

Maximizing the number of colorings

Date Wednesday, March 25

Time 3 pm

Location 303 Mudd

Abstract: Let $P_G(q)$ denote the number of proper q -colorings of a graph G . This function, called the chromatic polynomial of G , was introduced by Birkhoff in 1912, who sought to attack the famous four-color problem by minimizing $P_G(4)$ over all planar graphs G . Since then, motivated by a variety of applications, much research was done on minimizing or maximizing $P_G(q)$ over various families of graphs.

In this work, we study an old problem of Linial and Wilf, to find the graphs with n vertices and m edges which maximize the number of q -colorings. We provide the first approach which enables one to solve this problem for many nontrivial ranges of parameters. Using our machinery, we show that for each $q \geq 4$ and sufficiently large $m < \kappa_q n^2$ where $\kappa_q \approx 1/(q \log q)$, the extremal graphs are complete bipartite graphs minus the edges of a star, plus isolated vertices. Moreover, for $q = 3$, we establish the structure of optimal graphs for all large $m \leq n^2/4$, confirming (in a stronger form) a conjecture of Lazebnik from 1989.

Joint work with Oleg Pikhurko and Benny Sudakov.