Scott's Conjecture for Necklaces

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Abstract: A class $\mathcal{G}$ of graphs is χ-*bounded* if there is a function $f : \mathbb{N} \to \mathbb{N}$ such that for all $G \in \mathcal{G}$, $\chi(G) \leq f(\omega(G))$. χ-bounded classes were introduced in 1987 by András Gyárfás as a generalization of the class of perfect graphs. Gyárfás conjectured that for any tree T, the class of graphs that do not contain T as an induced subgraph is χ-bounded. In 1997, Alex Scott proved a 'topological' version of this conjecture: for any tree T, the class of graphs that do not contain any subdivision of T as an induced subgraph is χ-bounded; he then conjectured that for every graph H, the class of graphs that do not contain any subdivision of H as an induced subgraph is χ-bounded. While Scott's conjecture remains open, progress has been made in some special cases. In this talk, I will present a proof of Scott's conjecture for the case when the graph H is a *necklace* (i.e. a graph obtained from a path by choosing a matching such that no edge of the matching is incident with an endpoint of the path, and for each edge of the matching, adding a vertex adjacent to the ends of this edge). We remark that the *bull* (a five-vertex graph consisting of a triangle and two vertex-disjoint pendant edges) is a special case of a necklace, and so this result generalizes an earlier result (due to Chudnovsky, Scott, Trotignon, and the speaker) that the class of graphs that do not contain any subdivision of the bull as an induced subgraph is χ-bounded.

This is joint work with Maria Chudnovsky, Alex Scott, and Nicolas Trotignon