Spatial Price Discrimination with Heterogeneous Firms

Jonathan Vogel

Columbia and NBER

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Introduction
Motivation

- Theoretical economists tend to *hold fixed or abstract from* product characteristics/firm location.
- We know product locations in product characteristics space and firm locations in geography play central role determining price elasticities and therefore:
  - economic outcomes
  - responses to policy changes
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- We know product locations in product characteristics space and firm locations in geography play a central role in determining price elasticities and therefore:
  - economic outcomes
  - responses to policy changes
- Could argue we abstract from many aspects of reality.
- But we know from demand system estimation that product characteristics not of second order.
Introduction

Contribution 1

What determines the pattern of location (& firm entry, market share, and profit) in an environment in which heterogeneous firms have the ability to spatially price discriminate?
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  - Geographic space: if producer delivers the good or service, e.g. ready-mixed concrete, janitorial services, ...
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- Not the first to consider spatial price discrimination
  - See e.g. Hoover (1937), Lederer and Hurter (1986), Hamilton, Thisse, and Weskamp (1989), Hamilton, MacLeod, and Thisse (1991), and MacLeod, Norman, and Thisse (1992)
  - These tend to focus on existence
Introduction
Contribution 2

Implements upon empirical content of spatial competition literature in several dimensions

1. Does not impose restrictions on distribution of marginal costs across firms
   - Four-digit SIC industries reviewed in Bartelsman and Doms (2000) have 85th–15th TFP ratios in the range of 2:1 to 4:1
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2. Does not impose restriction on allocation of shipping/customization costs between firms and customers
   - If firms incur costs, consumers can arbitrage away cost differences
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1. Does not impose restrictions on distribution of marginal costs across firms
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3. Includes an entry stage to account for both
   - Within-market location; see e.g. Vogel (2008)
   - Between-market entry and exit decisions; see e.g. Syverson (2004), Jia (2008), and Melitz Ottaviano (2008)
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Contribution 3

Provides tractable model of location that confirms & extends results in different framework (Vogel 2008)
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Provides tractable model of location that confirms & extends results in different framework (Vogel 2008)

- Within a mkt, more productive firms are more isolated, all else equal
- Firm $i$ equilibrium outcomes depend on firm $j$’s characteristics *only* through a market-level parameter
Introduction

Other related literature

- Very little work investigating how *heterogeneous* firms choose their locations in geographic or product characteristics space *within* a market
  - For symmetric firms, see e.g. Hotelling (1929), d’Aspremont, Gabszewicz, and Thisse (1979), and Lancaster (1979)
  - For heterogeneous firms, see e.g. Vogel (2008)

- A large literature considers entry and exit, abstracting from within market locations, building on Bresnahan and Ries (1990, 1991) and Berry (1992)
Introduction

Important simplifying abstractions

- Single dimensional space
- Uniform demand density w/in a market
- Static game
Setup

Consumers

- A mass 1 of strategic consumers uniformly distributed along a unit circumference

- Consumers buy one unit of a homogeneous good from the lowest price source (reservation value, \( v > 0 \))
Setup
Firms

- A set $N$ containing $|N| \geq 2$ potential entrants each of which is endowed with a unique marginal cost of production $c_i \in [0, v - t/2)$
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- Firms play a three-stage game of complete information
  - Entry stage: to enter incur cost $f > 0$, $f \to 0$
    - $f \to 0$ important for uniqueness result
    - can obtain uniqueness without $f \to 0$ under assumption of ordered & sequential entry
  - Location stage: simultaneously choose locations
  - Price stage: simultaneously choose price schedule, $p_i(z)$ for each location $z$
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- Firm $i$ located at point $\xi_i$ selling to point $z$ incurs a \textit{delivered marginal cost} $k_i(\xi_i, z) \equiv c_i + t \|\xi_i - z\|
  - Throughout talk "transport/customization" cost allocated to firm
  - This is WLOG
Setup

Equilibrium concept

- Focus on (weakly) undominated pure strategy subgame perfect Nash Equilibria: "equilibria"

- Define "equilibrium characterization" as \( \{K, x, \pi\} \)
  - \( K \subseteq N \) the set of firms that enter the market
  - \( x \in \mathbb{R}^K \) the vector of market shares of the entrants
  - \( \pi \in \mathbb{R}^K \) the vector of variable profits of the entrants

- In talk focus on case in which \( K > 1 \), but allow \( K = 1 \) in paper
Price stage
Prices solved; equilibrium concept and strategic consumers explained

- Fix # of firms $n \geq 2$, marginal costs $c$, and locations $\eta$
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- Bertrand comp, identical goods, heterogeneous costs, and a continuum of prices $\implies$ two standard technical issues
  1. No pure strategy eqm if consumers not strategic for generic tie breaking rule ($\therefore$ assume strategic consumers)
  2. Continuum of equilibria in which some firm(s) uses weakly dominated strategy ($\therefore$ focus on undominated eqm)
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- The unique equilibrium price at point $z$, denoted $p(z)$, is

$$p(z) = \min_{i \neq \chi(z)} k_i(\eta_i, z)$$

with $\chi(z) \equiv \arg \min_{j=1,...,n} k_j(\eta_j, z)$
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Given locations, can then solve for mkt shares and profits
Price stage

Graphical representation of prices and sales

Slope is $t$

$Y$'s price

$C_X$

$C_Y$

$C_Z$

$X$

$Y$

$Z$

$Y$'s customers

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Location stage

Notation

Define \( \lambda(K) \equiv \frac{1}{|K|} + \frac{2}{t} \overline{c}(K) \)

- \( K \) is a set of \( |K| \) firms with average marginal cost \( \overline{c}(K) \)
- In eqm \( \lambda(K) \) serves as an inverse measure of market toughness (if all firms in \( K \) supply positive measure of consumers)
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  - $K$ is a set of $|K|$ firms with average marginal cost $\bar{c} (K)$
  - In eqm $\lambda (K)$ serves as an inverse measure of market toughness (if all firms in $K$ supply positive measure of consumers)

- Firm $n$ satisfies Condition C relative to $K$ if
  $$c_n < \frac{t}{2} \lambda (K)$$
  i.e. if
  $$c_n - \bar{c} (K) < \frac{t}{2} \frac{1}{|K|}$$
Location stage
Outline of what’s to come...

1. Solve for unique equilibrium characterization if all firms $n \in K$ satisfy Condition C relative to $K$
   - Each firm earns positive var profit
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2. Prove that an equilibrium to the subgame exists if not all firms $n \in K$ satisfy Condition C relative to $K$
   - Steps 1 and 2 $\implies$ an eqm exists to any location-stage subgame
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1. Solve for unique equilibrium characterization if all firms $n \in K$ satisfy Condition C relative to $K$
   - Each firm earns positive var profit

2. Prove that an equilibrium to the subgame exists if not all firms $n \in K$ satisfy Condition C relative to $K$
   - Steps 1 and 2 $\Rightarrow$ an eqm exists to any location-stage subgame

3. If at least one firm does not satisfy Condition C relative to $K$, then at least one firm earns zero variable profit
   - Steps 1 and 3 and $f > 0$ $\Rightarrow$ In any eqm to full game, each entrant must satisfy Condition C relative to set of entrants
Location stage

Step 1: Lemma 1

Suppose the set of firms in the location stage is $K$, $|K| \geq 2$, and each $n \in K$ satisfies Condition C relative to $K$

Then $\exists$ an eqm to the location-stage subgame. In any such eqm, the distance btw any two neighbors $i$ and $i + 1$ is

$$ d_{i,i+1}(K) = \lambda(K) - \frac{2}{t} \left( \frac{c_i + c_{i+1}}{2} \right) , $$

and firm $i$’s mkt share and variable profit are

$$ x_i(K) = \lambda(K) - \frac{2}{t} c_i $$

$$ \pi_i(K) = \frac{t}{2} x_i(K)^2 $$
Location stage

Step 1: Key property - firms "centered in mkt share"

Consider a non-eqm firm Y location btw X and Z:

![Diagram showing X, Y, and Z with Y maintained mkt share, reduced cost, and increased revenue by moving left.](image-url)
Location stage

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Consider a non-eqm firm Y location btw X and Z:

By moving left, Y (i) maintains mkt share, (ii) reduces cost, and (iii) increases revenue
Location stage

Step 1: Implications of "centered" result

Firms "centered in their market shares" yields two key results

1. Firm $i$’s equilibrium outcomes depend on firm $j$’s marginal cost only through $j$’s impact on $\lambda$
   - delivered marg cost at boundary customers determines prices, mkt shares, and therefore var profits
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1. Firm $i$’s equilibrium outcomes depend on firm $j$’s marginal cost only through $j$’s impact on $\lambda$
   - delivered marg cost at boundary customers determines prices, mkt shares, and therefore var profits

2. In eqm, there is a unique $x_i$ and $\pi_i$ for all $i$
Location stage
Step 1: Permissible asymmetries

What is the meaning of restriction on asymmetry, $c_n < \frac{t}{2} \lambda(K)$?
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  \[ g(z) \equiv \min_{j \in K} \{ k_j(z) \} \]
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\[ \frac{t}{2} \lambda(K) \]

\[ g(z) \]

\[ X \quad Y \quad Z \]

- If \( j \) violates Condition C, then \( c_j \geq \frac{t}{2} \lambda (K) \geq g(z) \)
Location stage
Step 2: Existence

Suppose \( \exists j \in K \) s.t. \( j \) does not satisfy Condition C relative to \( K \):
Then there exists an eqm to the location-stage subgame
Location stage

Step 2: Existence

Suppose $\exists j \in K$ s.t. $j$ does not satisfy Condition C relative to $K$:

Then there exists an eqm to the location-stage subgame

- $\exists$ a unique non-empty

$$K^* (K) \equiv \left\{ i \in K | c_i < \frac{t}{2} \lambda [K^* (K)] \right\}$$
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There exists an eqm in which:

- All \( i \in K^* (K) \) locate as Prescribed by Lemma 1 if only \( K^* (K) \) were in the mkt
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  - $g(z)$ unaffected by any $j \in K \setminus K^*(K)$
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  - \( g(z) \) unaffected by any \( j \in K \setminus K^*(K) \) \( \Rightarrow \) no \( i \in K^*(K) \) has an incentive to deviate
  - \( c_j \geq g(z) \) for all \( z \) and all \( j \in K \setminus K^*(K) \)
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  - $g(z)$ unaffected by any $j \in K \setminus K^* (K)$ $\Rightarrow$ no $i \in K^* (K)$ has an incentive to deviate
  - $c_j \geq g(z)$ for all $z$ and all $j \in K \setminus K^* (K)$ $\Rightarrow$ no $j \in K \setminus K^* (K)$ has an incentive to deviate
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Step 3: If Condition C violated, then at least one firm has zero mkt share

If $K \setminus K^* (K)$ not empty, then in any eqm $\exists$ at least one firm that supplies a mass zero of consumers.
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If $K \setminus K^*(K)$ not empty, then in any eqm $\exists$ at least one firm that supplies a mass zero of consumers

- If all firms have $x_i > 0$, then in any eqm distances, mkt shares, and variable profits given by Lemma 1
  
  $\Rightarrow x_i(K) = \lambda(K) - \frac{2}{t} c_i$
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  \[ \Rightarrow x_i(K) = \lambda(K) - \frac{2}{t} c_i \]
- If $K \setminus K^*(K)$ not empty, then $\exists i$ s.t. $x_i(K) \leq 0$, contradiction
Entry Stage

Starting from the full set of pot. entrants $N$, there is a unique set $K^*(N)$ s.t.

- if $K^*(N)$ enter, then $\pi_i[K^*(N)] > 0$ for all $i \in K(N)$
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  1. $\exists j \in V$ s.t. $\pi_j(V) = 0$
  2. $\exists j \notin V$ s.t. $\pi_j(V \cup j) > 0$

There exists an $f(N) > 0$ such that for all $f < f(N)$ an equilibrium exists, and the unique equilibrium characterization is given by $f_K(N), x, \pi$, where $d_i, i+1, \pi_i, and x_i$ are as given in Lemma 1 w/ $K = K(N)$. 

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  \( \{ K^* (N), x, \pi \} \), where $d_{i,i+1}$, $\pi_i$, and $x_i$ are as given in Lemma 1 w/ $K = K^* (N)$. 
Conclusions

- Provided a tractable model of endogenous product differentiation
  - amenable to extensions, including elastic demand
- Extended spatial competition to better accord with empirical regularities
- Confirmed previous results obtained in a different framework
  - differences in productivity are reflected in location decisions through isolation