Lecture 11: Logistic Regression III—Ordered Data

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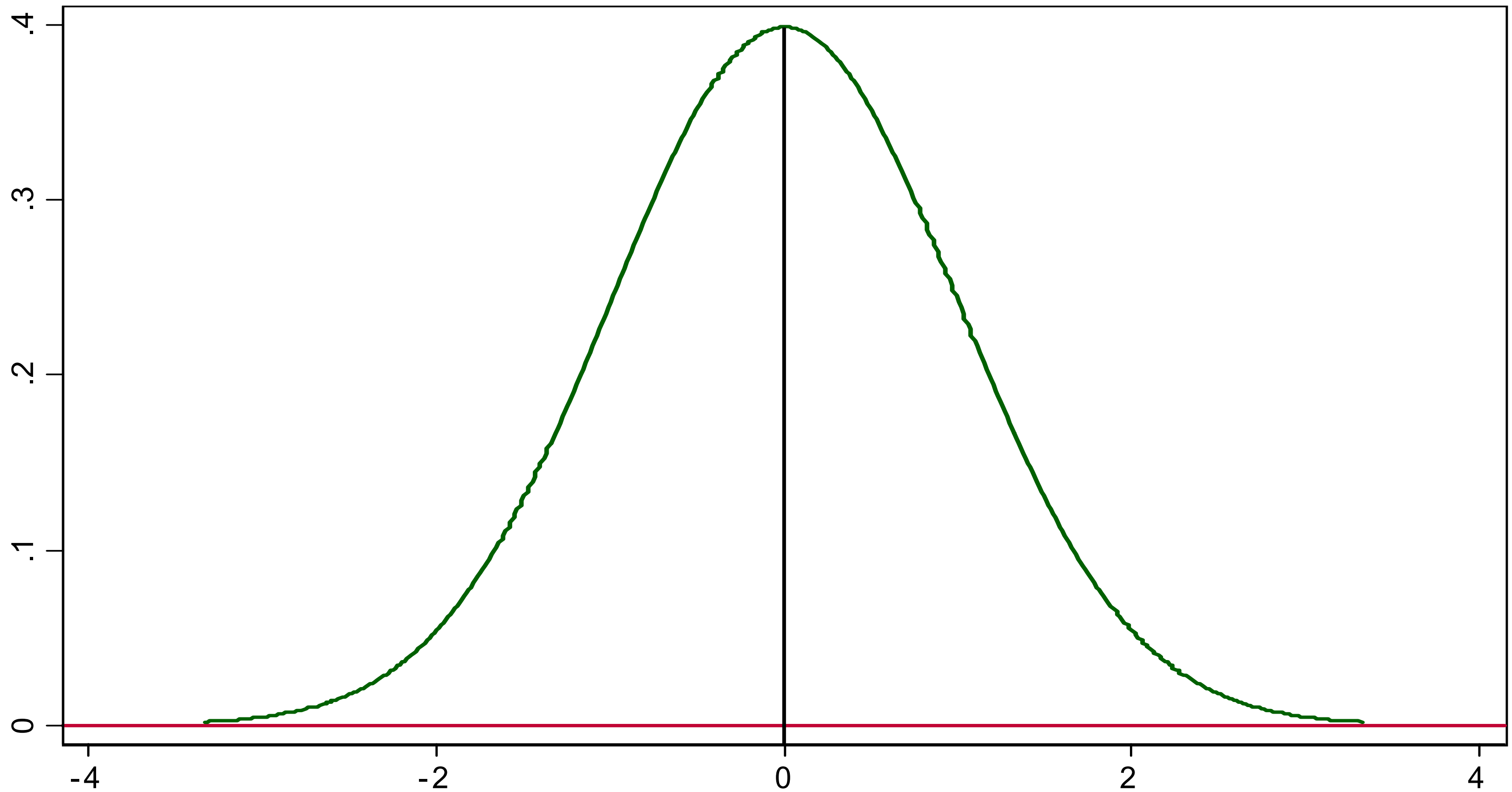
Sustainable Development U9611

Econometrics II

Logit/Probit Review

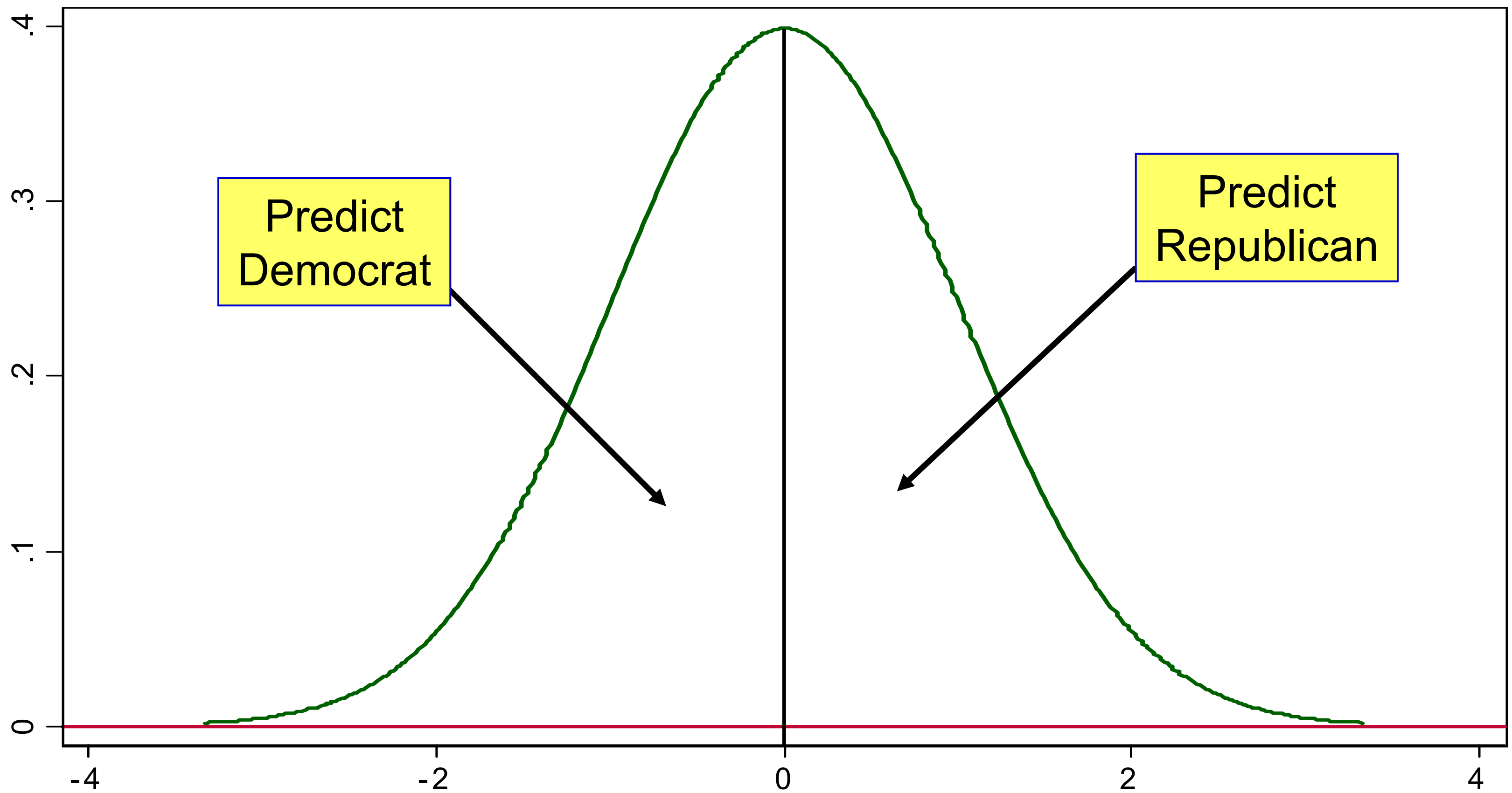
- We first looked at logit and probit estimation in the context of a binary dependent var.
- Then we added the possibility of 3 or more <u>unordered</u> categories for the dependent var.
  - You estimate these using multinomial logit
- Now we'll turn to the case of 3 or more <u>ordered</u> categories
  - Partisan attachment: Strong Rep. → Strong Dem.
  - Educational Attainment: < High School → High School → College → Graduate Degree

Binary Probit Estimation



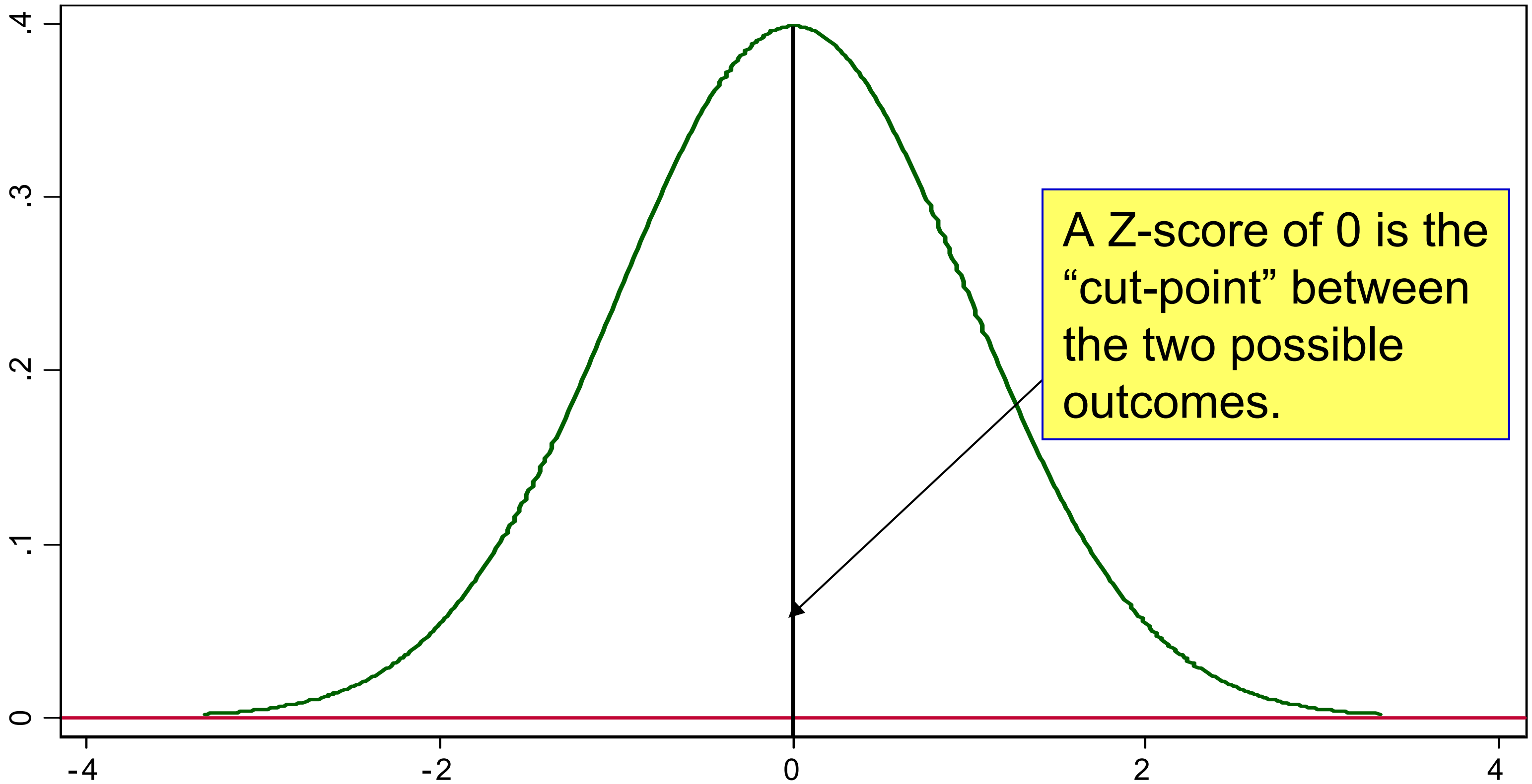
- Binary dependent variable Y = 0 or 1 (e.g., elect Democrat or Republican)
- Estimate Probit(Y) = $\beta_0 + x_1{*}\beta_1 + x_2{*}\beta_2 + \ldots$ with Stata, get β coefficients
- Calculate $X_i\beta = \beta_0 + x_{1i}{*}\beta_1 + x_{2i=}{*}\beta_2 + \ldots$ for each observation

Binary Probit Estimation



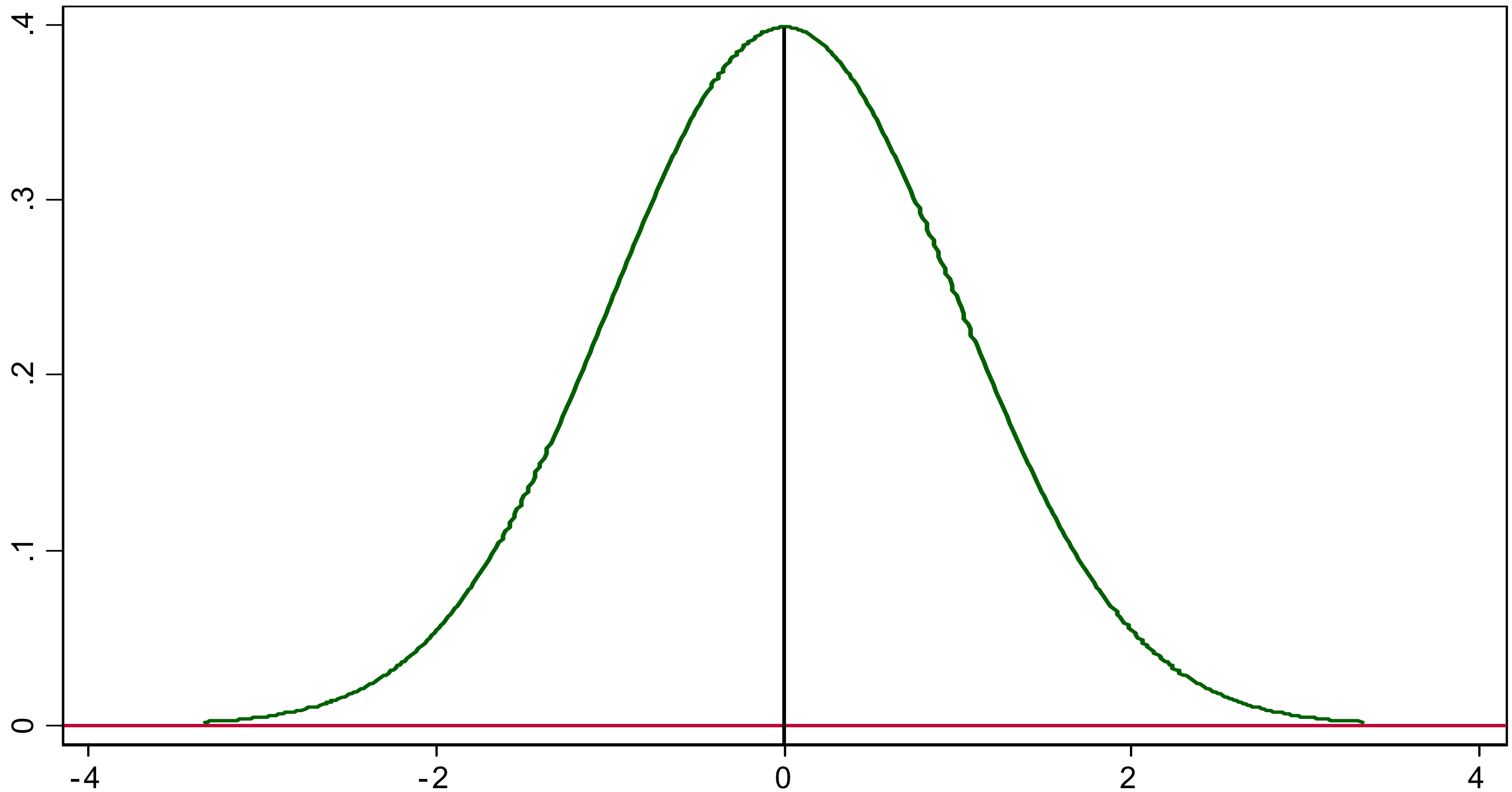
- For observation i, if $X_i\beta < 0$ then predict $Y_i = 0$; if $X_i\beta > 0$ predict $Y_i = 1$

Binary Probit Estimation



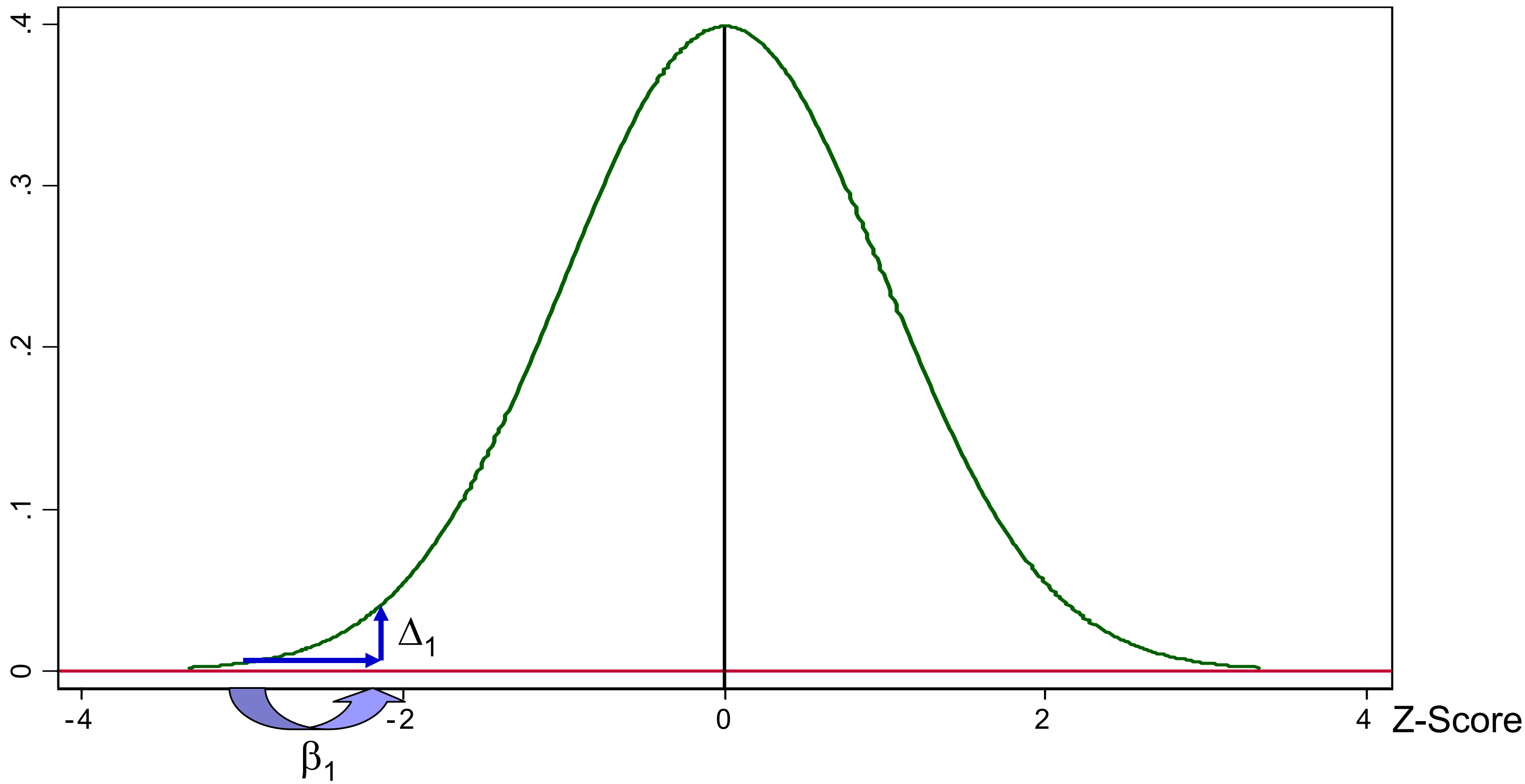
- Since 0 is the dividing line, we call this the "cut-point" of the estimation.
- Note: with logit instead of probit, just use an extreme value distribution instead of normal

Binary Probit Estimation



- Interpretation of β_1: increasing x_1 by one unit changes the Z-score by β_1 units
- The impact of this on Prob(Y=1) depends on your starting point

Binary Probit Estimation



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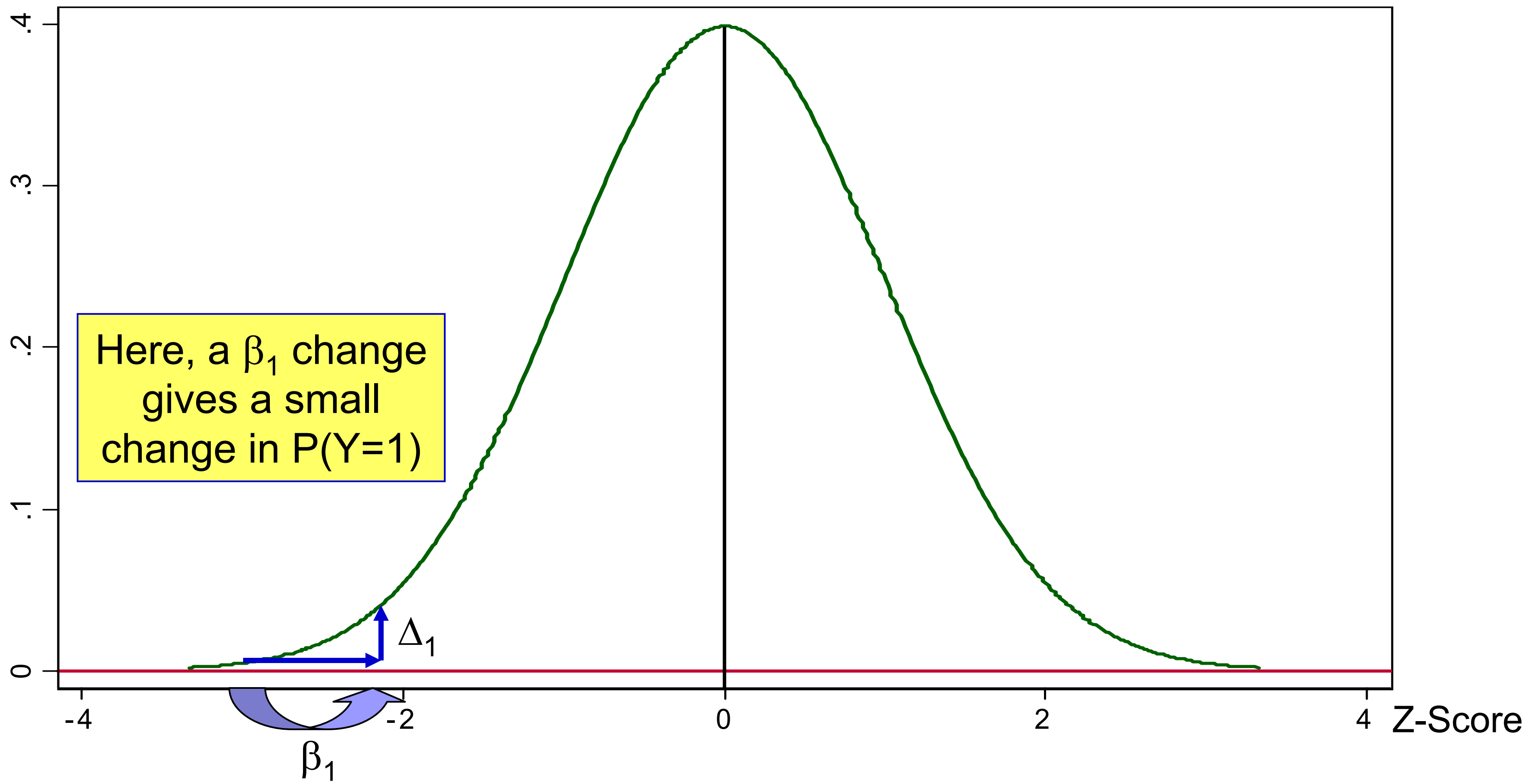
Binary Probit Estimation



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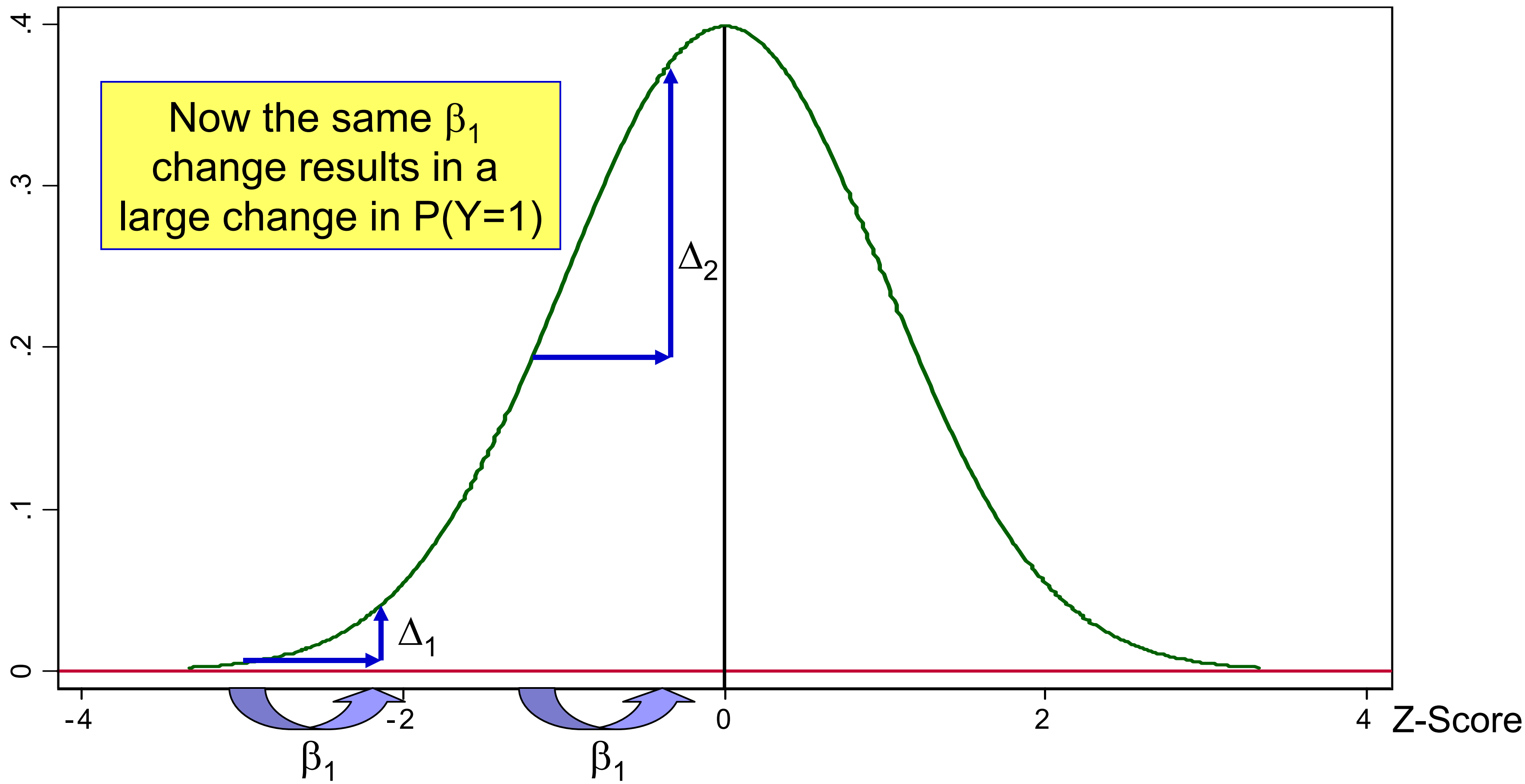
Binary Probit Estimation



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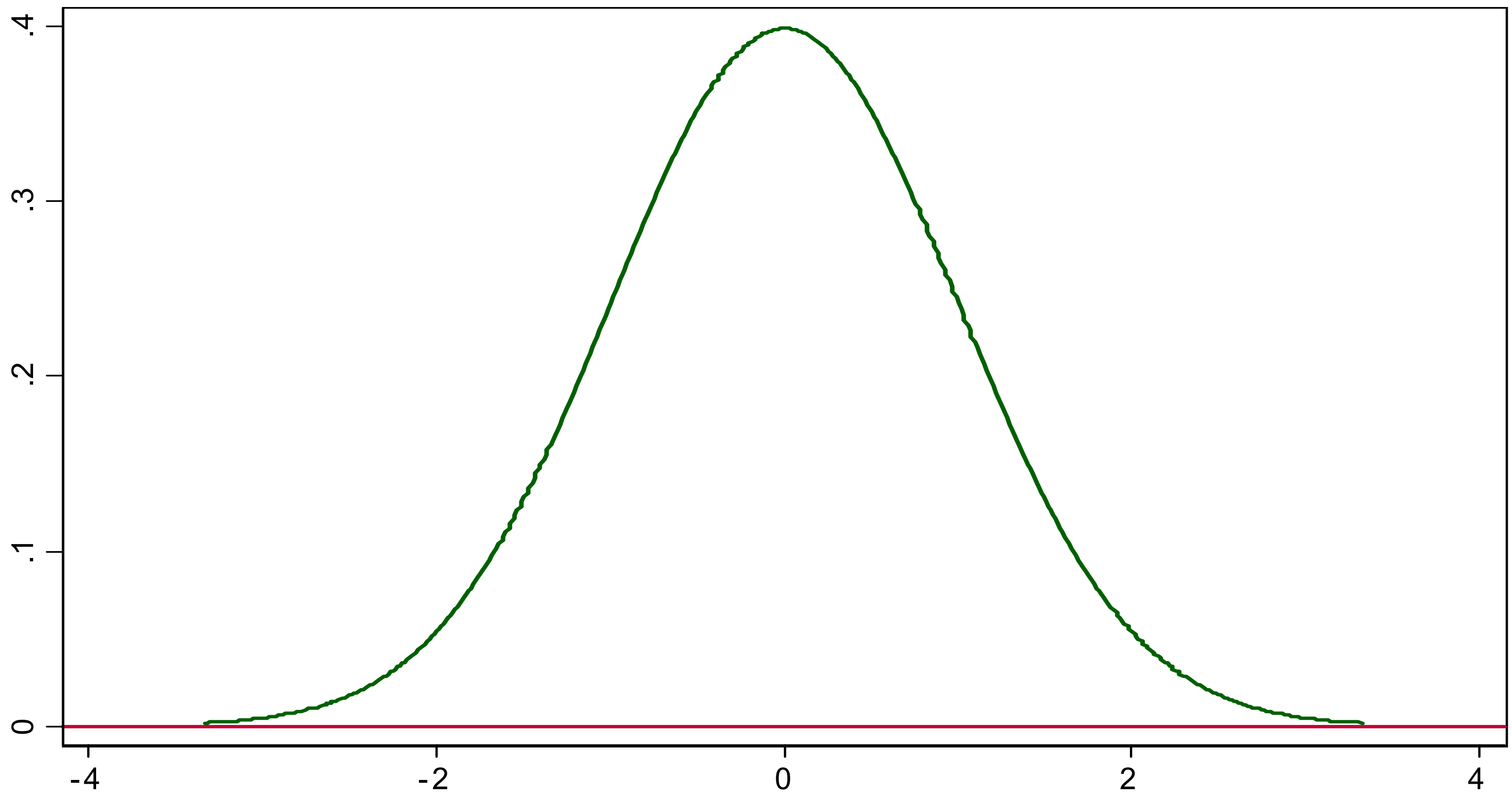
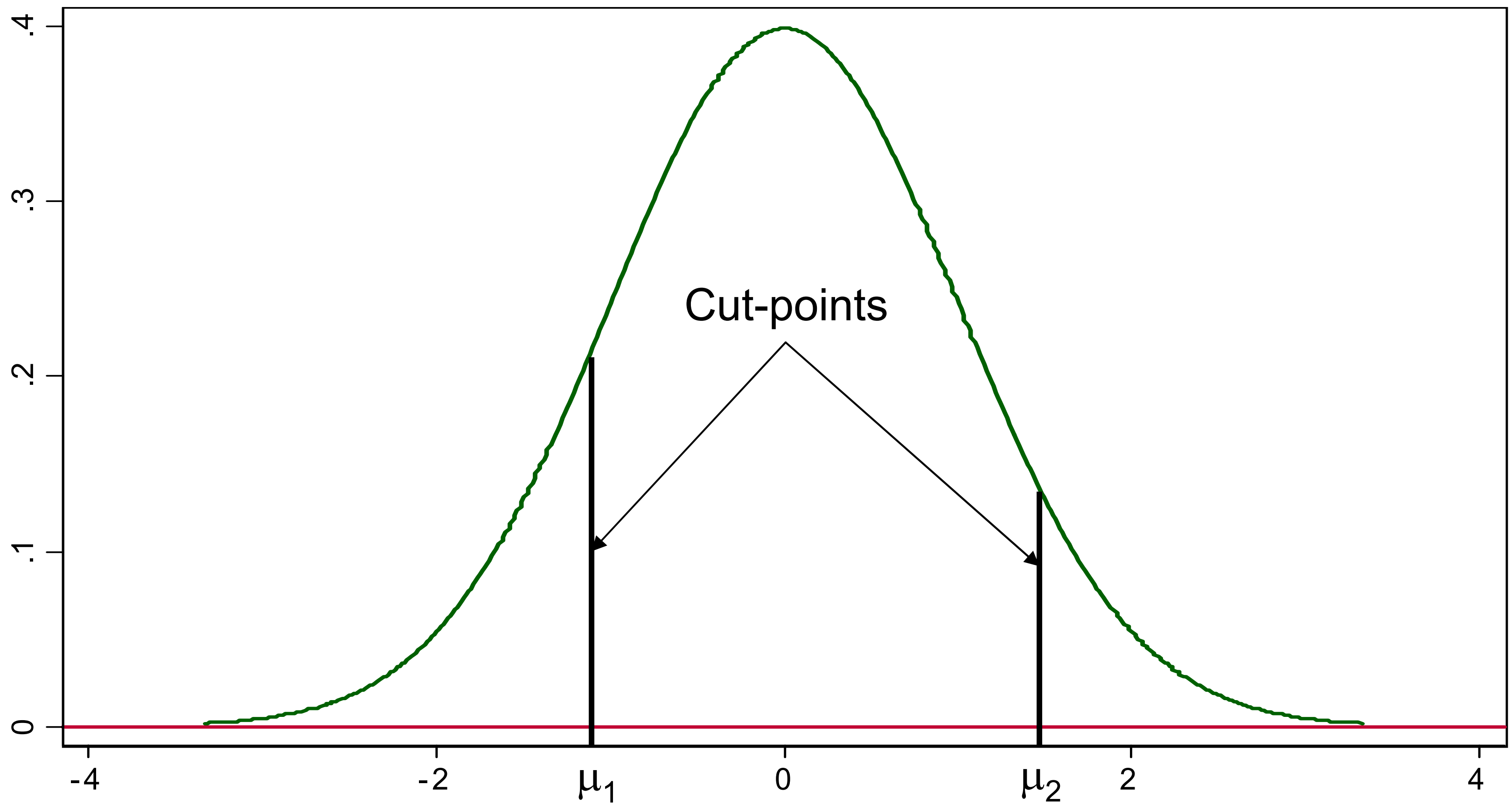
Ordered Probit Estimation



- Assume Y has <u>more</u> than two ordered categories (for instance, Low, Medium, High)
- We now need <u>two</u> cut-points to divide the curve into three sections
- Stata will estimate these as μ_1 and μ_2 by the maximum likelihood procedure

Ordered Probit Estimation



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•We now need <u>two</u> cut-points to divide the curve into three sections

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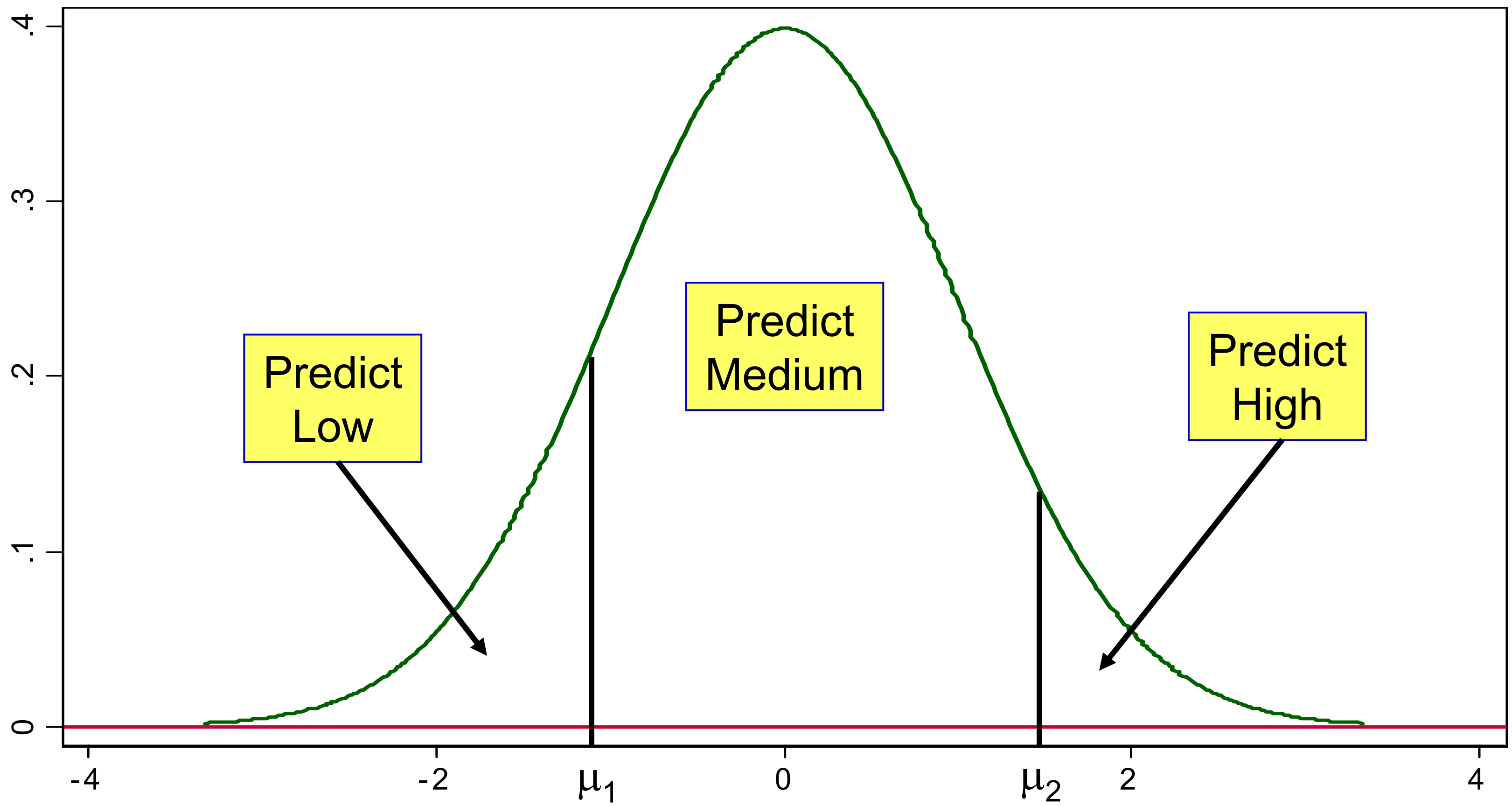
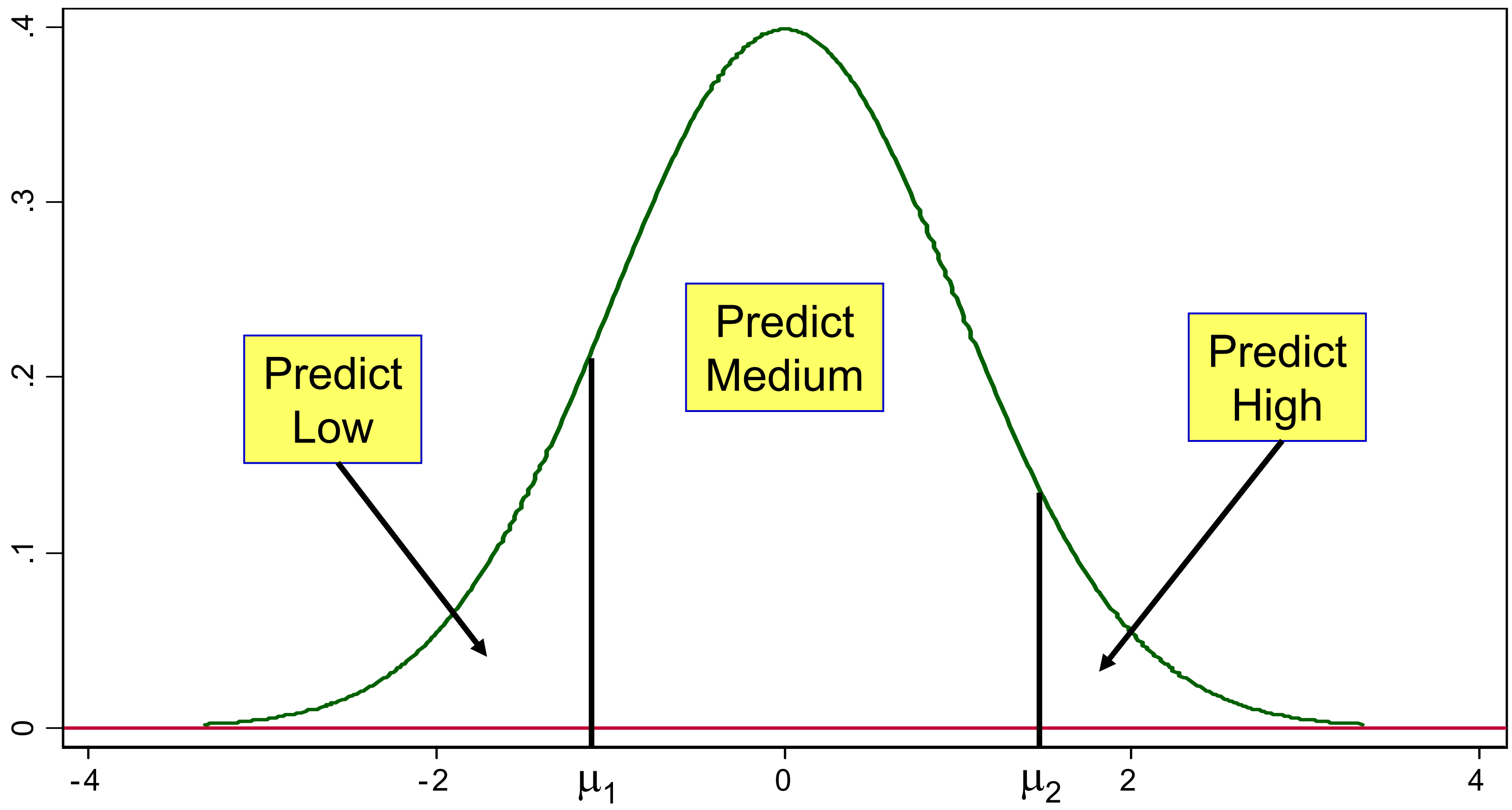
Ordered Probit Estimation



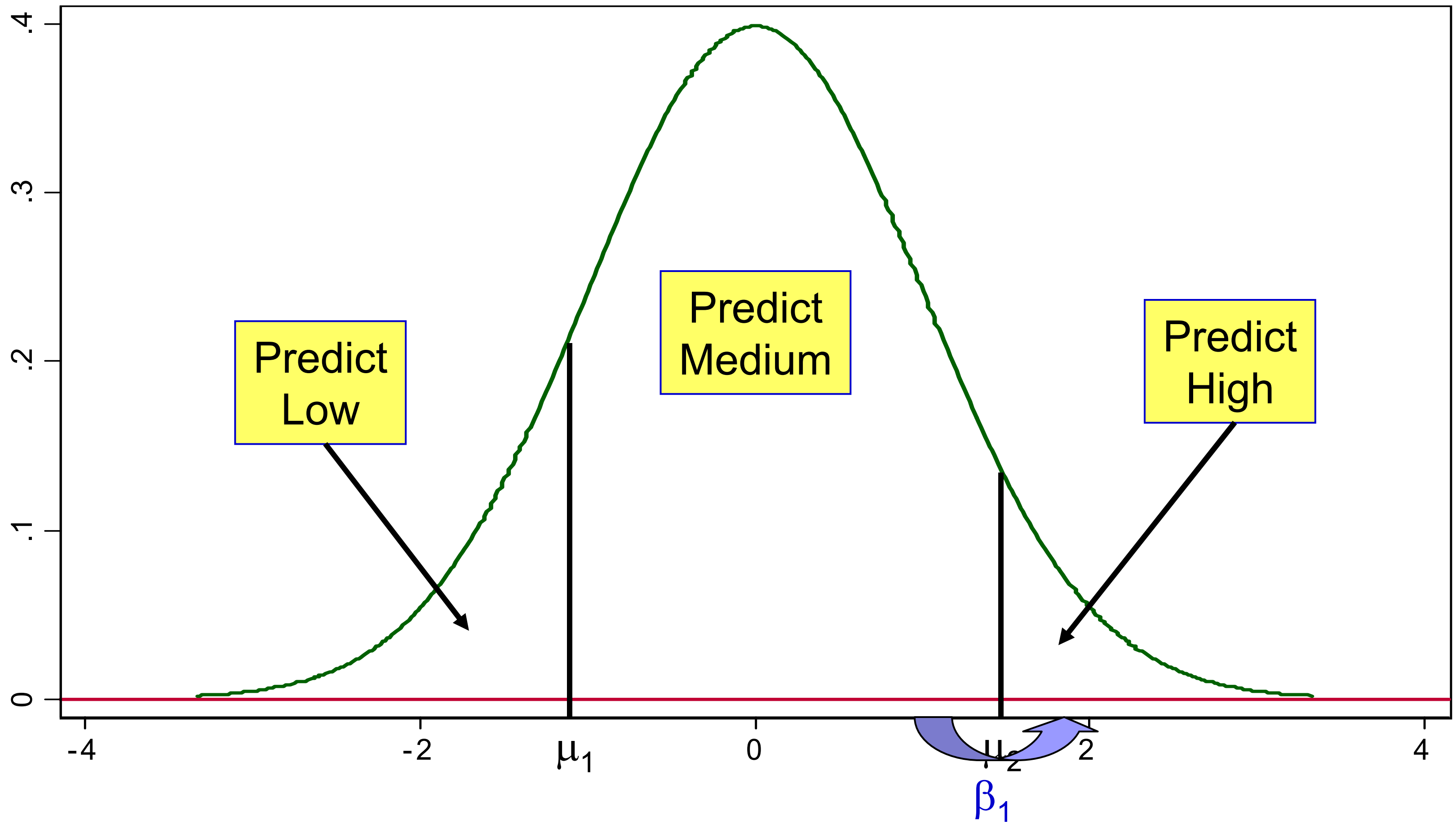
- If $X_i\beta < \mu_1$ then predict Y_i = Low
- If $\mu_1 < X_i\beta < \mu_2$ then predict Y_i = Medium
- If $X_i\beta > \mu_2$ then predict Y_i = High

Ordered Probit Estimation



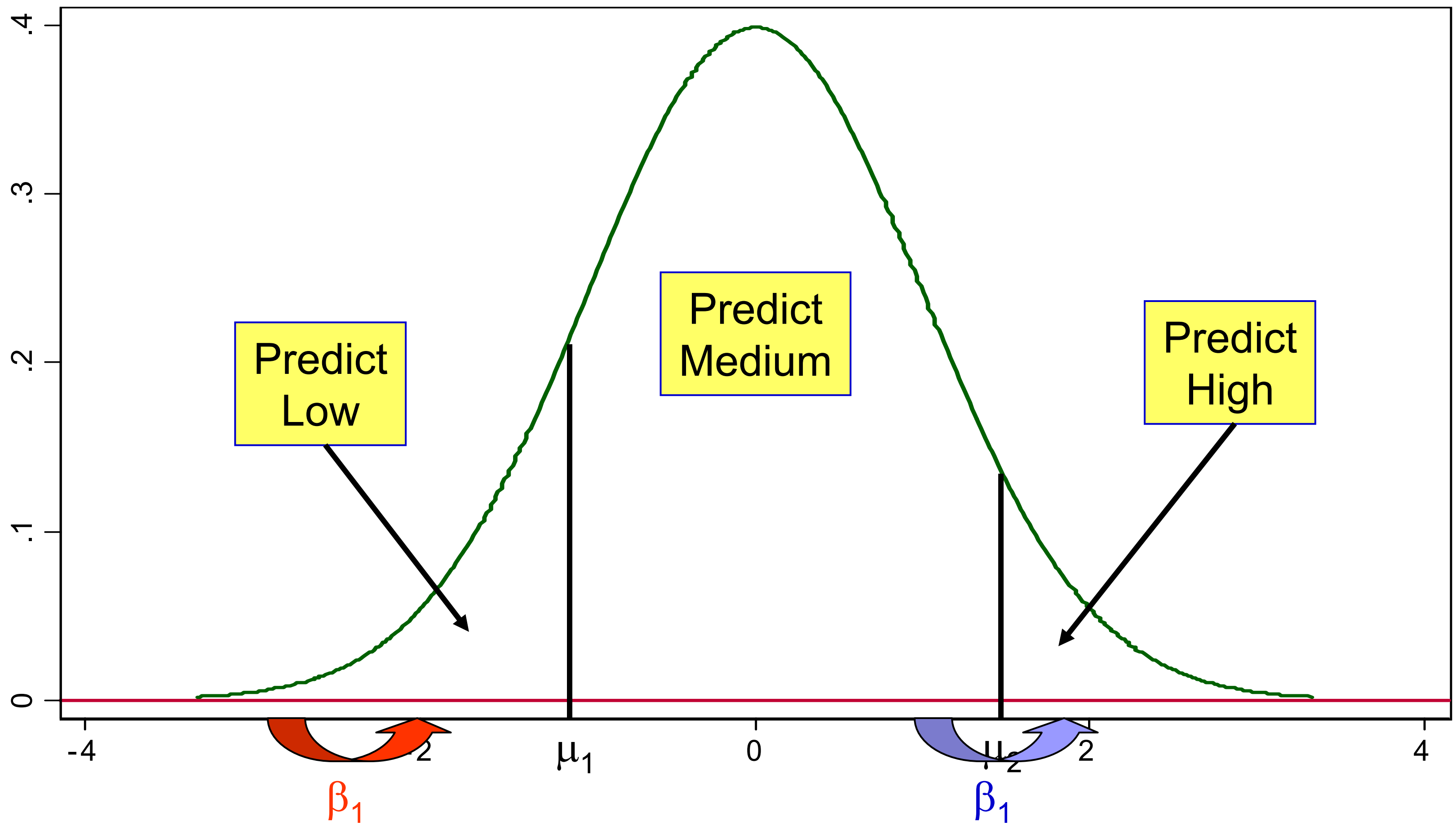
- As before, a β_1 coefficient shifts the Z-score by that amount

Ordered Probit Estimation



- As before, a β_1 coefficient shifts the Z-score by that amount
- This may change the prediction on Y…

Ordered Probit Estimation



- As before, a β_1 coefficient shifts the Z-score by that amount
- This may change the prediction on Y, or it may not

Example: Swedish Partisanship

```
. tab leftright
                      |      Freq.     Percent        Cum.
----------------------+-----------------------------------
   Clearly to the left |      1,373       13.29       13.29
  Somewhat to the left |      2,438       23.59       36.88
Neither left nor right |      3,335       32.28       69.16
 Somewhat tot he right |      2,314       22.39       91.55
  Clearly to the right |        873        8.45      100.00
----------------------+-----------------------------------
                 Total |     10,333      100.00

. recode leftright (1=1) (2=1) (3=2) (4=3) (5=3), gen(lr)

. tab leftright lr
     On the left-right |   RECODE of leftright (On the
      political scale, |   left-right political scale,
 where would you place |     where would you place yo
              yourself |         1          2          3 |     Total
----------------------+---------------------------------+----------
   Clearly to the left |     1,373          0          0 |     1,373
  Somewhat to the left |     2,438          0          0 |     2,438
Neither left nor righ |         0      3,335          0 |     3,335
Somewhat tot he right |         0          0      2,314 |     2,314
  Clearly to the right |         0          0        873 |       873
----------------------+---------------------------------+----------
                 Total |     3,811      3,335      3,187 |    10,333
```

Example: Swedish Partisanship

```
. lab def lr 1 "Left" 2 "Middle" 3 "Right“

. lab val lr lr

. tab gender lr, row


               |                  lr
        Gender |      Left     Middle      Right |     Total
---------------+---------------------------------+----------
        Female |     1,993      1,751      1,363 |     5,107
               |     39.02      34.29      26.69 |    100.00
---------------+---------------------------------+----------
          Male |     1,740      1,513      1,765 |     5,018
               |     34.68      30.15      35.17 |    100.00
---------------+---------------------------------+----------
         Total |     3,733      3,264      3,128 |    10,125
               |     36.87      32.24      30.89 |    100.00
```

Seems that women are more left-wing

Example: Swedish Partisanship

```
. tab union lr, row

Are you a member of a |                 lr
          labor union |      Left     Middle      Right |     Total
----------------------+---------------------------------+----------
Yes, LO-union (Blue-c |     1,199        962        379 |     2,540
                      |     47.20      37.87      14.92 |    100.00
----------------------+---------------------------------+----------
Yes, TCO-union (WHite |       761        505        576 |     1,842
                      |     41.31      27.42      31.27 |    100.00
----------------------+---------------------------------+----------
Yes, SACO-union (Acad |       503        344        486 |     1,333
                      |     37.73      25.81      36.46 |    100.00
----------------------+---------------------------------+----------
                   No |     1,210      1,401      1,658 |     4,269
                      |     28.34      32.82      38.84 |    100.00
----------------------+---------------------------------+----------
                Total |     3,673      3,212      3,099 |     9,984
                      |     36.79      32.17      31.04 |    100.00
```

Seems that union members are more left-wing as well

Example: Swedish Partisanship

Not much of a birth-year effect.

Slightly more likely to be liberal if you are young.

```
twoway lowess lr birthyear, bwidth(8) ylab(1 2 3) ysc(range(1 3))
```

Example: Swedish Partisanship

```
. ologit lr birthyear gender union
Iteration 0:   log likelihood = -10439.011
Iteration 1:   log likelihood = -10209.678
Iteration 2:   log likelihood = -10209.315
Iteration 3:   log likelihood = -10209.314

Ordered logit estimates                           Number of obs   =       9524
                                                  LR chi2(3)      =     459.39
                                                  Prob > chi2     =     0.0000
Log likelihood = -10209.314                       Pseudo R2       =     0.0220

------------------------------------------------------------------------------
          lr |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   birthyear |  -.0023528   .0011821    -1.99   0.047    -.0046695    -.000036
      gender |   .2848512   .0381107     7.47   0.000     .2101555    .3595469
       union |   .3047193   .0153667    19.83   0.000      .274601    .3348375
-------------+----------------------------------------------------------------
       _cut1 |  -3.922106   2.314659          (Ancillary parameters)
       _cut2 |  -2.538721   2.314431
------------------------------------------------------------------------------
```

As predicted, gender and union are significant predictors of ideological affiliation. Birthyear is also significant.

Example: Swedish Partisanship

```
. prchange

ologit: Changes in Predicted Probabilities for lr

birthyear
             Avg|Chg|        Left      Middle       Right
Min->Max    .02652879   .03979316  -.00230613  -.03748706
   -+1/2    .00036143   .00054216  -.00004053   -.0005016
  -+sd/2    .00580573    .0087086  -.00065121  -.00805739
MargEfct    .00036145   .00054217  -.00004055  -.00050162

gender
             Avg|Chg|        Left      Middle       Right
Min->Max     .0437066  -.06555989   .00489005   .06066984
   -+1/2    .04370405  -.06555608   .00488138    .0606747
  -+sd/2    .02187435  -.03281152   .00245112    .0303604
MargEfct     .0437606  -.06564089   .00490888   .06073202

union
             Avg|Chg|        Left      Middle       Right
Min->Max    .14116572   -.2117486   .02408424   .18766433
   -+1/2    .04674365  -.07011548   .00521764   .06489784
  -+sd/2    .05817041   -.0872556   .00647545   .08078018
MargEfct    .04681284  -.07021927   .00525127     .064968

               Left      Middle       Right
Pr(y|x)   .36014012   .33167729    .3081826

         birthyear      gender       union
    x=     1959.48     1.50073     2.74139
sd(x)=     16.0629     .500026     1.24547
```

Use Stata **"prchange"** command to obtain marginal effects of each independent variable, holding the others constant at their mean.

Example: Swedish Partisanship

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ologit: Changes in Predicted Probabilities for lr

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gender
             Avg|Chg|        Left      Middle       Right
Min->Max     .0437066  -.06555989   .00489005   .06066984
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        birthyear      gender       union
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Use Stata **“prchange”** command to obtain marginal effects of each independent variable, holding the others constant at their mean.

We see that the Marginal Effect of birthyear

Example: Swedish Partisanship

```
. prchange

ologit: Changes in Predicted Probabilities for lr

birthyear
             Avg|Chg|        Left      Middle       Right
Min->Max    .02652879   .03979316  -.00230613  -.03748706
   -+1/2    .00036143   .00054216  -.00004053   -.0005016
  -+sd/2    .00580573    .0087086  -.00065121  -.00805739
MargEfct    .00036145   .00054217  -.00004055  -.00050162

gender
             Avg|Chg|        Left      Middle       Right
Min->Max     .0437066  -.06555989   .00489005   .06066984
   -+1/2    .04370405  -.06555608   .00488138    .0606747
  -+sd/2    .02187435  -.03281152   .00245112    .0303604
MargEfct     .0437606  -.06564089   .00490888   .06073202

union
             Avg|Chg|        Left      Middle       Right
Min->Max    .14116572   -.2117486   .02408424   .18766433
   -+1/2    .04674365  -.07011548   .00521764   .06489784
  -+sd/2    .05817041   -.0872556   .00647545   .08078018
MargEfct    .04681284  -.07021927   .00525127     .064968

              Left      Middle       Right
Pr(y|x)  .36014012   .33167729    .3081826

        birthyear      gender       union
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sd(x)=    16.0629     .500026     1.24547
```

Use Stata "**prchange**" command to obtain marginal effects of each independent variable, holding the others constant at their mean.

We see that the Marginal Effect of birthyear is much less than either gender or union.

Example: Swedish Partisanship

```
. ologit lr birthyear gender union

Ordered logit estimates                           Number of obs   =       9524
                                                  LR chi2(3)      =     459.39
                                                  Prob > chi2     =     0.0000
Log likelihood = -10209.314                       Pseudo R2       =     0.0220

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          lr |      Coef.   Std. Err.      z     P>|z|     [95% Conf. Interval]
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------------------------------------------------------------------------------
```

- These are the estimates of the cut-points μ_1 and μ_2
- The standard errors compare the cut-points to 0
- But we really don't care about that; we want to know if they are different from each other.

Example: Swedish Partisanship

```
. ologit lr birthyear gender union

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------------------------------------------------------------------------------
```

- If they are equal, then we can eliminate the middle category.
- So we should test for the equality of these cut-points.

Example: Swedish Partisanship

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Ordered logit estimates                           Number of obs   =       9524
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       _cut2 |  -2.538721   2.314431
------------------------------------------------------------------------------
```

```
test _b[_cut1] = _b[_cut2]

 ( 1)  _cut1 - _cut2 = 0
           chi2(  1) = 3829.68
         Prob > chi2 =    0.0000
```

So we reject the null that the two cut-points are equal.