‘Interpreting’ Quantum Mechanics

- As Ismael writes, “Quantum mechanics is, at least at first glance and at least in part, a mathematical machine for predicting the behaviors of microscopic particles — or, at least, of the measuring instruments we use to explore those behaviors [SEP Online].”
- Nevertheless, there is tremendous disagreement over why the ‘machine’ works.
    - *Carroll*: “At a workshop attended by expert researchers in quantum mechanics… Max Tegmark took an...unscientific poll of the participants’ favored interpretation ….The Copenhagen interpretation came in first with thirteen votes, while the many-worlds interpretation came in second with eight. Another nine votes were scattered among other alternatives. Most interesting, eighteen votes were cast for “none of the above/undecided.” And these are the experts [2010b, 402, n. 199].”
- There are two reasons why this is not really disagreement over the interpretation of quantum mechanics, in any ordinary sense. First, at stake is not what people happen to mean by pertinent technical terms, like ‘state vector’, ‘collapse’, and so on. This would be a question of (presumably empirical) natural language semantics, and would tell us nothing about the nature of quantum reality. (Indeed, some ‘interpretations’ deny that the ‘mathematical machine’ means anything -- like austere formalists about mathematics.)
- Second, the ‘interpretations’ do not even all agree on the machine! Bohmian and GRW theories, for instance, amend the equations, and GRW makes different predictions.
- *Compare*: Despite the name, metaethics is not just about what we mean by “ought”!
- So, before we can ask about ‘interpretations’ of quantum mechanics -- i.e., ask why the ‘mathematical machine’ works -- we need to outline central aspects of the formalism.

Intuitive Summary

- The basic idea of quantum mechanics can be summarized as follows. However, to understand the summary, we will need to define the terms that I have underlined.
- Every quantum state can be expanded in terms of many different bases. Each basis corresponds to a superposition of states of a given quantity (e.g., position, momentum, spin, polarization). Once expanded, we can read off the (complex) amplitudes for the

various eigenstates of that quantity. And the amplitudes can then be used to compute the probabilities of obtaining corresponding eigenvalues upon measurement of the quantity.

Vectors

- Vectors, you will recall, are quantities with length and direction. The simplest examples are given by the arrows in Euclidean space. For every point in that space, there is exactly one arrow which reaches from the origin to that point. In quantum theory, one talks about vectors in a more abstract, complex-valued, space, and writes them as follows: |A>.
- Vectors of any kind can be added according to the Parallelogram Rule. Vectors can also be multiplied by *scalars*. Scalar multiplication yields another vector pointing in the same, or the opposite, direction (depending on whether the scalar is positive or negative).
- Vectors can be multiplied by other vectors too. One way is to define the product, called the **inner product**, of vectors, |A> and |B>, written **<A|B>** whose result is a (typically complex) *number*, not a vector. It is obtained by multiplying the complex conjugates of the expansion coefficients of |A> with the expansion coefficients of |B> in a basis and adding up the results. We will say that √<A|A> is the *length* or **norm** of |A>.
- Two vectors, |A> and |B>, of non-zero length, are **orthogonal** when <A|B> = 0. A vector space is *N-dimensional* when the *maximum number of mutually orthogonal* vectors in it is exactly N. And an **orthonormal basis** of an N-dimensional vector space is a set of N mutually orthogonal vectors, $|A_k\rangle$, each with *unit* length -- i.e., $\sqrt{\langle A_k|A_k\rangle} = \langle A_k|A_k\rangle = 1$.
    - *Note*: Any vector, |V>, in an N-dimensional vector space can be written, or *expanded*, as the linear sum of vectors from a given orthonormal basis. That is:
        - $|V\rangle = v_1|A_1\rangle + v_2|A_2\rangle + \dots$ (where each $|A_i\rangle$ is a unit length basis vector, and the **expansion coefficient**, v_i is the (complex) number, $\mathbf{\langle A_i|V\rangle}$).
- Vectors can be represented as *columns* or *rows* of their expansion coefficients in a basis. While columns correspond to ordinary vectors, |A>, rows correspond to a kind of dual vector, written <A|. To turn column vector, |A> into a row vector, <A|, take the complex conjugates, c*, of the expansion coefficients, c, so that c = x + iy becomes c* = x - iy.
- The <A|s are known as **bras**, and the |A>s as **kets** (so combing them yields a **bra-ket**).
    - *Note*: The *same* vector has *different* representations in different bases! Vectors, like tensors of which they are an example, have a life independent of coordinates.

Operators

- An *operator* on a space acts on a vector to give another vector. Quantum mechanics uses *linear* operators, **O**, which satisfy the condition: **O**(n|A> + m|B>) = n(**O**|A>) + m(**O**|B>).
- Like vectors, linear operators can be represented by their *coefficients* in a basis. The result is a *matrix*. Two linear operators, **O** and **M**, can be multiplied if the number of *columns* of **O** is equal to the number of *rows* of **M**. The components of linear operator, **O**, $\mathbf{O}_{ij}$, in a basis $|A_1>, |A_2>, \ldots |A_n>$ ($i, j \in N$) are then given by the quantity, $<A_i|\mathbf{O}|A_j>$.
    - *Note*: $<A_i|\mathbf{O}|A_j>$ is a *number* since it is the **inner product** of $|A_i>$ and $\mathbf{O}|A_j>$.
- **Hermitian (self-adjoint) linear operators**, **O**, are central to quantum mechanics. The *eigenvalues* of these are **real**, and their *eigenvectors* form an **orthonormal basis**.

Eigenvectors and Eigenvalues

- An **eigenvector**, $|\lambda_i>$, of the operator, **O**, is any vector such that there is a (perhaps complex) scalar, λ_i with $\mathbf{O}|\lambda_i> = \lambda_i|\lambda_i>$. **O** leaves fixed the ray along which $|\lambda_i>$ lies.
- The **eigenvalue** of **O**, acting on eigenvector, $|\lambda_i>$, is simply defined to be the scalar λ_i.
- It is important that eigenstates of one Hermitian operator fail to be eigenstates of infinitely-many others. Indeed, if the **commutator**, **[A, B]** = **AB** - **BA** ≠ 0 (so **A** and **B** do not *commute*), then **A** and **B** lack a common basis of simultaneous eigenvectors.

Quantum States

- The **state** of a physical system at a time, t, is, intuitively, all there is to know about it at that time. In classical (pre-quantum) physics, the state of a physical system at t is given by the specification of its position and momentum at t (its coordinates in a so-called **statespace**). Consequently, the state of a classical system is a measurable thing.
- By contrast, the state of a quantum system, written $|\Psi>$, is not directly observable. We get a grip on it *via* its mathematical representation. It is represented by a vector in a particular complex vector space, called a **Hilbert Space**. (This is the state space of the quantum system.) By convention, the state vector is **normalized**, meaning $<\Psi|\Psi> = 1$.
- *Note*: Since a Hilbert space is a vector space, we can represent any vector in it as the *linear sum,* i.e. ***superposition*****,** of *pairwise orthogonal* vectors. So, for any $|\Psi>$, we have:

- $|\Psi\rangle = c_1|\Psi_1\rangle + c_2|\Psi_2\rangle + \ldots$ (where, for all i and k, c_i is a scalar and $\langle\Psi_i|\Psi_k\rangle = 0$).

- *Note*: The two canonical "**wavefunctions**" are the wavefunction for position, $\langle x|\Psi\rangle$, and for momentum, $\langle p|\Psi\rangle$. One can translate between these representations (they represent the same state vector in different coordinates -- think reference frames) using Fourier transformations. Spin is an independent degree of freedom. So, to fully represent the state of a particle at a time, one specifies its position (momentum) and its spin states. One can combine this information into a single object using so-called **spinor** notation.

Observables

- Quantum mechanics connects up with experiment by way of **observable quantities**, such as position, momentum, and spin (in a technical sense peculiar to quantum objects).
- These are represented mathematically as **Hermitian (self-adjoint) linear operators**, **O**.
- When such operators do not commute, the corresponding properties are **complimentary**.
- *Bohr*: "[W]e are not dealing with contradictory but complementary pictures of the phenomena, which only together offer a...generalization of the classical...description."
  - *General Uncertainty Principle*: $\Delta A \Delta B \geq (½)|\langle\Psi|[\mathbf{A}, \mathbf{B}]|\Psi\rangle|$
    - *Note*: It follows from this principle that the product of the 'uncertainties' (more on this idea) of the position and momentum of a particle must be greater than ½ times Planck's constant, ħ ('h-bar'), since $[x, p_x] = i\hbar$, $[y, p_y] = i\hbar$, and $[z, p_z] = i\hbar$. This is **Heisenberg's Uncertainty Principle**.
    - *Heisenberg*: "The path of the electron through the cloud chamber obviously existed; one could easily observe it. The mathematical framework of quantum mechanics existed as well, and was much too convincing to allow for any changes. Hence it ought to be possible to establish a connection between the two, hard though it appeared to be."

Measurement

- Suppose we measure the observable quantity represented by **O**. Then the *only* possible results of our measurement are the *eigenvalues*, λ_i, of **O**. For certain such operators, **O**, the eigenvalues form a discrete set, or **spectrum**. Hence, the modifier "quantum".

- Moreover, whether we obtain eigenvalue, λ_i, when the system is in state, $|\Psi>$, and we measure the observable quantity represented by **O**, is generally a matter of *probability*.
    - **Born's Rule**: The probability of obtaining λ_i when the system is in state $|\Psi>$ and we measure the observable represented by **O**, is $P_i = |<\lambda_i|\Psi>|^2$ (where $|\lambda_i>$ is the eigenvector of the eigenvalue equation, $\mathbf{O}|\lambda_i> = \lambda_i|\lambda_i>$, and $|\lambda_i>$ is normalized).
        - *Note*: $|<\lambda_i|\Psi>|^2 = |c_i|^2 = (cc^*)^2$ where c_i occurs in the expansion: $c_1|\lambda_1> + c_2|\lambda_2> + ... = |\Psi>$ (and we require that $|c_1|^2 + |c_2|^2 + |c_3|^2... = 1$).
        - *Note*: Born's Rule means that probabilities are gauge invariant in that all observable quantities are unaffected by a global change of phase of the state vector. Sending $|\Psi> \rightarrow e^{i\theta}|\Psi>$ leaves the probabilities the same. (Differences between phases can, however, have physical significance.)
- *Note*: Although the *probability* of obtaining value λ_i when the system is in state $|\Psi>$ and we measure the observable represented by **O**, is real (since $(x + iy)(x - iy) = x^2 + y^2$), it is determined by the *complex* quantities which are the coefficients of the vectors when the state is expanded as eigenvectors of **O**. It is fixed by the c_is in $= |\Psi> = c_1|\lambda_1> + c_2|\lambda_2> + ...$

Copenhagen Interpretation

- According to (an interpretation of!) the Copenhagen Interpretation, a state, $|\lambda>$, has value, λ, of the quantity represented by **O**, if/f $|\lambda>$ is an *eigenvector* of **O** with eigenvalue λ.
    - *Note*: In this case, one says that $|\lambda>$ is an **eigenstate** of the quantity in question.
- Since eigenstates of one operator *fail* to be eigenstates of infinitely-many others, this means: quantum states lack values for infinitely-many properties at any given instant!
- *Example (Spin)*: The eigenstates of the operator representing the property of being spin → or ← (with respect to some chosen axis) are **superpositions** of the eigenstates of the operator representing the property of being spin ↑ and ↓ (with respect to that axis). So, on the Copenhagen Interpretation, a spin ↑ particle **fails** to have a spin → or ← property!
    - *Details*: $|\uparrow> = \sqrt{(1/2)}|\rightarrow> + \sqrt{(1/2)}|\leftarrow>$. Let the spin ↑ operator be $\mathbf{O}_\uparrow$. Then problem is that spin → and ← states are not eigenstates of the operator $\mathbf{O}_\uparrow$.
        - *Note*: If we *were* to measure the → spin of a spin ↑ particle, we can see that we would get spin → with 50% probability -- since $(\sqrt{(1/2)})^2 = (1/2)$.

- *Example (Position & Momentum)*: Although the details are more involved, the situation is the same. By **Heisenberg's Uncertainty Principle**, the eigenstates of the operator representing the property of having some position fail to be eigenstates of the operator representing the property of having some momentum (and vice versa). So, on the Copenhagen Interpretation, a particle with a position at t **fails** to have a momentum at t!
- Suppose that we obtain λ_i when measuring the observable represented by **O**. Then on the Copenhagen Interpretation, the state of the quantum system instantaneously and discontinuously "collapses" to $|\Psi\rangle = |\lambda_i\rangle$. This is the **Collapse of the Wavefunction**.
- The system is thereby supposed to acquire a value of the property corresponding to O.
    - *Note*: "Measurement" is a misleading term on the Copenhagen Interpretation!
    - *Problem*: Special Relativity says that is no objective order to distant events. So, what does "instantaneously" mean here? If the "measurement" induces collapse (and this cannot happen across time), then, if there are objective facts about collapse, frames in which $|\Psi\rangle$ collapsed before or after measurement are wrong. (The same is thereby true *vis a vis* different observers' judgments of entanglement -- see below.) The Copenhagen Interpretation reintroduces a privileged foliation!
- By contrast, *between* measurements the state of the system, $|\Psi\rangle$, evolves linearly and deterministically (and unitarily), according to the time-dependent **Schrödinger equation**:

$$i\hbar\frac{d}{dt}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle$$

    - *Note:* $\hat{H}$ is the *total energy* operator of the system, called the Hamiltonian.
    - *Note*: Linearity is the requirement that if $|A\rangle$ evolves to $|A'\rangle$, $|B\rangle$ evolves to $|B'\rangle$, and so on, then $c_1|A\rangle + c_2|B\rangle + \ldots$ must evolve to $c_1|A'\rangle + c_2|B'\rangle + \ldots$. Hence, a superposition of $|A\rangle$ and $|B\rangle$ evolves into a superposition of $|A'\rangle$ and $|B'\rangle$.
    - *Note*: This equation too is manifestly not *Lorentz invariant*. Hence, it is at best an approximation of the relativistic equation for the evolution of the state vector.
- *Question (Theoretical Equivalence)*: We are assuming the standard **Schrödinger picture** of quantum mechanics according to which the state vector evolves in time while the observables do not change. (A version of this exists appropriate to quantum field theory as well.) However, there is a **unitary transformation** (giving an empirically equivalent theory) from time-dependent state vectors to time-independent ones, trading

time-independent observables for time-dependent ones. The result is **Heisenberg's picture**, obeying the dynamical equation obtained by replacing **|Ψ(t)>** on the left of Schrödinger's equation with **L(t)**, for some observable, **L**, and **H|Ψ(t)>** on the right by the commutator of L with the Hamiltonian**, [L, H**]. (There is even a kind of mix of the pictures, called the **Dirac-Tomonaga picture**, according to which the time dependence is distributed between states and observables.) And this is just sticking only to Hilbert space pictures! There are also algebraic formulations in terms of so-called **C*-algebras**, **Feynman's path-integral** formulation, **density matrix** formulations, and more. What are we to make of this plethora of formulations? Might they all represent the same thing?

- *Problem*: This is *prima facie* inconsistent! There are two dynamical laws, for two different contexts of observation. When *not* being measured, a state vector evolves in accord with the Schrödinger equation, and it is generally not in an eigenstate of the operator of any property of interest. But when we *measure* the state for that property, the state obeys a different law, instantaneously taking on a value (with probability obtained by expanding the state in a basis of eigenstates of that operator and applying *Born's Rule*). And then the evolution starts all over again -- this time from the new state.
- *Response:* The different laws concern different contexts, so there is no inconsistency.
- *Rejoinder*: In order to distinguish those contexts, we need to know what counts as a measurement. Must a human being sense the system? Must a sentient being? Would a video camera suffice? Whatever answer we proffer, the resulting criterion would seem to be indeterminate, with the result that all manner of states of the world are indeterminate.
    - *Note*: This problem is at the heart of the **Schrödinger's Cat** thought experiment.
- *Clarification*: It is sometimes suggested that mere appeal to **decoherence** allows one to explain collapse using only the linear Schrodinger equation (or a relativistic surrogate). But this is incorrect. The latter preserves superpositions. So, Schrodinger's Cat evolves into a superposition of dead and alive no matter how entangled the system becomes.
- *Question*: What exactly is objectionable about the thought that the fundamental physical laws might be indeterminate? After all, the view that the fundamental mathematical laws -- e.g., concerning all subsets of natural numbers -- might be so is commonly endorsed.
- *Question*: We noted that the probabilities of obtaining measurements are determined by the complex amplitudes. This makes it hard to treat the latter as fictions, despite the

Copenhagen Interpretation's silence on them. But if they are *not* fictions, what are they? Might they be the real entities, and the observables of the Born Rule not fundamental?

Prospects for Realism

- The Copenhagen Interpretation (insofar as it can be pinned down!) is a quintessentially antirealist view. It says that the facts about quantum states counterfactually depend on us.
    - *Compare*: Intuitionism about mathematics or constructivism about morality.
- *Note*: On another interpretation, the Copenhagen Interpretation is instrumentalism, so is antirealist in the way that formalism about mathematics and emotivism about ethics are.
- *Dirac*: "[O]nly questions about the results of experiments have real significance and it is only such questions that theoretical physics have to consider (*Principles*, 4th Edition)."
- Einstein was not satisfied with that interpretation for just this reason. Indeed, he thought that he could prove using the 'mathematical machine' that quantum mechanics was incomplete -- i.e., failed to represent measurement-independent facts about the world.
- In order to understand Einstein's worries, let us introduce formalism for states of collections of quantum systems (these could be multiple systems of a single particle, like position and spin, but we will focus on multiple systems involving multiple particles).
    - Product State: When one system is in state, $|\Omega>$, and another system is in state $|\Phi>$, we write the state of the composite system like this: $|\Psi> = |\Omega>_1|\Phi>_2$.
        - *Example*: If $|\Psi> = |\uparrow>_1|\downarrow>_2$ is the state of two *electron spins*, then the first is spin up (along an axis), and the second is spin down (along the same axis). Such a state is just the conjunction of states of the sub-systems.
        - *Note*: In quantum field theory one gives up on the idea of particle identities, so the labeling of particles here does not really make sense. However, there is a surrogate form of the argument in that context.
    - Since the components of a product state are independent, the probability of getting any combination of results when measuring each is what we would expect: the joint probabilities are the product of the individual ones. (If we toss a die and flip a coin, the probability of getting a 3 and getting heads is $(1/6)(1/2) = (1/12)$.)
    - Composite State: The more interesting, and characteristically quantum, states do not have independent components. They instead have so-called **entangled** states.

- Consider the <u>product states</u>, $|\Omega\rangle = |\uparrow\rangle_1|\downarrow\rangle_2$ and $|\Phi\rangle = |\downarrow\rangle_1|\uparrow\rangle_2$. Since these are each possible states of the system, and since the space of such states forms a vector space, the following must *also* be be a possible state of the system:
  - **Singlet: $|\Psi\rangle = \sqrt{(½)}(|\Omega\rangle - |\Phi\rangle) = \sqrt{(½)}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2)$** (where the $\sqrt{(½)}$s ensure that the probabilities — squared amplitudes — sum to 1.)
  - The state **$|\Psi\rangle$** is a **superposition** of a one in which the first electron is spin up and the second is spin down, and one in which the first is spin down and the second is spin up. (One <u>cannot</u> obtain <u>the same</u> spin results for the two particles because the coefficients for **$|\uparrow\rangle_1|\uparrow\rangle_2$** and **$|\downarrow\rangle_1|\downarrow\rangle_2$** are zero.)
  - *Note:* Such a state can be prepared by emitting them from an event <u>conserving angular momentum</u> (if the particles are left undisturbed).
  - *Note*: There is <u>nothing more to know</u> about the <u>subsystems</u> of this state, as it cannot be written as a product state. Indeed, it is <u>maximally entangled</u>.
- *Observation*: The state $|\Psi\rangle$ is peculiar. It is <u>symmetric under rotations</u>. It has exactly the same form when expressed using <u>any pair of orthogonal spin bases</u>.
  - *Note*: This is very puzzling! According to the Copenhagen Interpretation, it means that there is no axis of spin along which we can say that either particle is spin up or spin down along it. So, any spin quantity (correlate of an operator of which $|\Psi\rangle$ is an eigenstate) must somehow be "holistic".

- What if we decide to measure the spin of electron 1, along the chosen spin axis? It will either pass the test or not, each with 50% probability, and when it does, the state $|\Psi\rangle$ will **collapse** to either $|\Omega\rangle =$ **(**$|\uparrow\rangle_1|\downarrow\rangle_2$ or $|\Phi\rangle = ||\downarrow\rangle_1|\uparrow\rangle_2$. But these are <u>product</u> states — i.e., states which can be expressed as <u>conjunctions</u> of states of the individual systems.
  - Hence, immediately on measurement, electron 2 will <u>also</u> go from having no spin along any axis (according to the Copenhagen Interpretation) to having one.
- <u>Technical Context</u>: In order to combine systems, S_A and S_B, like that of two electrons, one needs a way of **combining their state spaces**. This is done by forming their **tensor product**, **$S_A \otimes S_B$**. The **orthonormal basis vectors** of the combined system are built from possible combinations of the basis vectors of the subsystems (so that if $|ab\rangle$ is a vector in the combined space of $|A\rangle$ and $|B\rangle$, then $\langle ab|a'b'\rangle = \delta_{aa'}\delta_{bb'}$). The **dimension** of the state space of a composite system is thus the <u>product</u> of the dimensions of the state spaces of its component systems. (One cannot add basis vectors from the two spaces.)

EPR Argument

- States like the second above were used by Einstein, Podosky, and Rosen (EPR) to argue that quantum mechanics is incomplete. There are facts of the matter that are independent of our measurements that quantum mechanics fails to represent. Here is one formulation:
  - (1) Suppose that at time, t, the composite system described above were in state **$|\Psi\rangle = \sqrt{(½)}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2)$**, where electrons 1 and 2 are very far apart.
  - (2) Suppose that Alice were to measure the ↑ spin of particle 1, and got YES.
  - (3) Then, immediately after t, at t + dt, Alice (who has studied quantum mechanics!) could know that particle 2, very far away, is spin ↓.
  - (4) Alice would not have disturbed particle 2 as of t + dt.
    - Criterion of Reality: "If without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity (Einstein *et al*., 1935, 777)."
    - (5) So, particle 2 would have already been spin ↓ at t, before particle 1 had been measured.
  - (6) Quantum mechanics does not represent particle 2 as being spin ↓ at t in this situation (it represents its state as a superposition of being spin ↓ and spin ↑).
  - (7) So, quantum mechanics is incomplete.
- Argument (1) — (7) is quite compelling. The key premise is (4), but this seemed to have *a priori* and empirical support, the latter coming from the Theory of Special Relativity. This implies that there are no superluminal -- *a fortiori* instantaneous -- causal influences.
- In fact, since state |Ψ> is symmetric under rotations, (7) can be strengthened. According to the Copenhagen Interpretation, particle 2 has no determinate spin properties before particle 1 is measured. But Alice could have chosen to measure the spin of particle 1 along any axis. Had she, she would have instantaneously known particle 2's spin along it. Since Alice could not have instantaneously disturbed particle 2, quantum mechanics is "infinitely incomplete" — i.e., particle 2 has *infinitely-many* measurement-independent properties which quantum mechanics fails to represent it as having. (This is the case whether or not we could simultaneously measure a particle's spin along different axes.)

- *Note*: If this is right, then the whole apparatus of quantum mechanics seems fundamentally misconceived. No vector can point in two directions at once! So, no vector could represent electrons as having spin properties along even two of the axes.
- *Note*: If one accepts the EPR argument, we would seem to have to reject the <u>disturbance account of uncertainty</u> relations suggested by Feynman and others (following Heisenberg). By measuring particle 1 we could instantaneously determine facts about particle 2 <u>without disturbing it</u>. So, if we <u>cannot</u> measure particle 2's spin along different axes simultaneously, this cannot be <u>merely</u> because we would have to disturb it.
- *Question*: The argument (understandably!) assumes that measurements have unique outcomes. How would giving up this assumption, *a la* Everett, affect the argument?
- *Question*: The EPR argument makes heavy use of **counterfactuals**. Are these avoidable?