General Relativistic Spacetime

- We have been discussing spacetime theories. Last time we focused on Minkowski spacetime, the arena for Special Relativity, and associated philosophical problems.
- However, Minkowski spacetime is just a particular General Relativistic Spacetime. It amounts to one solution to Einstein's Field Equations in which there is no mass-energy.
- More importantly, all General Relativistic Spacetimes (GRS) approximate Minkowski spacetime in the limit. Special Relativity applies because GRS are locally Minkowskian.
    - *Details*: General Relativity replaces the Lorentz (Minkowski) metric, $ds^2 = \eta_{uv}dx^u dx^v$, with a metric for a curved spacetime, $ds^2 = g_{uv}dx^u dx^v$ The metric, g_{uv}, must *approximate* the metric, η_{uv}, in small regions, so that each (pseudo-Euclidean) tangent space, T_pM, carries Lorentz metric depending smoothly on p.

Einstein's Ambitions

- Einstein's ambitions for General Relativity and what the theory says are distinct. He aspired to extend the 'Principle of Relativity' (principle (2) from last time) to accelerating reference frames -- to show that "both a body moving with uniform motion in a straight line and an accelerating body may with equal right be looked upon as 'stationary'."[1]
    - (2) The laws are the same in all inertial (non-accelerating / straight-line) frames.
- If (2) could be extended to 'accelerating frames', then just as there is no distinction between stationary and constantly moving states of motion (in Minkowski spacetime), there would also be no distinction between constantly moving and accelerating states.
- *Note*: As David pointed out to me, it is not obvious what Einstein could have had in mind by this, given that an accelerated observer in Minkowski spacetime cannot, in general, consistently assign global coordinates using the kinds of protocols we discussed last time.
- This ambition was not fulfilled. There is still a distinction between inertial and accelerated trajectories in General Relativity if the accelerations are not due to gravity.

[1] Quoted in Kennedy, *Space, Time, and Einstein*, p. 163.

- *Note*: Einstein appears to have conflated the claim that General Relativity can be given a generally covariant formulation with the claim that even accelerated motion is relative.
    - *Einstein*: "Should the independence of physical laws of the state of motion of the coordinate system be restricted to the uniform translatory motion of coordinate systems...? What has nature to do with our coordinate systems…?...*[T]he laws ought to be entirely independent of this choice (general principle of relativity)*."[2]
- This is a conflation because general covariance is cheap (as Erich Kretschmann showed in 1917). Even Newtonian Gravitation theory has a generally covariant formulation. But this explains the import of tensors. If a tensor equation holds in one frame it holds in all.
- However, this was not Einstein's only ambition. He also sought to include Newton's Law of Universal Gravitation among the 'laws' mentioned in the Principle of Relativity (2).
    - *Newton's Law of Universal Gravitation*: $F = G\frac{m_1 m_2}{r^2}$, where F is the gravitational force acting between the objects, m_1 and m_2 are their masses, r is the distance between their centers of masses, and G is Newton's gravitational constant.
- This ambition was fulfilled. Gravitational acceleration is relative in General Relativity, just like velocity. There is no frame-invariant fact as to one's gravitational acceleration.
- Einstein linked the above ambitions via his *(Strong) Principle of Equivalence*: A freely falling frame in a gravitational field is locally equivalent to an inertial frame. (The modifier 'locally' is needed since a *region* of the field will always betray tidal effects.)
- *Details*: Given a metric, $g_{uv}(x)$ with coordinate system, x, for any point in spacetime, p, there is a coordinate system, x', called a local inertial frame, such that $g'_{uv}(x') = \eta_{uv}(x')$, where x and x' refer to p, and the 1st -- but not 2nd! -- derivative of $\eta_{uv}(x')$ is 0.
- *Note*: If there are coordinates, x', with $g'_{uv}(x') = \eta_{uv}(x')$ globally, then the space is flat.
- *Upshot*: General Relativity inherits locally the lightcone structure of Special Relativity

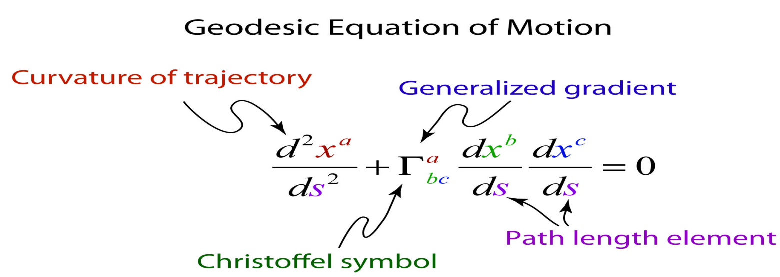
Peculiarity of Gravity

[2] Quoted in Norton "Philosophical Significance of the General Theory of Relativity."

- General Relativity avoids a force of gravity in favor of a new geometry. It says that what we previously took to be inertial observers -- like people standing on Earth, near enough -- are actually accelerating ones, and the likes of objects falling to the ground are inertial.
- The idea to General Relativity is that gravity can be traded for spacetime curvature works because gravity is peculiar. Unlike other forces, all objects are affected by gravity in the same way, independent of their composition. This follows from $F = G\frac{m_1 m_2}{r^2}$, since also F = $m_i a$, where m_1 is the is the gravitational mass and m_i is the inertial mass.
- All experimental data suggests that $m_i = m_1$. So, the force due to gravity felt by an object from m_2 is $(Gm_2)/(r^2)$, independent of the object's mass (and other physical properties).
    - *Recall*: Newtonian Gravity acts instantaneously, contravening Special Relativity.
- *Upshot (Galileo)*: In a gravitational field, all objects fall with the same acceleration.

Geometricized Gravity

- In General Relativistic Spacetimes, gravitationally accelerated trajectories are among the inertial, 'straight line' -- i.e., geodesic -- paths in some curved spacetime (not space!).
- *Details*: Geodesics are lines of 'supremal distance' and 'constant direction', like straight lines in Euclidean or Minkowski space (minimal distance in the former, and maximal distance in the latter). The only difference is that the space they inhabit need not be flat.
- *Note*: In General Relativity, geodesics are only locally maximal, perhaps not globally.
    - Consequently, the equation for geodesic motion in flat spacetime must be complicated from Newton's equation $d^2x/dt^2 = 0$ ($d^2x/dt^2 = a = 0$, with $F = 0$), to:

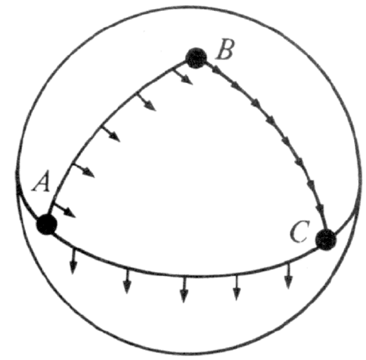


[3]

    - The Christoffel symbol (affine connection coefficients) represents 64 functions encoding the curvature, each depending on g_{ab}. *If it equals 0, then the space is flat,* but not conversely. We could be using curved coordinates on flat spacetime.

[3] Image taken from https://galileounbound.files.wordpress.com/2019/07/geodesiceq.jpg

- However, there is an object that gives a necessary and sufficient condition for flatness, the *Riemann Curvature Tensor*, $R^a{}_{bcd}$. This measures the change in direction of a vector (corresponding to the index, b) 'parallel transported' (keeping it as *constant as possible*) around an infinitesimal curve defined by two others (corresponding to indices c and d), and gives a final vector (corresponding to the index, a). $R^a{}_{bcd}$ is inter-definable with g_{ab}. This is because $R^a{}_{bcd}$ is itself defined in terms of derivatives and products of the Christoffel Symbols, which we saw above depend on the metric.



- *Note*: g_{ab} is *not* fixed by the geometry, since it also depends on the coordinate system.
- Hence, in order to 'geometricize' gravity, the source of gravity (mass-energy) must be related to the $R^a{}_{bcd}$. But the object that represents the former, the *Energy-Momentum Tensor*, T_{ab}, has too few indices to be equated with $R^a{}_{bcd}$. So, one constructs a tensor with two indices out of $R^a{}_{bcd}$, the *Einstein Tensor*, G_{ab}, and makes it proportional to T_{ab}. (G_{ab} says how a sphere of test particles in free fall at a point changes [Baez 2006].) If we do not add a Cosmological Constant (e.g., to register the accelerating expansion of the universe), then, in units with c = 8π = 1, the Field Equations become:

$$G_{ab} = T_{ab} \quad \text{(where G is Newton's Gravitational Constant)}$$

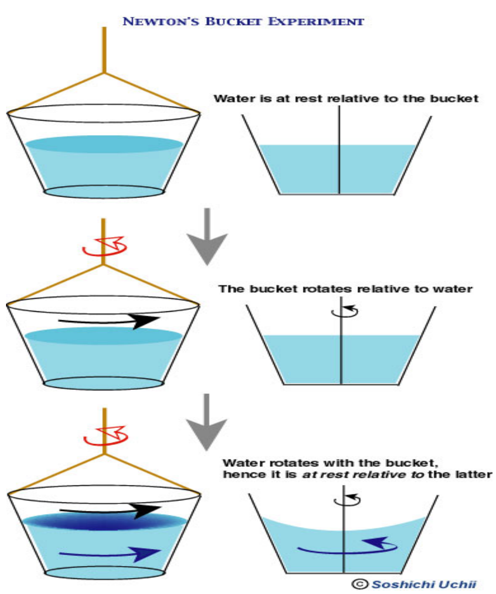
- *Note*: Since *gravitational* energy contributes to curvature, the equations are nonlinear.
- *Note*: Since G_{ab} depends on g_{ab}, the metric remains the key player of General Relativity.

The Substantivism Debate

- Einstein had a deeper philosophical aim with General Relativity. This was to vindicate a view of Ernst Mach. Newton held that spacetime (space) exists over and above relations between physical bodies. This view is called substantivalism. Mach argued that it does not. This view is called relationalism. Einstein took his theory to vindicate relationalism.
    - *Einstein*: "In Newton's mechanics, and no less in the special theory of relativity, there is an inherent...defect which was...pointed out by Ernst Mach ….Newtonian mechanics does not give [an account of inertial effects] since it makes the

unobservable cause of absolute space responsible for [them]. *The general theory ...takes away from space and time the last remnant of physical objectivity*."[4]

- As stated, substantivalism is ambiguous between elimitivism and reductionism. It is ambiguous between the view that spacetime does not exist, and the view that although it does, it is in some sense an epiphenomenon on relationships between physical bodies.
- The 'in some sense' qualifier does a lot of work. A *traditional* reductionist would claim to define spacetime in terms of relations among physical bodies. However, if one could do that, then the difference between elimitivism and reductionism would be verbal.
- *Compare*: A traditional nominalist about mathematical entities, such as Goodman and Quine [1947], claims to define these entities in terms of uncontroversially physical ones.
- Most contemporary relationalists are different. Even if we cannot *define* spacetime in terms of relations among physical bodies, the former might still fail to be 'fundamental', or anything 'over and above' such relations. It might be 'constituted' by such relations.
- *Compare*: Contemporary formulations of materialism and dualism about consciousness.
- Newton himself made the most influential arguments for substantivalism. Here are two.
  - *The Bucket*: If one were to release a bucket of flat water hanging from a twisted rope, then it would begin to spin. Moreover, when the bucket was in relative motion to the water, the water would remain flat. But when the water had 'caught up' to the bucket, and so was no longer in relative motion to it, the water would be concave. So, no relative motion would cause the concavity. Hence, the concavity would be caused by absolute motion (motion relative to space itself). Consequently, space is fundamental (not an epiphenomenon on relations between bodies), and substantivalism is true.



  - *Note*: This counterfactual is also a counter-actual, so can be (and has been) tested! But the inference seems untestable, since we cannot spin a bucket in empty space.

[4] Quoted in Kennedy, *Space, Time, and Einstein*, p. 163

- *Spinning Spheres*: The following physical situations are distinct. (1) an otherwise empty world in which two spheres are connected by a chord and not rotating, and (2) an otherwise empty world in which two such spheres *are* rotating. (This is guaranteed by the fact that the cord would be tense in scenario (2) but not (1).) However, the *relative positions* of all physical bodies are the same in both worlds. Hence, the tension must be due to rotation relative to space -- i.e., absolute rotation (acceleration). So, space is fundamental, and substantivalism is true.

- Notwithstanding Einstein's insistence that "the general theory...takes away from space and time the last remnant of physical objectivity", it actually seems to strengthen Newton's case. If spacetime curvature is 'constituted' by the distribution of mass-energy according to the Field Equations 'without residue' (as Einstein put it), then that distribution should fix the spacetime curvature. But, on the contrary, that is only *logically fixed* by the distribution of mass-energy plus independent 'boundary conditions' (≈ the shape of the universe as a whole). The Field Equations just give local constraints.
- *Question*: What happens if we regard the metric field as itself just another physical body (which may be reasonable given that gravitational waves carry energy and momentum)?

Problems of Pluralism

- *Problem*: I used 'epiphenomenon' to characterize relationalism. But this is a filler word. The natural way to understand it is modally. Suppose that F-facts are *not* reducible in the *traditional* sense to G-facts - i.e., definable in terms of them. Then F-facts are epiphenomenal on G-facts whenever F-facts *supervene* on G-facts and are causally inert.
    - *Example*: epiphenomenal dualism or non-naturalist moral realism.
- *One* problem with this analysis is that it assumes a notion of causality which would itself need to be explained non-modally in order for the characterization to be non-circular.
- A deeper problem is that of pluralism: whether something is an epiphenomenon in this sense is relative to a notion of supervenience - and there are myriad such notions!
- *Note*: A totally parallel problem applies if one analyses 'epiphenomenon' in terms of hyperintensional ideology, like *grounding*, since there are myriad such notions as well.

- *Example 1 (Huggett & Hoefer)*: “Absolutists and relationalists are...left in a frustrating and perplexing quandary by [General Relativity]. Considering its anti-Machian models, we are inclined to say that motions such as rotation and acceleration remain absolute...according to the theory….[C]onsidering its most Mach-friendly models, which include all the models taken to be good candidates for representing the actual universe, we may be inclined to say: motion in *our world* is entirely relative….But even if we agree [with this]..[m]any philosophers ... would be happy to acknowledge the Mach-friendly status of our spacetime, and argue that nevertheless that...spacetime [is] a real thing [SEP, 2016].”

- *Problem*: We can all agree that it is logically consistent with the Field Equations that there is absolute motion -- motion relative to absolute space (since, e.g., General Relativity has models in which there is a single rotating neutron star.) If we assume that <>(there is absolute motion) → [](there is absolute motion) for logical possibility, then there is absolute motion. Alternatively, we might appeal to a notion of possibility, <*>, according to which either ~<*>(there is absolute motion) or ~[<*>(there is absolute motion) → [*](there is absolute motion)]. But there seems to be no non-semantic question to dispute. What could we learn by answering the question of whether ‘spacetime [is] a real thing’ in the relevant sense except how we are using ‘real’?
    - *Example 2 (Maudlin)*: “The heart of the hole argument is evidently...that S and S_{WARP} can be interpreted as representing two metaphysically distinct and yet...possible situations….[If they do], then [determinism is violated] because *outside the hole, the physical situations are identical*….The Field Equations… must be radically indeterministic. But it is not at all clear that S_{WARP} represents a… metaphysically *possible* situation….[W]e can argue that if the particular ...individual events *p* and *q* are light-like related , then it is not metaphysically ...possible for *those very events* to have been space-like related: the spatio-temporal relations among space-time events are essential to their identity (151).”[5]

- *Problem*: In *some* sense of ‘possible’ it is certainly possible for p and q to be light-like related in one world and space-like related in another -- e.g., classical (first-order) logical possibility. Maudlin might reply that this is not a ‘real’ sense of possibility. But what

[5] Maudlin, *Philosophy of Physics: Space and Time*. (Text rearranged for readability.)

does that mean? Not that it isn't *counterfactual* or *alethic* (it satisfies the (T) axiom). So, *as a matter of logical possibility*, the Field Equations are indeterministic. But we can always introduce another sense of possibility -- call it 'metaphysical' -- according to which it is *not* possible for p and q to be light-like related in one world and space-like related in another. Then *as a matter of metaphysical possibility* the Field Equations are not indeterministic. What is left to dispute except what we mean by 'deterministic'?

- *Objection*: Even if the problem of pluralism infects debates about substantivalism and offshoot debates about determinism, it does not infect debates about counterfactuals. Suppose that there were two identical spheres, tied to each other by a rope, rotating with respect to empty space (something that even I must grant is at least logically possible!). Then we can ask the counterfactual question: *would the string exhibit a tension*? This depends only on the ordering of worlds across the 'modal pluriverse'. In particular:
    - Counterfactual Absoluteness: Suppose that $\forall P([N+]P \rightarrow [N]P)$, but not conversely. Then, if $(A\ [] \!\rightarrow B)$ is non-vacuously true with respect to a model, $N = \langle D, S, V, @\rangle$, where $w \in D$ just in case w is N-possible, then $(A\ [] \!\rightarrow B)$ remains true with respect to a model, $N+ = \langle D', S', V', @\rangle$ where $w \in D'$ just in case w is N+-possible, whenever N is a submodel of N+ (where N and N+ are models of propositional conditional logic, S is a class of relations, one for each formula of the language, D is a domain, V is a valuation, and @ is the actual world).

- *Response*: This is indeed a different matter. However, *Counterfactual Absoluteness* is suspect. Is it true that the laws of mathematics and logic would have been the same if the laws of physics had been radically different? It must be if Counterfactual Absoluteness is true (and the mathematical and logical truths are metaphysically necessary). But, given the indispensability of mathematics and logic to the physical laws, this is surely dubious!