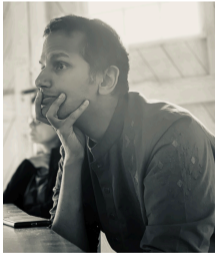


# Sequential Veto Bargaining with Incomplete Information

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## Veto bargaining important in politics & orgs

- Legislatures send bills to Executives
- Executives need legislatures to confirm appointments
- Search committees put forward candidates to their higher-ups
- Boards of Directors require sign-off from shareholders



*"If Congress returns the bill having appropriately addressed these concerns, I will sign it. For now, I must veto the bill."*

# Veto Bargaining

**Veto bargaining:** (bilateral) bargaining with **single-peaked prefs** and **one-sided offers**

- **Proposer** and **Vetoer**
- 1-dimensional policy

**Romer and Rosenthal (1978)**

- TIOLI offer with complete information
- Proposer targets Vetoer precisely
  - no vetoes, but Vetoer's ideal point affects outcome, even if she doesn't obtain any surplus

# This Paper

Analysis omits two (related) features:

- Proposer doesn't know Vetoer's ideal point  
→ *Cannot target precisely*
- Sequential proposals  
→ *Proposer can learn from past rejections*  
→ *But Vetoer may now strategically reject*

## Results

- **Commitment payoff is achievable**
- Such eqa exploit **leapfrogging**  
→ owes to single-peaked prefs  
→ unlike usual monopolist
- Other eqa can coexist  
→ with Coasian dynamics

◇ How much does Proposer benefit from sequential proposals?

◇ **Does lack of commitment (significantly) hurt Proposer?**

*Coasian Conjecture:* Proposer cannot avoid moderating proposals after rejection, so much so that he is at the mercy of Vetoer's private info

## Existing Work

### Sequential veto bargaining

- Romer & Rosenthal 1979; Cameron 2000; Rosenthal & Zame 2019; Chen 2021
- Cameron & Elmes 1995; Evdokimov 2022

### Coase Conjecture in seller-buyer settings

- FLT 1985; GSW 1985; AD 1989

### Non-Coasian logic in seller-buyer settings

- Board & Pycia 2014; Tirole 2016
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Model

# Model

At each  $t = 0, 1, \dots$ , **Proposer** makes a proposal  $a_t \in \mathbb{R}$  that **Vetoer** can accept or reject

Game ends when Vetoer accepts

If agreement is reached in period  $T$ , payoffs are

$$\delta^T u(a_T) \quad \text{and} \quad \delta^T u_V(a_T, v)$$

- Until agreement, flow utility from **status quo**,  $a = 0$ ; normalize this utility to 0
- After agreement, flow utility from  $a_T$
- So utility measured as gain over status quo

## Single-peaked preferences

- **Proposer's ideal point** known to be 1
- **Vetoer's ideal point** is  $v$ , her type, which is **private info**

Study PBE

Nb: can interpret Vetoer as a voting group, so long as Proposer only observes outcome, not vote profile

Example



## Two-Type Example

**Proposer**  $u(a, v) = 1 - |1 - a|$

(constants normalize  $u(0) = u_V(0, v) = 0$ )

**Vetoer**  $u_V(a, v) = v - |v - a|$

Vetoer type  $v \in \{l, h\}$ , with

$$0 < l < 1/2 < h < 2l < 1$$

Under complete information,  $a(h) = 1$  and  $a(l) = 2l$

But this violates IC for  $h$

Proposer's **optimal delegation set** (deterministic static mechanism) is either

- Pooling menu  $\{2l\}$
- Separating menu  $\{a^*, 1\}$ , with  $h$  indiff between  $a^*$  and 1 ✓ **more interesting case**

# The Sequential Rationality Problem

In our dynamic game without commitment, when players are patient, can Proposer obtain action **1** from type  $h$  and  $a^*$  from  $l$ ?

Standard “skimming” recipe:

- Propose 1 at  $t = 0$ , which is accepted by  $h$
- If rejected, propose  $a^*$  at  $t = 1$ , which is accepted by  $l$

(perhaps modulo some discounting adjustments)

**But not an equilibrium!**

- After rejection at  $t = 0$ , Proposer believes Vetoer type is  $l$
- Sequential Rationality  $\implies$  at  $t = 1$  propose  $2l > a^*$
- But **anticipating  $2l$** , type  $h$  **rejects 1 at  $t = 0$**

# The Leapfrogging Solution

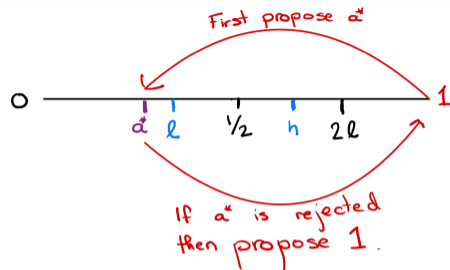
Modulo discounting adjustments:

**First propose  $a^*$**  (receives ! in chess annotation)

- Accepted only by type  $l$

**Only then propose 1 forever**

- Accepted by type  $h$



**Key idea:** By first securing agreement with  $l$ , sequential rationality no longer impels Proposer to moderate should  $h$  subsequently reject

Owes to single-peaked Vetoer prefs

→ Futile in monopoly pricing; indeed, all equilibria there have skimming

# A Non-Constructive Argument

## Result (Two types)

Assume the optimal delegation set has separation. When players are patient, Proposer can achieve (at least) approximately the delegation payoff.

### Proof:

Let  $a^\delta$  be lowest action s.t.  $h$  is indifferent between  $a^\delta$  today and 1 tomorrow.

Note that  $a^\delta \rightarrow a^*$  as  $\delta \rightarrow 1$ .

- If Proposer proposes  $a^\delta$  in first period,  $l$  accepts and  $h$  rejects. After rejection of  $a^\delta$ , Proposer believes  $\Pr(h) = 1$  and proposes 1 forever.
- If Proposer proposes  $a \neq a^\delta$  in first period, play some continuation equilibrium.
- In first period, Proposer chooses an optimal proposal.

So either Proposer uses  $(a^\delta, 1)$  on path, or follows another path that is even better.

# An Equilibrium Construction

An equilibrium construction is quite involved

## Natural construction

- First propose  $a^\delta$
- If rejected, propose 1 ever after
- Type  $l$  accepts  $a^\delta$   
and type  $h$  accepts 1

## Potential deviation

- First offer a high action
- Type  $h$  may accept, and  
Proposer may be better off

Resolved by **Proposition 1**, which distinguishes three cases:

- Skimming.**  $\Pr(h)$  low: skimming approximates the pooling outcome, which is optimal.
- Leapfrogging.**  $\Pr(h)$  moderate: on path offers  $(a^\delta, 1)$ .
- Delayed leapfrogging.**  $\Pr(h)$  high: first offer 1; in second period mix between leapfrogging and skimming. Type  $h$  mixes in the first period to justify Proposer's indifference.

## Wrap-up of Example

Example illustrates why **leapfrogging** works

and how it delivers a high payoff by weakening seq rationality constraint

**Limitations** of example, beyond specificity

- are there equilibria that attain even higher or lower Proposer payoffs?
- why is the optimal delegation payoff the right benchmark?
  - **commitment in dynamic game?**

## General Analysis

## Payoffs and Types

Proposer's  $u(a)$  is (weakly) concave with a unique maximum at 1; and  $u(0) = 0$

Vetoer's  $u_V(a, v) \equiv -(a - v)^2 + v^2$

- Normalized so that  $u_V(0, v) = 0$
- Single-crossing expectational differences (SCED); Kartik, Lee, Rappoport (2019)
- **Interval choice**: set of types willing to accept any offer is an interval

Vetoer's type  $v \sim F \in \mathcal{F}$

- $\mathcal{F}$ : CDFs with density bounded away from 0 and  $\infty$  on an interval support
- Denote support of  $F$  by  $[\underline{v}, \bar{v}]$
- $\bar{v} \leq 1$  (for simplicity)



## Auxiliary Static Problem

Auxiliary **static mechanism design** problem:

$$\mathcal{S} \equiv \{m : [\underline{v}, \bar{v}] \rightarrow \Delta(\mathbb{R}) \text{ s.t. IC and IR}\} \quad (+ \text{ integrable; finite mean and variance lotteries})$$

$$U(F) \equiv \max_{m \in \mathcal{S}} \int u(m(v)) dF(v) \quad \text{Proposer's optimum}$$

- **Stochastic mechanisms** are allowed
- This problem studied by Kartik, Kleiner, Van Weelden (2021)

### Assumption (Interval delegation is optimal)

An interval delegation set  $[c^*, 1]$  solves Proposer's static problem.

- Simple, deterministic mechanism
- Types above  $c^*$  get ideal point, types in  $(c^*/2, c^*)$  get  $c^*$ , types below  $c^*/2$  get the SQ 0
- KKVW derive sufficient conditions: e.g.,  $f$  logconcave and  $u$  linear-quadratic

## An Upper Bound

Why is the static problem relevant to our dynamic game?

**Lemma (Upper bound on Proposer's payoff)**

Proposer's payoff from any strategy, given a Vetoer best response, is at most  $U(F)$ .

Invoking an auxiliary static problem is familiar from seller-buyer bargaining

Here, absent transfers, important that static problem allow for stochastic mechanisms

# An Upper Bound

Why is the static problem relevant to our dynamic game?

## Lemma (Upper bound on Proposer's payoff)

Proposer's payoff from any strategy, given a Vetoer best response, is at most  $U(F)$ .

Proof idea:

- Time-stamped allocation  $(a, t) \mapsto$  static lottery ( $a$  w.pr.  $\delta^t$ ; 0 w.pr.  $1 - \delta^t$ )
- Payoff equivalent for Proposer and all Vetoer types
- Because Vetoer is playing a best response, resulting static mechanism is IC and IR

Lemma holds even if game form allowed cheap talk, menus, etc.

Lemma  $\implies$  we can refer to  $U(F)$  as **commitment payoff** (at least upper bound on)

# Main Result

## Theorem (Commitment payoff is achievable)

Assume an equilibrium exists for all  $\delta$  and beliefs in  $\mathcal{F}$ .

When players are patient,  $\exists$  eqm with Proposer payoff approx. his commitment payoff.

- Lack of commitment does not hurt Proposer, given his favorite eqm
- Unless  $c^* = 1$  (“no compromise”), sequential proposals strictly better than just TIOLI
- **Non-Coasian**: if  $0 < 2\underline{v} < c^*$ , Coasian dynamics suggest compromising down to  $2\underline{v}$ ; not seq rational to stop at  $c^*$  when there are pos-surplus types for whom  $c^*$  is unacceptable  
→ note that  $\underline{v} > 0$  is the “gap case”

# Main Result

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Assume an equilibrium exists for all  $\delta$  and beliefs in  $\mathcal{F}$ .

When players are patient,  $\exists$  eqm with Proposer payoff approx. his commitment payoff.

### Proof ideas:

- $[c^*, 1]$  remains an optimal mech  $\forall$  beliefs  $F_{[v,c]}$  with  $c \geq c^*$  and for  $F_{[c^*/2, c^*]}$  (Lemma 2)
  - Uses SCED and interval delegation structure
- If belief is  $F_{[v, c^*]}$ , use **option to leapfrog** to obtain commitment payoff (Lemma 3)
  - Option to follow path of first offering 0 and then  $c^*$  forever
  - If all types below  $c^*/2$  accept first offer 0, then  $c^*$  is an optimal second offer by Lemma 1 (static mech is upper bound) and Lemma 2, given that it is accepted by all remaining types
- More involved: use induction to extend from  $F_{[v, c^*]}$  to  $F_{[v, \bar{v}]}$ , applying Lemmas 1 & 2

Note: we do not construct a commitment-payoff eqm (cf. two types)

## Coasian Equilibria

So far: maximum Proposer payoff. But can other eqa coexist, perhaps with a Coasian flavor?

**Full Delegation:** interval delegation set  $[2 \max\{0, \underline{v}\}, 1]$

- Vetoer gets much discretion; if  $\underline{v} = 0$ , every Vetoer type gets her first best
- Proposer only minimally exploiting his bargaining power
  - Caveat: full delegation can sometimes be an optimal mech

## Coasian Equilibria

So far: maximum Proposer payoff. But can other eqa coexist, perhaps with a Coasian flavor?

### Proposition (Coasian dynamics)

If  $\underline{v} \leq 0$  or  $\bar{v} \leq 1/2$ ,  $\exists$  **skimming eqm**; at patient limit, **outcome is full delegation**.

- Resolves eqm existence
- Construction adapts “dynamic programming” arguments from seller-buyer analyses
- But single-peakedness necessitates some differences
- When  $\underline{v} > 0$ , have to deter low-offer deviations (**leapfrogging** is salient!);  
 $\bar{v} \leq 1/2$  ensures that any such deviation can be accepted by all types, hence unattractive

→ **Norms** can matter in veto bargaining: requires sequentiality and incomplete info

## Related Literature

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Conclusion

# Conclusion

Bilateral bargaining over policy: **single-peaked preferences**

Proposer is **uncertain** of Vetoer's ideal point, and can make sequential proposals

**Takeaway #1:** Leapfrogging behavior

- First secure agreement with low types
  - weaken subsequent sequential rationality constraints
  - thereby extract surplus from high types
- Absent when dividing a dollar/monopoly pricing

**Takeaway #2:** Commitment payoff can be achieved

- Fundamentally non-Coasian

**Takeaway #3:** Other equilibria can coexist

- Coasian intuition has some merit: full delegation can arise
- Norms can matter

**Thank you!**