simplifying quantum mechanics; and what difference is there in principle between such a shift and the shift whereby Kepler superseded Ptolemy, or Einstein Newton, or Darwin Aristotle?"
- Although Quine was the great champion of **epistemological holism**, and continued to regard even pure mathematics as an empirical science until the end, he appeared to abandon the view that logic admits of revision, empirical or otherwise, late in his life.
    - *Quine*: "[W]hat if someone were to reject the law of non-contradication and so accept an occasional sentence and its negation both as true?....Perhaps...we can so rig our new logic that it will isolate its contradictions and contain them…. [B]ut… the notion ceased to be recognizable as negation when they took to regarding some conjunctions of the form 'P & ~P' as true, and stopped regarding such

sentences as implying all of the others. Here, evidently, is the deviant logician's predicament: when he tries to deny the doctrine he only changes the subject."

- *Note*: This is a very strange objection coming from Quine! Quine was the deepest critic of the (widely held) view that there is a useful distinction between change in subject and change in theory about a subject, as this would afford an **analytic/synthetic distinction**.
    - *Question*: What is wrong with the following reasoning? "In order to rationally change our logic, we would have to use logic. But if we use our current logic, then we will conclude that no change is warranted. If we use a new logic, then our conclusions will be baseless. So, rational change in logic is impossible."
- Putnam responds to this objection that "[o]nly if…[the **Distributive Law** - see below] is 'part of the meaning' of 'or' and/or 'and' (which and how does one decide?) can it be maintained that quantum mechanics involves a 'change in meaning' of...the connectives."
- However, the real response to the argument, to which Putnam alludes first, is: who cares? Maybe 'line' as it occurs in General Relativity does not mean *line*. That at most tells us that the line-like entities postulated by the theory are not lines. It does not tell us that the theory is false! Similarly, maybe 'quantum disjunction' is not disjunction. If Putnam is right about the analogy between logic and physical geometry (something that we will be discussing shortly), then this argument just shows that there are no disjunction facts.

The Distributive Law

- The characteristic difference between quantum logic and classical logic is that the following principle is valid (a tautology) in classical logic, but invalid in quantum logic.
    - **Distributive Law**: P & (Q v R) ⟵⟶ [(P & Q) v (P & R)]
      [Generally: P_1 & (R_1 v R_2 v… R_k) = (P_1 & R_1) v (P_1 & R_2) v…(P_1 & R_k)]
- Birkhoff and von Neumann gave an early argument against the left-to-right direction based on the so-called **wave-particle duality** of quanta, like electrons and photons.



- Let P = that we observe the experiment without observing which slit the particle went through, Q = that we observe the particle pass through the left slit, and R = that we observe the particle pass through the right slit. Then, Birkhoff and von Neumann argue: $P \& (Q \lor R) = P \& \top = P \neq (P \& Q) \lor (P \& R) = \bot \lor \bot = \bot$.
  - *Objection (Popper)*: Distinguish $(Q \lor R)$ from $(Q \lor {\sim}Q) = \top$. Since ~Q is classically consistent with P, the scenario satisfies the Distributive Law! $P \& (Q \lor {\sim}Q) \longleftrightarrow P \& \top \longleftrightarrow P \longleftrightarrow \bot \lor P \longleftrightarrow (P \& Q) \lor (P \& {\sim}Q)$.
- Putnam's argument is more involved. It is based on a new (sketch of a) semantics..
- *Note*: Putnam's later view is different. It resembles Russell's approach to the paradoxes in ruling out certain sentences syntactically as ill-formed (as with $x \in x$ in type theory). According to it "we do not *allow* the conjunction of certain statements in quantum logic".
- Recall that quantum states are represented as unit vectors (or rays) in a Hilbert space. So, the question arises how to represent the disjunction of two claims of the form **m(s) = r**, meaning that magnitude, **m**, has value **r** in system, **s**. Two options suggest themselves.
  - (P v Q) = the **union** of the spaces corresponding to P and Q
  - (P v Q) = the **span** of the spaces corresponding to P and Q (i.e., all vectors that can be got by adding a vector from the first space to a vector from the second).
- Putnam sides with the span interpretation. This means that (P v Q) can be true although neither P nor Q is (since a vector can lie in the space spanned by those corresponding to P and Q while being parallel to neither). This implies the failure of the Distributive Law because a subspace corresponding to a proposition P may lie in the span of the spaces corresponding to two others, (Q v R), while lying in the subspace of neither individually.
  - *Note*: Putnam retains the idea that (P & Q) represent the intersection of the spaces corresponding to P and Q (which is empty when P and Q are orthogonal), and he understands ~P to be the orthocomplement of the space corresponding to P.
- Quantum disjunction differs from classical disjunction: if $V_1, V_2 ... V_n$ span the state space of the system, then their disjunction is a quantum, but not classical, tautology. (The state vector need not be parallel to any of the V_is, but must lie somewhere in the state space.)
- Putnam's example of the failure of the Distributive Law corresponds to **Heisenberg's Uncertainty Principle**. Let S_i and T_i be (nondegenerate) eigenstates of position and momentum, respectively. Then: $(S_1 \lor S_2 \lor \ldots S_j) = (T_1 \lor T_2 \lor \ldots T_k)$ = span of spaces S_1,

$S_2,\ldots S_j$ = span of spaces $T_1, T_2,\ldots T_k$ = entire space = ⊤. But (S_l & T_m) = intersection of spaces S_l and T_m = ∅ = ⊥, for all values of l and m. Consequently, we have that:

  - S_i & (T_1 v T_2 v… T_k) = $S_i \neq$ (S_i & T_1) v (S_i & T_2) v… = ⊥ v ⊥… = ⊥.

- *Note*: Like Birkhoff and von Neumann, Putnam is rejecting the **left-to-right direction**.
- *Upshot*: While classical logic is isomorphic to a Boolean Algebra -- i.e., a set equipped with two operations, satisfying *Associativity*, *Commutativity*, *Distributivity*, *Identity*, and *Complementation* -- Putnam's logic is not. His system forms a so-called 'ortholattice'.

Boolean Objects	**Classical Logical Correlates**
Union	Disjunction, v
Intersection	Conjunction, &
Complementation	Negatin, ~
Identity	Tautologies, ⊤ and Contradictions, ⊥
Members of the algebra	Equivalence classes of statements

Logic and Fact

- What problem is quantum logic trying to solve? Putnam suggests that if we replace classical with quantum logic then we can avoid appeal to hidden variables and collapse. Accordingly, quantum logic might let us hold onto locality in the face of Bell's Theorem!
- *Putnam*: "If one does not believe (1) that the laws of quantum mechanics are false, nor (2) that there are 'hidden variables', nor (3) that the...'cut' between the observer and the observed exists; one perfectly possible option is this: to *deny* the properties classically attributed to 'and' and 'or'....[I]t is *more likely that classical logic is wrong* than that there are hidden variables, or "cuts between the observer and the system", etc."
- *Putnam*: "*[A]ll* so-called 'anomalies' in quantum mechanics comes down to the *non-standardness of the logic*." With quantum logic "every single anomaly vanishes."
- *Problem (Maudlin)*: It seems pointless to say only that, e.g., the electron had a position before it was measured, given that it did not have the position that was revealed by

measurement. If the state vector is complete, then it (typically) did not have that position. To suppose otherwise would require giving up that, e.g., (S_i & T_2) is a logical falsehood!

- *Note*: We could introduce indeterminate determinables (like being of momentum T_1 or T_2 or… T_k, but failing to be any particular T_i). But then the Measurement Problem could be rephrased: why does measuring an indeterminate determinable result in a determinate?
- *Deeper Problem:* Unlike physical geometry, it is hard to understand what could be meant by the claim that quantum logic is true to the exclusion of classical logic. We can stipulatively introduce the classical connectives, and ask all the old questions using them. The situation is more analogous to a geometer advocating, e.g., hyperbolic geometry to the exclusion of Euclidean *as a pure mathematical theory*. What could be meant by *that*?
- It could mean that as a matter of natural language semantics 'line' refers to hyperbolic lines. This would at least be a factual debate. Indeed, by 'semantically descending', the debate could even be made to sound metaphysical: "are lines really hyperbolic or Euclidean, elliptic, or something else?" But that is a very boring disagreement! We could avoid it altogether by just stipulating how we will use words. This is what we do.
- Putnam himself might be taken to hold that classical disjunction does not 'exist'.
    - *Putnam*: "There are operations approximately answering to the classical logical operations, vis. The v, &, and ~ of quantum logic. If these are not the operations of disjunction, conjunction, and negation, then no operations are."
- *Note*: There is precedent for taking questions of what ideology to accept as analogous to scientific existence questions. This was, again, Quine's position, and Sider follows Quine fully to the point of claiming that it is factual whether $\exists$ or $\forall$ is 'joint carving'. However, not even Sider maintains that some alternative or-like connective might fail to 'exist'!
- *Problem*: Classical disjunction is just a truth-function. If there are truth-functions at all, then surely there are some that satisfy the Distributive Laws. We can construct them!
- The only sensible view in the neighborhood would seem to be that, although all the nonstandard connectives 'exist' if any connectives do, the quantum connectives are particularly useful for empirical science -- namely, for modeling the structure of Yes/No questions in quantum mechanics. This is analogous to the (standard) view that although

Euclidean, hyperbolic, elliptic, etc. geometric spaces exist if any spaces do, *as pure mathematical structures*, Riemannian space is particularly useful for modeling spacetime.

- If this is right, then Putnam is making a **normative** claim: we *ought* to reason in accord with quantum logic. But there is no metaphysical remainder, as in the geometrical case.
- *Problem*: This claim, factually construed, admits of the same kind of pluralist deflation! We ought$_{\text{quantum}}$ use quantum logic, ought$_{\text{classical}}$ use classical, and so forth for all the various logics. Just as it makes little sense to say that quantum disjunction exists to the exclusion of classical, it makes little sense to say that, e.g., quantum ought exists to the exclusion of classical ought. The only factual question is what language we happen to speak. But what to infer from what cannot be resolved by natural language semantics!
- *Conclusion*: What it means to say that quantum logic, or classical logic, or any of the logics with which we began, is the 'true logic of the world' must just be: 'use this logic!'
- *Compare (Carnap)*: "[T]here has been only a very slight deviation...from the form of language developed by Russell which has already become classical. For instance, certain sentential forms (such as unlimited existential sentences) and rules of inference (such as the Law of Excluded Middle), have been eliminated by certain authors. On the other hand, a number of extensions have been attempted, and several interesting, many-valued calculi analogous to the two-valued calculus of sentences have been evolved….The fact that no attempts have been made to venture still further from the classical forms is perhaps due to the widely held [but fallacious] opinion that...the new language-form must be proved to be 'correct' and to constitute a faithful rendering of 'the true logic' [1937]."