Special Relativistic Spacetime

- We discussed last time how all contemporary theories of space, time, and motion share a common mathematical framework. They all assume that spacetime is a Riemannian, or rather *pseudo*-Riemannian, *manifold*, M, equipped with a *metric*, g, written <M, g>.
- Such an object is ∞-differentiable. Hence, paths along it have velocity, acceleration, and so on. Also, there is a well-defined *distance* between nearby points. However, the 'distance' can be negative, and this is the reason for the qualification, 'pseudo'.
- If we ignore gravity, then the proper arena for spacetime is the simplest such manifold, a *Lorentzian* (i.e., *Minkowskian*) manifold, <R^4, η>, where η is the Lorentz metric. Minkowski noticed that this is the forum appropriate for Einstein's *Special Relativity*.

The Spacetime Interval

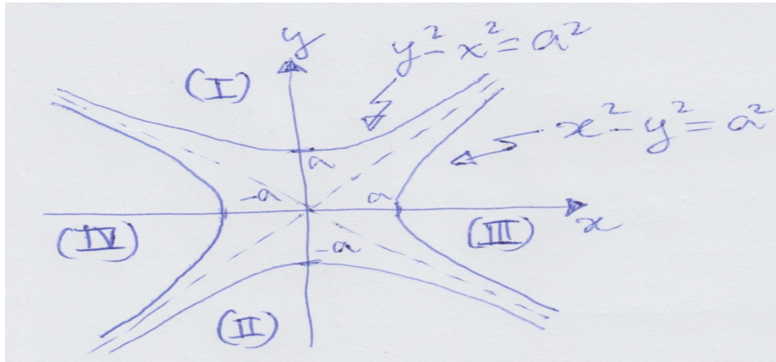
- Spacetime coordinates are arbitrary. We can label events how we like. There is no serious question as to *where the origin* (in the sense of coordinate axes) is, for instance.
- Hence, the physical reality lying behind a coordinate representation of spacetime must be what is *invariant* between different coordinizations. It is what all observers can agree to.
- Special Relativity says that much less is invariant than we would think. Duration, distance, shape and even order vary by *frame of reference*, i.e., coordinate assignment.
- The theory of Special Relativity rests on two claims:

  (1) The speed of light (in vacuum) is the same for all observers, c. (Or, avoiding 'speed': light in a vacuum emitted at an event, e, lies along e's future lightcone [as in Galilean spacetime, nothing has a speed simpliciter in Minkowski spacetime].)
  (2) The laws are the same in all inertial (non-accelerating / straight-line) frames.
  Background: *Maxwell's Equations* say that electromagnetic waves (light) travel at c. But these are not invariant under *Galilean* transformations, i.e., the class of coordinate transformations that leave Newton's Laws invariant. They are instead invariant under the *Lorentz* transformations. Relativistic mechanics amends Newtonian mechanics so as to be invariant under Lorentz transformations too.

- Despite the modesty of these assumptions, the world must be a strange place if they are true! While (2) is familiar from Galileo, (1) -- which was famously experimentally corroborated by Michelson and Morely -- says that light does not behave like familiar particles *or* waves. If I am on a train moving at a constant velocity, k m/s, and throw a baseball with velocity k* m/s, as measured by me, then you on the ground will measure the baseball I throw to travel at k + k* m/s. But when the baseball is a beam of light, you and I will measure it to have the same velocity -- no matter how fast my train moves! Similarly, the speed of an ordinary wave depends on the speed of the *medium* through which it moves. But, again, there is no medium peculiar to electromagnetic waves.
- Speed is just distance divided by time. So, if relatively moving observers agree on the speed of light, they must *disagree* on the interval between the events in space or time.

    Note: By (2), differently moving observers must make *reciprocal* judgments. If S judges that S*'s clock runs slow, then S* must judge that S's clock runs slow.
- However, it is *not* the case that they disagree on everything! How the distance between two cities on a map of Ohio factors into x and y components is frame relative. *But the distance itself is not*. In a similar way, how the distance in *spacetime* between two *events* factors into space and time components is frame-relative. But the distance itself is not.
    - *Spacetime Interval*: s^2 = (difference in time)^2 - (difference in space)^2

      = (ct)^2 - x^2 - y^2 - z^2
    - *Note*: This is like the Pythagorean Theorem, but with a - instead of +. The former gives the metric for Euclidean space. The latter gives it for Lorentzian space.
    - *Note*: This is another way of writing the *Lorentz metric signature*, η = <1, -1, -1, -1>, where the choice of signs is conventional, but the *difference* in signs is not.
        - *Notation*: An arbitrary metric for a spacetime manifold is written: $ds^2 = g_{uv}dx^u dx^v$. In this format, the metric of Minkowski spacetime becomes: $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 = \eta_{uv}dx^u dx^v$.
- The equation for the spacetime interval is that of a hyperbola. (For a fixed distance, s, the equation gives all points a distance s from the origin.) All observers agree about the hyperbola on which an event lies. They disagree on its time and space coordinates.

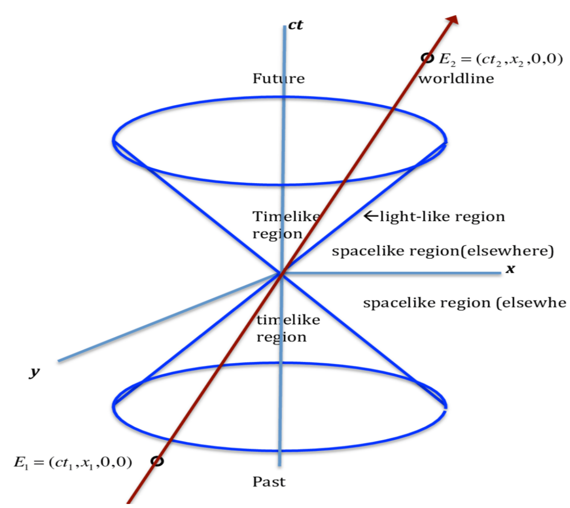


[1]

- *Note*: A feature of Minkowski spacetime is that the distance between two points in it is *maximized* by straight lines (geodesics). These are just the lines of <u>inertial</u> motion.
  - *Example*: A spacecraft moving inertially between two events will cover *more spacetime distance* than a spacecraft that accelerates between the same events. This is why the 'Twins Paradox' is not a paradox. It is a feature of the geometry.
  - *Note*: The 'paradox' is not exactly the result of the younger twin's *accelerating* (though someone must accelerate if the twins are to reunite with different ages). We can fix things so the older twin accelerate *more*, but covers more distance. (In General Relativity, there are 'Twins Paradoxes' involving *only inertial* travelers.)

<u>Light Cone Structure</u>

- We can *measure* the spacetime interval between two events by passing through them with a time-keeping device. This gives the *proper time* elapsed, as (difference in space) = 0, so (difference in time)^2 = (Spacetime Interval)^2 = s^2. Watches are spacetime-oders!
  - *Note*: The conjecture that watches measure proper time -- and, therefore, the spacetime interval on their worldlines -- is *prima facie* an <u>physical</u> one, subject to *empirical* test.



- *Note*: Corresponding to proper time is *proper distance*, where (difference in time) = 0.
- Any two events in spacetime are related by an interval of exactly one of three kinds:

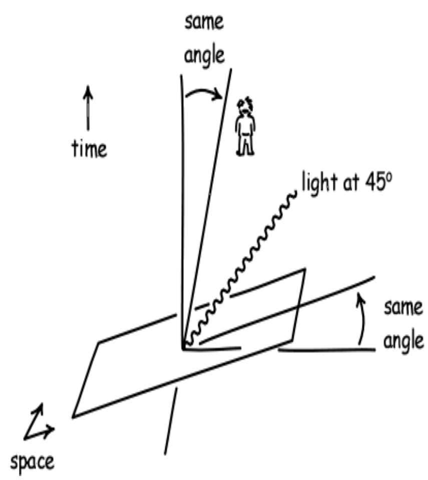
[1] Image taken from https://miro.medium.com/max/1050/1*vnfPAr29NE4tUZ0VsxTwzg.jpeg

Timelike: (difference in time)^2 > (difference in space)^2

Spacelike: (difference in space)^2 > (difference in time)^2

Lightlike: (difference in time)^2 = (difference in space)^2

- *Note*: Lightlike intervals are 0, *even when the spatial and temporal distances are large.*[2]



Lightcone Structure into which *Minkowski* spacetime folliates. Such a cone exists at every event, and all inertial observers can agree to it.

Spacelike separated events have a peculiar feature: if B is any event spacelike separated from event A, then B is earlier than A, later than A, or simultaneous with A, *depending on the frame of reference we measure it with respect to*.

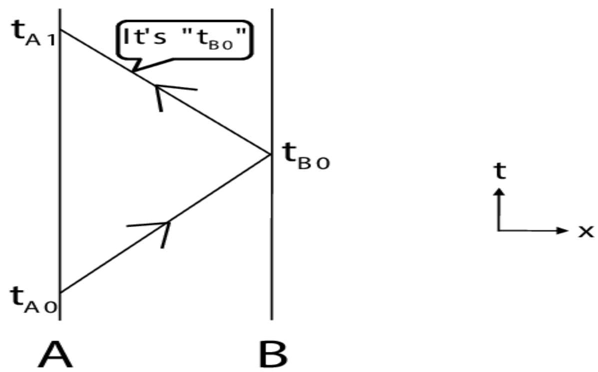
- Events A and B can only be simultaneous in a frame of reference if they are spacelike separated (on a line parallel to the space axis). Hence the Relativity of Simultaneity.
- However, if A and B are timelike separated, then although they can be made closer or farther away in time by choice of inertial frame, their order cannot vary by inertial frame.
- *Upshot*: If causal signals cannot travel faster than light, then causal relations must be objective (frame-invariant), agreed upon by all inertial (non-accelerating) observers.
- *Question*: What does 'causal' mean here, and what *non-semantic* question is at stake?

Conventionality of Simultaneity

- Some problems associated with Special Relativity, such as that of substantivalism (to be discussed), arise equally, or more perspicuously, in the context of General Relativity.
- *Example*: One objection to spacetime substantivalism is that it violates Newton's 3rd Law. But this objection seems no longer to apply in the context of General Relativity. 'Space-time tells matter how to move' AND 'matter tells space-time how to curve'.
- *Problem 1 (Reichenbach)*: Special Relativity postulates that *one-way* the speed of light is constant. But in order to measure it, we need to know when distant events are

[2] Image taken from: https://www.pitt.edu/~jdnorton/teaching/HPS_0410/chapters/spacetime_rel_sim/rel_sim_4.gif

simultaneous. And in order to know *that*, we need to know the speed of light! How do we get started? This raises the specter of a new kind of conventionalism about simultaneity -- in addition to the kinds engendered by eternalism and Special Relativity.[3]



N.B: The Relativity of Simultaneity allows there is a perfectly objective fact as to what is simultaneous with what relative to an inertial frame. The Conventionality of Simultaneity would say that there fails even to be objective facts about frame-relative simultaneity!

- *Question*: What does 'objective' mean as it occurs in the second sentence? In the first, it means frame-invariant. But it means something different in the second.

- *Clarification*: Reichenbach himself might say that formulations of Special Relativity incorporating different simultaneity definitions are the same theory because they are empirically equivalent. So, the difference is like the one between a formulation of mechanics that uses Cartesian rather than polar coordinates. But this is just Logical Positivism, which has well known problems.

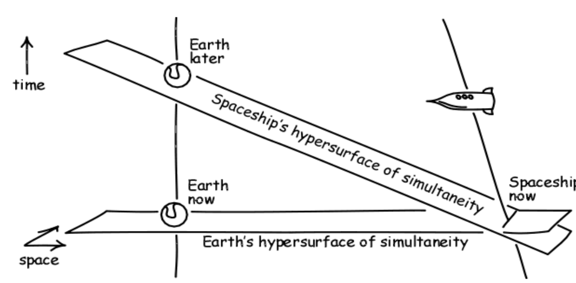
- *Details*: In order to judge the simultaneity of distant events, we need synchronized clocks. In order to synchronize clocks, we need to send a light pulse from one to the other, and bounce it back If light has the same speed in both directions, then the reflection happens at a time halfway between the emission and receipt of the pulse.
    - *Einstein-Poincaré Simultaneity Definition*: Clocks t_A and t_B are synchronized relative to inertial frame F just in case $\mathbf{t_B(B_2) - t_A(A_1) = (½)[t_A(A_3) - t_A(A_1)]}$ (where A_1, B_2, and A_3 are the events of light pulses leaving, reaching, and arriving back at three corresponding points on the two stationary wordlines, A and B).

[3] Image taken from https://www.researchgate.net/figure/The-Einstein-synchronization-An-observer-A-Alice-needs-to-be-told-the-arriving-time-tB0_fig4_314942977

- *Problem*: There seems to be no way to check the antecedent without appealing to prior knowledge that our clocks are synchronized -- i.e., knowledge of simultaneity of events.
- *Note*: Alternative synchronization definitions will lead to surprising results! The length of objects, and the slowing of clocks, will depend on the direction in which they move. However, anything that is measurable will come out the same under the new definition.
    - *Motivation*: Any theory can be given a *generally covariant* formulation, according to which its laws hold in any coordinate frame whatever. Coordinate frames resulting from the replacement of **(½)** with another, ε, 0<ε<1, are special cases.
- *Note*: Typical advocates of the Conventionality of Simultaneity base their arguments on the causal theory of time. Malament [1977] threatens to undercut these arguments. But perhaps there are independent reasons to believe in the Conventionality of Simultaneity.
- *Response 1*: Standard synchronicity is 'simpler'. It uses only the relation of *equality*.
- *Rejoinder*: Simplicity is not so simple!
- *Response 2*: Standard synchronicity is independently confirmed insofar as isotropic spacetime theories -- namely, Friedman-Robertson-Walker (FRW) spacetimes -- are.
- *Rejoinder*: We cannot justify belief in an isotropic spacetime theory independent of a justification of the standard simultaneity definition, since the former implies the latter!
- *Response 3*: At most this shows that we should be skeptics about the one-way speed of light -- not that there is no 'objective' fact of the matter -- whatever that might means.

Chronogeometrical Fatalism

- *Problem 2 (Rietdijk-Putnam-Penrose)*: For any event, E, in our future, there exists a frame of reference (a coordinate system whose origin has some relative position and motion) such that an observer, O, in that frame would judge in our present that E is in O's present. Hence, E must be determinate in our present, even though it is in our future![4]



[4] Imagine taken from Norton, "Philosophical Significance of the Special Theory of Relativity".

- *Details*: The argument assumes that that simultaneity implies determinateness and that determinateness is transitive. With respect to the figure below, one may argue as follows.
    - Spaceship Now is simultaneous with respect to Earth Now (vis a vis the Earth frame).
    - Earth Later is simultaneous with respect to Spaceship now (vis a vis the spaceship frame).
    - [Determination] If A is simultaneous with respect to B (vis a vis some frame), then A is determinate with respect to B.
    - [Transitivity] If A is determinate with B, and B is determinate with respect to C, then A is determinate with respect to C.
    - Hence, Earth Later is determinate with respect to Earth Now.
- *Question*: Distinguish fatalism from determinism. The former says that you could not have failed to do what you will do. The latter says that it is necessary that: if the actual laws and conditions hold, then you will do what you in fact do. A compatibilist about free will can accept the latter, but not the former. How could the argument establish the former?
- *Response*: Determination assumes that, although simultaneity is relative to an observer, determinateness is not. The following principle seems more in keeping with Relativity.

    [Determination*] If A is simultaneous with respect to B (vis a vis some frame), then A is determinate with respect to B (vis a vis that frame).
- But, then, in order to make the argument valid, [Transitivity] must be reformulated thus:
    - [Transitivity*] If A is determinate with B (vis a vis frame E), and B is determinate with respect to C (vis a vis frame F), then A is determinate with respect to C (vis a vis frame E).
- This is quite unmotivated! Much more plausible is the following.
    - [Transitivity**] If A is determinate with B (vis a vis frame E), and B is determinate with respect to C (vis a vis frame E), then A is determinate with respect to C (vis a vis frame E).
- *Note*: I have granted for the sake of argument that simultaneity implies determinateness. But this is far from obvious. Simultaneous events are after all outside of our lightcones.

- *Clarification*: There are *other* reasons to believe that the future is determinate -- e.g., that classical logic is correct. The question is whether Special Relativity gives a new reason.

Return of the A-Series?

- It might be wondered if the breakdown of the Rietdijk-Putnam-Penrose argument suggests a rehabilitation of McTaggart's A-series. Instead of defining past, present and future with respect to a plane of simultaneity, we could define it with respect to an event in (Minkowski) spacetime. Then, 'an event's present is constituted by itself alone [Stein 1968, 15]' whereas its past and future are the events in or on its past or future lightcones.
- *Problem 1*: If an event's present really contains no other events, then spatiotemporally extended objects like ourselves seem to have little claim to experiencing it! Perhaps we could avoid this problem by appealing to 'causal diamonds' encompassing the specious present. But, either with way, there are infinitely-many world lines through a given event, and each would seem to determine a different sequence of experiences [Lee 2012]!
- *Problem 2*: Changing definitions of 'past', 'present', and 'future' from those appropriate to Galiliean spacetime to those better suited to Minkowski spacetime does little to mitigate the worry that there is no real change in spacetime array. If one can add 'moving spotlights' along worldlines, why can't one add them along slices of Galilean spacetime?
- *N.B*: Lack of 'real change' in a diagram does not preclude change in what it represents!