Whom to Lobby? Targeting in Political Networks*

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Abstract

We study lobbying in a setting in which decision-makers share resources in a network. Two opposing interest groups choose which decision-maker they want to target with their resource provision, and their decision depends on the decision-makers' ideologies as well as the network structure. We characterize the lobbying strategies in various network settings and show that a higher resource flow as well as homophily reinforce decision-makers' ideological bias. We highlight that competing lobbyists' efforts do not neutralize each other and their payoffs and competitive advantages depend on the networks they face. Our findings are consistent with empirically established lobbying activities.

Keywords: Networks, Lobbying, Targeting, Flow of Resources, Ideology, Centrality, Homophily, Colonel Blotto, Externalities

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"The real puzzle of its [a lobbies] P[ublic]A[ffairs] management is to select the most useful actors to approach, the best connections . . . " van Schendelen (2010) p.113.

"You cannot be too aggressive in pursuing your own ends. These documents must be honestly prepared. The [member's] office will be interested in both sides." Levin (2009) p.133.

1 Introduction

Lobbying activities have become an essential part of the decision-making process of political actors. This is evident from the sheer number of lobbyists employed in Washington, DC, Brussels, London, Berlin and other political epicenters.[1] But the importance of lobbyists becomes even more indisputable when considering the range of tasks lobbyists undertake. They engage in direct advocacy, legislative drafting, campaign fundraising, testimonies, and political consulting for both policymakers and their clients. Additionally, they target their activities at certain decision-makers.[2] In U.S. Congress, for example, several bi-partisan committees and subcommittees meet regularly, listen to the testimonies of experts and special interests, submit statements to the Congressional Records and provide recommendations to their colleagues who have parliamentary voting rights but are not members of relevant committees.[3]

These examples show that lobbyists do not face isolated decision-makers but members of parties and committees, staffers of politicians and government agencies, and decision-makers' political contacts who all have a say on policy issues. This multi-agent structure implies that lobbyists can also influence political actors indirectly, i.e., through the lobbied decision-maker's network of direct and indirect contacts. These observations are well-established which makes it surprising to discover that little attention has been paid to the lobbyists' optimal strategy of whom to lobby, how to maximize influence in political networks and how to minimize wasteful lobbying attempts in such multi-agent policy environments.

We therefore propose a model of lobbying with ideologically heterogeneous decision-makers who share resources in political networks. We allow for decision-makers to be neutral or ideo-

[1] There have been on average 13,000 federally registered lobbyists in Washington, DC during the last years (OpenSecrets.org), and estimates for Brussels are in the proximity of 30,000 active lobbyists (Corporate Europe Observatory).

[2] It is a common practice and has been recently illustrated by changes in health care legislation and financial regulation – e.g., Pear (2009) and Lipton and Protess (2013) – that lobbyists provide legislative drafting to policymakers that is shared among many politicians and Congressional Records such that "in House, many spoke with one voice: lobbyists".

[3] In the European Parliament, there exist several so-called inter-groups. Any member of the European Parliament can be a member of an inter-group independent of his party affiliation. To the meetings of inter-groups many lobbyists are invited to provide information about the issues discussed. Then, the members of the inter-group work on putting these issues on the official agenda of the European Parliament, which illustrates how the information of lobbyists disperse among other politicians. It is also fairly common for firms to lobby delegates of the districts they have headquarters or plants in. These delegates try to prevent an unfavorable outcome by talking to other delegates, including foreign governments. Loewenberg (2003a,b) reports how American firms were lobbying U.S. government officials in early 2000s to approach EU bureaucrats and change the planned regulation of 30,000 chemicals in the European Union. From the early 2000s to mid-2000 the planned regulations were substantially water-downed.

logically biased in favor of one of the lobby groups, implying that they are leaning towards its position. Lobby groups compete for a policy outcome, and they consider the decision-makers' ideologies as well as their positions in the political network when they form an optimal lobbying strategy. We show that these strategies depend not only on the decision-makers' ideology and centrality but also on the characteristics of decision-maker's characteristics.[4] Furthermore, our analysis highlights that lobbyists' resource provision may or may not be socially wasteful depending on the network and lobbies' strategies. Our model allows us therefore to provide novel explanations for empirically observed lobbying patterns.

In our model, we consider two lobby groups with opposing goals who approach policymakers in order to advance their objective. Lobbies compete for a share of a budget such as a grant, subsidy, or local public goods, and aim to maximize their share of this budget. To do so lobbyists provide resources to their lobbying targets.[5] This implies that our setup shares features of a Colonel Blotto game in which favored and unfavored lobbyists choose defensive or attacking lobbying strategies to influence decision-makers. The key difference, and theoretical innovation of our model, is that targets are connected through a network, and therefore, there are externalities between those lobbying targets, which implies a discontinuity in the analysis of the political network and lobbies' optimal strategies and a well-known challenge for network models.

We assume that both lobbyists have the same amount of resources and decide which decision-maker to address. We argue that access to decision-makers is limited and that lobbyists undertake a carefully planned process to identify their optimal lobbying target. In equilibrium each lobby targets either one decision-maker or follows a mixed strategy in choosing its targets, which indicates under what circumstances decision-makers are the most attractive target. The constrained access to decision-makers is supported by the vast literature that documents the importance of interpersonal relationships in securing access to politicians (Blanes i Vidal et al. (2012), Bertrand et al. (2014), Cohen and Malloy (2014), Kerr et al. (2014)).[6] Lobbyists therefore address only a limited number of decision-makers directly and rely on their direct contacts to

[4]Technically, there are various concepts of centrality such as the betweenness, degree and eigenvector centrality for the analysis of networks but all three concepts coincide for the cases we consider in our analysis.

[5]In our framework we generalize the specific resources that lobbyists provide. Interest groups provide financial resources either in exchange for policy favors (Bernheim and Whinston (1986), Grossman and Helpman (1994), Besley and Coate (2001)) or as legislative subsidy (Hall and Wayman (1990), Hall and Deardorff (2006)). Informational resources are either private, nonverifiable information (Crawford and Sobel (1982), Potters and van Winden (1992), Austen-Smith (1994, 1995, 1998) and Lohmann (1995)) or private, verifiable information (Milgrom and Roberts (1986), Bennedsen and Feldmann (2002), Bull and Watson (2004), Bull and Watson (2007)). For reviews see Persson and Tabellini (2000), Grossman and Helpman (2001), Hall and Deardorff (2006) and Congleton et al. (2008).

[6]Blanes i Vidal et al. (2012) illustrate the revolving door phenomenon in which former staff members and politicians become lobbyists. They show that past work experience can explain how a subset of lobbyists enjoys a significant advantage in establishing lobbying relationships with politicians. Their results imply that relationships are important and that they can be traded. Cohen and Malloy (2014) analyze education and alumni networks and show that such networks affect US politicians' voting behavior. Both illustrate the scarcity in political access but focus on networks between politicians and lobbyists that were formed years or decades ago. Our analysis focuses on the networks between decision-makers and the lobbyists' strategies who want to establish new network links to decision-makers today.

forward resources through the network, which occurs in our model with a certain probability.

Based on the arguments decision-makers hear from both lobby groups, they update their initial position on the issue and then decide on the optimal policy. In our analysis we abstract from a specific voting rule for collective decisions. Instead, we consider a more general form of political consensus for balancing decision-makers' interests when voting rules may vary or decision-makers depend on cooperative behavior with each other.[7] In particular, the policy implemented is the average of the positions taken by the decision-makers after lobbying has occurred. This reflects that in practice no decision-maker is forced to go too far away from his bliss point as decision-makers interact repeatedly and pushing one of them too far away from his bliss point might lead to uncooperative behavior in the future. In other words, decision-makers with equal power find a compromise that reflects their individual positions, a feature that has been documented in the legislative bargaining literature.[8]

The optimal lobbying strategy aims at identifying the decision-maker who allows for the greatest influence. Decision-makers are characterized by the following three features: (i) they can be ideologically biased in favor of one the lobby groups, (ii) more or less central in the network of decision-makers and (iii) connected to decision-makers that are similar in their ideological bias or rather different, that is they can exhibit homophily.[9] The notion that decision-makers exhibit homophily, that is that they have a preference for like-minded colleagues, has an impact on how decision-makers are selected and on the overall possible gains for lobby groups.

Our analysis highlights that lobbyists can gain political influence well beyond their own lobbying activities and that their strategies, payoffs and advantages depend on the political networks they face. We show that lobbyists' lobbying efforts by targeting decision-makers may not neutralize each other and may not be wasteful when they actually increase a lobby's payoff, which is in stark contrast to a classic Tullock (1980) contest or Becker (1983, 1985) pressure group competition. Furthermore, we show that the competing lobbyists' relative benefits from targeting decision-makers compared to no lobbying depend on both the biases of the decision-makers as well as the flow of resources in the network.[10]

We also show that if decision-makers share resources with a high probability, then this reinforces the ideological bias. Put differently, facing decision-makers who favor a lobby group strongly and a low flow of resources through the network is equivalent to fairly neutral political

[7]For example, in the European Union voting rules within its various institutions range from majority to supermajority and unanimity depending on the topics. For an institutional review see Jorgensen et al. (2006). Alternative models such as Groseclose and Snyder (1996) and Dekel et al. (2008, 2009) focus on interest groups' vote buying incentives when they offer sequentially or repeatedly funds in order to gain a supermajority.

[8]Morelli (1999) provides a theoretical foundation in form of a demand legislative bargaining model with an endogenous order of play and shows that this in line with empirics and experiments – Fréchette et al. (2005, 2012).

[9]See McPherson et al. (2001) for an overview of the literature on homophily.

[10]For detailed reviews of contests and their success functions see Skaperdas (1996), Epstein and Nitzan (2006), Konrad (2009) and Sisak (2009).

actors and a high flow of resources. This has implications for lobbying strategies that will differ depending on whether the issues at hand are of high political interest and are therefore widely discussed or whether there is little debate about this specific matter.

Further, we find that decision-makers that are more central within the network are more attractive lobbying targets. This implies that lobby groups specifically approach decision-makers who are well-connected and thus adept at spreading resources they obtain from special interests among their colleagues. However, there are circumstances in which a more central decision-maker is not the favored target and we document that this is in particular the case if the most central decision-maker is strongly biased.

Last, we show that if decision-makers exhibit homophily, then the interest group that is a priori favored by politicians will obtain greater gains. Therefore, the structure of political networks has an impact on the success of different lobby groups and should therefore be taken into account when assessing the possible gains of lobbying.

Our findings shed light on different empirical patterns of lobbying strategies. There are various studies that address the question whether interest groups lobby like-minded, neutral or opposing decision-makers and they provide mixed evidence. Our model shows that this question is misleading as not only the ideological bias, but also the network is crucial to determine lobbying patterns. We show that in many circumstances it is optimal to choose a neutral decision-maker, in particular when the flow of resources is high. This is in line with de Figueiredo and Richter (2014)'s review that illustrates a common agreement that too biased decision-makers are not lobbied but influential ones are attractive targets. We are also able to explain how different lobbying strategies relate to decision-makers' ideologies when we consider their centrality and the flow of resources in networks.[11]

Other studies have illustrated that opposing or supporting decision-makers are more likely targets.[12] Our model shows that these lobbying patterns can emerge when the sharing of resources in the network of decision-makers is taken into account. Specifically, we find that unbiased and opposing decision-makers are lobbied when the flow of resources is low and the goal is to neutralize their influence in the network. On the other hand, lobbying of supporters is more likely to occur if they take a central position in the network of decision-makers who work on the issues of lobbies' interest.

However, the role of centrality has its limits, when the most central decision-maker is biased and other less central decision-makers are less biased. A real world example is provided by Loewenberg (2003a,b)'s case study of American lobbying in the European Union. When the

[11] Recent studies also show that undecided decision-makers are more likely to be lobbied (Eggers and Hainmueller (2009), Bertrand et al. (2014)).

[12] For example, Austen-Smith and Wright (1994, 1996) and de Figueiredo and Cameron (2008) show that lobbyists target their opponents; whereas Kollman (1997), Hojnacki and Kimball (1998), Mian et al. (2013) and Igan and Mishra (2014) document that lobbyists target their supporters.

EU Commission wanted to introduce heavy safety standards for chemicals in the early 2000s, American companies did not lobby the central players of the EU Commission. Instead they targeted officials of member states that were less central but also less biased.

We also show that homophily in a political network lead to greater lobbying gains. It is well documented that the divide between Democrats and Republicans has increased, but the positions within parties have become more similar, which has implications on the political network. Our analysis incorporates this notion of polarization by comparing decision-making networks when there is homophily and when there is not.[13] We show that if like-minded decision-makers are connected, then lobby groups reap greater benefits and lobbying key players in networks becomes more attractive. Indeed, McCarty et al. (2006) show that there is a positive correlation between polarization and lobbying, in line with our predictions. Hence, we provide a novel explanation for the empirically documented escalation of lobbying in a polarized system.

Before turning to a formal development of our arguments, we illustrate how our study adds to the literature on Colonel Blotto games, targeting in networks, lobbying, and vote buying. The question of optimal targeting begins with the Colonel Blotto game, where two players simultaneously distribute forces across n battlefields. Within each battlefield, the player, who allocates the higher level of force wins. Shubik and Weber (1981) extend the Colonel Blotto game to situations, in which the battle fields are heterogeneous in terms of how hard they are to conquer. Our paper extends this literature by allowing for spillovers across battlefields. Recent contributions in the network literature focus on how a monopolist targets a network (Galeotti and Goyal (2009), Candogan et al. (2012), Bloch and Quèrou (2013), Dziubiński and Goyal (2013), Goyal and Vigier (2013, 2014)), how two competitors target a network of homogeneous consumers (Goyal and Kearns (2014)), or how multiple decision-makers inform and influence each other (Galeotti et al. (2013)).Here we incorporate the institutional features of both lobbying and political networks and analyze the optimal lobbying strategies of competing lobbyists who target heterogeneous decision-makers, which we observe in reality.[14] Hence, we provide a novel analysis of how competing agents target a network of heterogeneous decision-makers.

Our analysis contributes to the lobbying literature that focuses on costly access to a single decision-maker that is necessary to present interest groups' information (Austen-Smith (1995, 1998), Lohmann (1995), Cotton (2009, 2012)).[15] We follow Baron (2006) and generalize the provi-

[13]Other recent empirical studies have also emphasized the role of homophily among decision-makers in party and coalition networks (Grossman and Dominguez (2009); Koger et al. (2009)).

[14]Earlier, unpublished work considered political networks and lobbying with a single group (Groenert (2010)) and with two interests groups but no analysis of ideological biases (Lever (2010)).

[15]There contributions serve decision-makers as a sorting device for allocating access among lobbyists. Another strand focuses on the lobbyist's trade-off between providing financial resources for policy favors and verifiable information. Bennedsen and Feldmann (2006) and Dahm and Porteiro (2008a,b) focus on interest groups' incentives to provide either information or financial resources when their acquired information may reveal undesired information from their perspective. Groll and Ellis (2014, 2016) focus on commercial lobbyists' incentives to offer decision-makers their preferred mix of resources in exchange for political access that can be sold to clients.

sion of costly resources such as private, verifiable information or financial resources as lobbying and analyze lobbies' strategies of targeting connected decision-makers.[16] Our analysis is related to Bennedsen and Feldmann (2002)'s optimal information search and Dekel et al. (2009)'s vote buying analysis for entire legislatures. Bennedsen and Feldmann (2002) analyze a single lobby's strategic choice of acquiring and disclosing information about the desirability of a local public good in the policymaker's district. Their analysis focuses on the optimal number of information searches and disclosures when decision-makers vote collectively but are not heterogeneous or connected with each other, which we consider. Dekel et al. (2009) consider heterogeneous decision-makers and how lobbying targets change when lobbyists are and when they are not budget-constrained.[17] In general, the legislators lobbied are the opposing ones, specifically those that are the closest to the median. In our analysis we consider the role of a decision-maker's ideology and connections to other decision-makers. Our analysis highlights the externalities that arise when lobbies target connected decision-makers who share resources and then vote collectively. Unlike Dekel et al. (2009) we provide a lobbying model that manages to reconcile all relevant empirical cases of lobbying opposing, unbiased and supporting decision-makers who are connected and share resources.[18] Finally, our study contributes to the empirical literature on lobbying connected policymakers (Lazer (2011), Cohen and Malloy (2014), Do et al. (2015)) by providing an explicit network measure that would allow to identify a mechanism with which lobbyists influence decision-makers. Unfortunately, current data do not allow for identifying decision-makers' and lobbyists' contacts and, therefore, we have to postpone our empirical test.

The rest of this study is organized as follows: Section 2 presents the model. Section 3 analyzes the implications of lobbyists' payoffs and relative advantages. Section 4 characterizes the interactions of decision-makers' ideology and the resource sharing. Section 5 focuses on the centrality in the political network and Section 6 on the role of direct and indirect connections between decision-makers. Section 7 puts our findings in context of observed lobbying patterns. The last section concludes. All proofs and some simulations can be found in the Appendix A.[19]

2 Model

Two lobbyists, L_1 and L_2, influence a given group of decision-makers in order to maximize their share of a budget of size one.[20] The decision-makers are connected in a network described by

[16] His analysis considers explicitly the roles of the legislators as agenda setters as well as voters and lobbies' vote buying incentives to gain minimal majorities or supermajorities of individual legislators.

[17] Their analysis extends Groseclose and Snyder (1996)'s and Dekel et al. (2008)'s vote buying analysis and focuses on gaining supermajority votes depending on interest groups' budget constraints. If lobbyists are not budget constrained only the preferences of the legislators near the median matter, as only these will be lobbied.

[18] For a review of empirical studies see de Figueiredo and Richter (2014).

[19] The proofs for Proposition 5 and 6 can be found in the Supplemental Appendix.

[20] Alternatively to grants, subsidies, or local public goods, the setting described here can also be used to describe any political process where the goals of the lobbyists are opposing and decision-makers decide collectively.

an undirected graph and their set is given by $N = \{1, \ldots, n\}$. We assume that the network consists of a single component, that is all decision-makers are connected through a path in the network.[21] A link between two decision-makers implies that they share resources with each other. Each decision-maker has an a priori preference of how the budget should be divided among both lobbyists. Decision-maker D_i's preference is denoted by $\varphi_i \in (0, 1), i \in N$, where φ_i gives the share of the budget that the decision-maker wants to assign to L_1. A decision-maker with $\varphi_i > \frac{1}{2}$ is biased in favor of L_1, a decision-maker with $\varphi_i < \frac{1}{2}$ in favor of L_2. Last, a decision-maker, who assigns equal shares to both lobbyists – i.e., $\varphi_i = \frac{1}{2}$ is called neutral. The parameter φ_i can be interpreted as the personal, intrinsic preference of decision-maker i or, alternatively, as the preference of the decision-maker's constituency.

In order to influence how the budget is split, both lobbyists simultaneously provide resources to their targeted decision-makers.[22] These resources reveal how they are affected by the allocation of the budget. To be able to convey their resources to the decision-makers, the lobbyists have to be linked to the political network. We assume that lobbyists establish exactly one link. Without any link, no lobbying is possible. But once a lobbyist has made a connection to the network, he can always use the connections between decision-makers to access the other decision-makers, without establishing a second link. The single link follows the notion that decision-makers are time-constrained and access is costly and therefore it will not be feasible to lobby every one of them.[23] Both lobbyists have the same ability to transmit resources, which implies that both have identical resources and allows us to focus on the effects of networks.

Both lobbyists give their resources to the decision-maker they are directly connected with, who then spreads the resources through the network. We normalize the resources to one. The resources reach the neighboring decision-makers with probability δ, whereas the second-order neighbors receive the argument with probability δ^2, etc., which implies that the share of resources a decision-maker receives is decreasing in the distance to the lobby's target. Based on the arguments made, each decision-maker adjusts his preferences of how the budget should be split among the lobby groups. The share of the budget decision-maker $k \in N$ attributes to L_1 when L_1 chooses decision-maker $i \in N$ and L_2 chooses $j \in N$ is given by the following contest

[21] If the network does not consist of a single component, then our analysis can be done for each component separately and our results carry over.

[22] Our setup is that of a constant sum lobbying game that differs from lobbying models in which lobbying is wasteful – e.g., Grossman and Helpman (1994). Nevertheless, our solution strategy carries over to settings in which the lobbying prize is shrinking in efforts. The rational is that as lobbying is not continuous in our model and thus every wasteful set up can be represented as a non-wasteful one, once both lobbyists choose to enter they have to identify their optimal lobbying target.

[23] The truth lies probably somewhere between lobbying everyone and lobbying the most attractive decision-maker. The lobbyists' mixed strategies in choosing their targets, which we derive later, give a notion of whether lobbyists want to be more or less spread out across decision-makers and are therefore an indicator under what circumstances lobbyists want to target more or fewer decision-makers. For more detailed lobbying models with limited or costly access see Austen-Smith (1995, 1998), Lohmann (1995), Cotton (2009, 2012) and Groll and Ellis (2014, 2016).

success function:

$$p^1(i,j|k) = \frac{\varphi_k \delta^{l(i,k)}}{\varphi_k \delta^{l(i,k)} + (1-\varphi_k)\delta^{l(j,k)}}, \tag{1}$$

where $l(i,k)$ ($l(j,k)$) is the number of links between the node L_1 (L_2) chooses and node k.[24]

A decision-maker receives resources of L_1 with probability $\delta^{l(i,k)}$, those of L_2 with probability $\delta^{l(j,k)}$. He then weights the arguments according to his ideological bias. If a decision-maker favors L_1, then he is not easily convinced that L_2 should obtain a larger share of the budget. Further, the more biased a decision-maker is, the harder it is to change his preferences.

Based on this we now turn to the question of how decision-makers in a committee or legislature collectively agree on the shares of the budget. Suppose that every decision-maker can make a proposal and that the committee takes those proposals into account – i.e., the goal is to

$$\min_{\pi^1(i,j)} \sum_{k=1}^{n} (p^1(i,j|k) - \pi^1(i,j))^2,$$

where the functional form of a loss function captures a committee's or parliament's role in balancing interests amongst decision-makers. Then, L_1 is interested in maximizing

$$\pi^1(i,j) = \frac{1}{n}\sum_{k=1}^{n} p^1(i,j|k). \tag{2}$$

Our collective decision-making process allows us to generalize from specific voting rules that vary across topics and institutional settings in which lobbying takes place. The setup captures a form of weighted consensus among decision-makers with equal power who interact repeatedly and rely on cooperative decision-making behavior.[25]

For the given budget splitting rule, we are interested in mixed strategies. We denote by $\sigma^x(i)$, $x \in \{1,2\}$, $i \in N$ the probability lobbyist x assigns to decision-maker i. A pure strategy for lobbyists x is given by $s^x(i)$, where x assigns probability one to decision-maker i. Then, the payoff for L_1, is given by

$$\Pi^1(\sigma^1,\sigma^2) = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma^1(i)\sigma^2(j)\pi^1(i,j). \tag{3}$$

[24] Equivalently, the share of the budget k would award L_2 is then $p^2(i,j|k) = \frac{(1-\varphi_k)\delta^{l(j,k)}}{\varphi_k \delta^{l(i,k)} + (1-\varphi_k)\delta^{l(j,k)}}$.

[25] Alternatives explanations for this payoff function are that each decision-maker is chosen with equal probability to make a suggestion and that this suggestion is then implemented or that decision-makers in parties, ruling coalitions or committees of long-lived organizations maximize collective welfare. Or alternatively, the setup's principle is also shared with legislative demand bargaining. For example, Morelli (1999) analyzes legislative demand bargaining with an endogenous order of play and shows that policy outcomes are proportional to party seats. In most parliamentary settings the proportionality of seats carries over to parliamentary committees in order to reflect constituents' interests in the detailed policy-making.

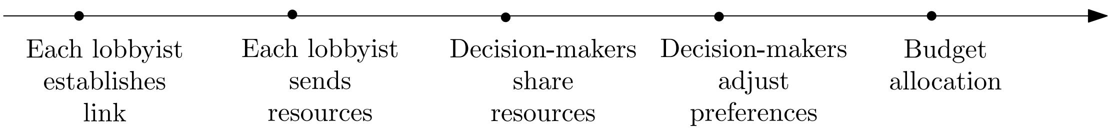


Figure 1: Sequence of Decision-Making.

whereas the payoff of L_2 is $\Pi^2(\sigma^1, \sigma^2) = 1 - \Pi^1(\sigma^1, \sigma^2)$.[26]

The sequence of play is summarized in Figure 1, which shows that we have specified a game G with two players, lobbyists L_1 and L_2. Both players have the same set of pure strategies $S = N$, which is finite. The payoffs are $\Pi^1(\sigma^1, \sigma^2)$ and $\Pi^2(\sigma^1, \sigma^2)$. Moreover, we have a constant sum game. Because of this we can use linear programming techniques to find the Nash equilibria of the specified game.

Properties of the Payoff Function Before solving the game, it is important to note some features of the contest success function. Suppose L_1 chooses decision-maker i and L_2 targets j. Then, we are interested in the payoffs of the lobby groups at decision-maker k.

We first show that only relative distance matters, that is only the difference between the distance that i has to k and the distance j has to k is relevant:[27]

$$\frac{\varphi_k \delta^{l(i,k)}}{\varphi_k \delta^{l(i,k)} + (1-\varphi_k)\delta^{l(j,k)}} = \frac{\varphi_k}{\varphi_k + (1-\varphi_k)\delta^{l(j,k)-l(i,k)}}.$$

Note that there is an upper bound on the relative distance. As there always exists a path from i to any k via j the relative distance in absolute terms can never be larger than the number of links of the shortest path between decision-makers i and j. This property implies that when lobbyists choose the same decision-maker, the flow of resources does not matter. Another feature of the contest success function chosen is that they have different properties, when decision-makers are biased or unbiased.

If there are two unbiased decision-makers k and k', i.e. $\varphi_k = \varphi_{k'} = \frac{1}{2}$, then

$$\frac{1}{1+\delta^q} + \frac{\delta^q}{1+\delta^q} = \frac{1}{1+\delta^r} + \frac{\delta^r}{1+\delta^r} = 1$$

with $q, r \in \mathbb{N}$. This implies that it does not matter whether the lobbyist is closer to one decision-maker and further away from the other or whether he has the same relative distance to both

[26] We do not take the costs of lobbying or voting into account, although we could easily incorporate a fixed cost of establishing a lobbying link or casting a vote. As long as costs are sufficiently small, lobbyists choose to lobby and decision-maker vote. An abstention from lobbying implies that they get no share of the budget, which cannot be optimal. In our setup neither lobbyist wants to abstain from lobbying as long as the costs of lobbying are sufficiently small and each lobbyist fears to lose the entire budget. For explicit models with continuous lobbying expenditures in which lobbyists find it optimal to abstain see for example Aidt (2002).

[27] This implies that if the node chosen by L_1 has a distance of 4 to the node we consider, and the node chosen by L_2 has a distance of 3, then this is the same as if L_1 had a distance of 2 and L_2 of 1.

compared to the other lobbyist.[28] This does not hold when there is a biased decision-maker.

When there is an ideological bias, which is the same for both decision-makers k and k', such that $\varphi = \varphi_k = \varphi_{k'} \neq \frac{1}{2}$, then

$$\frac{\varphi}{\varphi + \delta^q(1-\varphi)} + \frac{\delta^q \varphi}{\delta^q \varphi + (1-\varphi)} \neq \frac{\varphi}{\varphi + \delta^r(1-\varphi)} + \frac{\delta^r \varphi}{\delta^r \varphi + (1-\varphi)},$$

where $q \neq r$, $q, r \in \mathbb{N}$. Whether it is better to have a smaller relative distance to one decision-maker and a larger one to the other one or to have equal relative distance to both of them depends on the specific parameters.

Based on this we can now turn to addressing the question of how lobbyists target decision-makers in a network. We focus on three features, namely, (i) the role of ideology and how it interacts with the flow of resources, (ii) centrality as well as (iii) homophily, which we discuss in the following.

3 Lobbying

In a first step we want to focus on the lobbyists' payoffs and illustrate their lobbying incentives in equilibrium. In other words, we ask whether the lobbying of decision-makers' networks makes interest groups better off, worse off or the same compared to the case in which both lobby groups do not lobby. The following result highlights that in order to evaluate lobbyists' benefits from such lobbying contests, one has to consider the interplay of the magnitude and distribution of biases as well as the flow of resources within the decision-making network.

Proposition 1. *Let the strategies of the lobby groups be such that in equilibrium the distance to at least one biased decision-maker differs. Then the payoffs of both lobbyists differ generically compared to the case in which nobody lobbies.*

It holds generically that the equilibrium payoffs differ compared to the no lobbying case if lobbyists choose different decision-makers with a positive probability and at least one of the decision-makers is biased in the network. It might be the case that equilibrium payoffs are the same even if the relative distance to biased decision-makers differs, but changing the bias of one decision-maker marginally will result in different payoffs compared to the benchmark of no lobbying. Note that this is not true for a change in δ, that is if lobbying yields the same payoff relative to the benchmark, then changing δ marginally might still result in an equilibrium in which the payoff equals the benchmark payoff.

[28] Clearly, a lobbyist still has an advantage if he is closer to both decision-makers relative to the other lobbyist.

If lobbyists choose the same lobbying target, then

$$\pi^1(i,i) = \frac{1}{n}\sum_{k=1}^{n}\varphi_k,$$

which implies that their efforts neutralize each other and so lobbyists reap the same payoff as if there was no lobbying taking place. This is no longer true if both lobby groups target different politicians with positive probability such that their relative distance from at least one biased decision-maker differs. Then lobbying can lead to gains or losses compared to the no lobbying benchmark.

However, it is generally not clear which lobbyist can generate greater payoffs from such a lobbying contest. As an illustration, consider the ring of (b) in Figure 2 with $n = 3$ decision-makers. Further, let two of them be in favor of L_1 with $\varphi_1 = \varphi_2 = .6$ and the third decision-maker be in favor of L_2 with $\varphi_3 = .3$, which implies that L_1 has initially more supporters but L_2 has a more biased initial supporter. For a flow of resources of $\delta = .2$, we can show that L_1 reaps a higher payoff compared to the no lobbying benchmark; but for $\delta = .5$, L_2 reaps a higher expected payoff compared to the no lobbying benchmark.

Our first result implies that there is an interplay between the magnitude as well as the distribution of biases and the flow of resources within the network that determines which lobbyist benefits more from the lobbying competition between both. Our finding is also in contrast to most of the literature on contests, where efforts neutralize each other and are in essence wasteful as they do not generate higher payoffs compared to the benchmark case. Our model therefore provides a new motivation for why lobbying may take place, namely not only to counter the efforts of opponents, but also to enlarge one's share of the pie.

4 Ideology and Flow of Resources

In this section we consider the case, where all decision-makers have the same centrality and a decision-maker is directly connected to two other decision-makers, as it is illustrated by (a) in Figure 2. It is the simplest case as agents do not differ in their network centrality and therefore provides a natural benchmark, which also highlights some of the technical difficulties in analyzing such a Colonel Bloto game with externalities. A real world example would be any committee or advisory board that chooses a policy or makes a recommendation and in which all members have similar formal power and agenda influence. This case allows us to describe the interactions of the flow of resources and ideological biases, without any effects of centrality and indirect connections in networks, which we will consider in the next two sections.

We will show that if the sharing of resources is sufficiently high, only the most unbiased

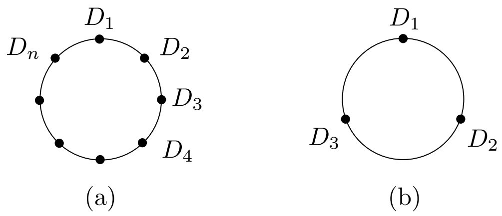


Figure 2: Initial Preferences of Interest.

decision-makers are being lobbied. Further, a high flow reinforces the bias and makes it harder for an initially disadvantaged lobbyist to influence the network. In general, the higher the flow of resources, the less likely a more biased decision-maker will be lobbied. On the other hand, if the flow is low, there are situations in which also a very biased decision-maker will be targeted with positive probability.

4.1 The Role of Ideology

First, we consider the implications of neutral or identically biased decision-makers when all n decision-makers are connected in a ring as illustrated by (a) in Figure 2.

Proposition 2. *For a ring with n decision-makers let $\varphi_i = \varphi$ for all $i \in N$. If*

1. *$\varphi = \frac{1}{2}$, then any (σ^1, σ^2) is a Nash equilibrium;*
2. *$\varphi \neq \frac{1}{2}$, then in the unique Nash equilibrium, $\sigma^1(i) = \sigma^2(i) = \frac{1}{n}$.*

The first case of Proposition 2 implies that whenever all decision-makers are neutral, then any strategy combination can be a Nash equilibrium. Given that all decision-makers have the same centrality and the same bias, they are an equally attractive lobbying target. Furthermore, due to the neutrality of the decision-makers, it does not matter whether the lobbyists choose the same decision-maker or different ones. This leads to the result that any strategy combination defines a Nash equilibrium. It implies that an observed lobbying pattern of choosing the same or a different decision-maker in this setting would be a coincident. This result does not hold if all decision-makers have a bias of the same magnitude such that one lobbyist is initially favored and the other one initially disadvantaged, as can be seen in the second case of Proposition 2.

For the second case now suppose all decision-makers are in favor of L_1. Then, L_1 prefers to lobby the same decision-maker as L_2, whereas L_2 prefers to target a different decision-maker. These strategic considerations lead to an equilibrium in mixed strategies, which is unique. L_1 has an incentive to target the same decision-maker as the initially disadvantaged L_2 and to counteract L_2's lobbying effort as a defensive lobbying strategy.[29] But as this is a constant sum game, it has to be the case that if L_1 is better off targeting the same decision-maker as L_2, then L_2 will

[29]Formally, if L_1 and L_2 address the same decision-maker, then the payoff of L_1 is φ_k, if L_1 and L_2 choose different targets then L_1 has a payoff of $\frac{1}{3}\left(\frac{\varphi_k}{\varphi_k+\delta(1-\varphi_k)} + \frac{\delta\varphi_k}{\delta\varphi_k+(1-\varphi_k)} + \varphi_k\right)$. As $\frac{\varphi_k}{\varphi_k+\delta(1-\varphi_k)} + \frac{\delta\varphi_k}{\delta\varphi_k+(1-\varphi_k)} < 2\varphi_k$, it is better for L_1 to target the same decision-maker as L_2.

prefer to lobby a different one than L_1. This leads to the mixing we observe and provides a rational for why lobbyists follow different lobbying strategies. Furthermore, it also implies that competing lobbyists have incentives to monitor each other but also to conceal their lobbying activities behind closed doors from competition and the public.

In order to understand who is being lobbied if a lobbyist can choose between an initial ally and opponent, we consider the implications of decision-makers who have a bias of the same magnitude but of different direction. For simplicity, suppose there are two decision-makers who are in favor of one lobbyist and one in favor of the other. In other words, we focus on a ring as illustrated by (b) in Figure 2.

Proposition 3. *For a ring with three decision-makers let $\varphi_i = \varphi_j = 1 - \varphi_k \equiv \varphi$. Then, in every Nash equilibrium the lobbyist who i and j favor lobbies k. The lobbyist favored by k is indifferent between all decision-makers.*

Proposition 3 says that a lobbyist who is already favored by the majority of the decision-makers will seek to extend his support by lobbying the opposing decision-maker directly. Given this strategy the lobbyist who is only favored by one decision-maker is indifferent between all decision-makers. To see the strategic considerations at play, suppose L_1 chooses a decision-maker who favors him before being lobbied. Then, L_2 would prefer to choose the decision-maker in favor of L_1, L_1 did not choose. In this case both lobbyists would have the same distance to the decision-maker in favor of L_2. This implies that L_2 would gain more from the decision-maker in his favor, namely $1 - \varphi_k$. Further, he manages to neutralize some of the bias of the opposing decision-maker he is directly connected to. But L_1 can easily prevent L_2 from gaining such an advantage, namely by attaching himself to the opposing decision-maker. Then, L_2 is indifferent between all decision-makers. In other words, the lobbyist who is initially favored by the majority follows expansive lobbying strategies, whereas the less favored lobbyist engages either in a counteractive strategy to neutralize the other lobby's effort and secure his ally's support or in non-competitive lobbying to gain an opponent's support.

4.2 Ideology and Flow of Resources

The previous results hold for all values of the flow of resources. However, this was due to the particular structure of the biases within the network. In general, the optimal strategy depends on the flow of resources. If the sharing of resources is high, only the most neutral decision-makers are targeted; whereas if the flow is low, then all decision-makers are lobbied with positive probability. This is highlighted by the following results.

Proposition 4. *For a ring with three decision-makers let δ be sufficiently large. Then, only the most neutral decision-makers are targeted by both lobbyists.*

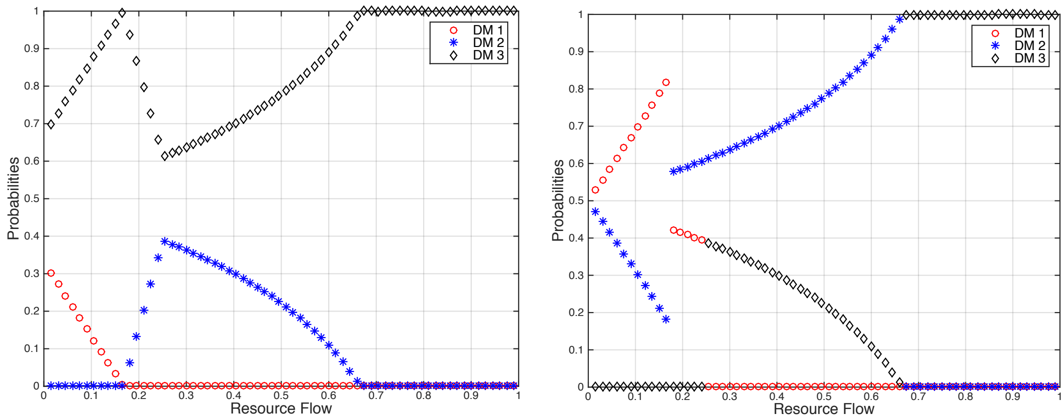
If resources flow well through the network, it does not matter much whether a lobbyist is directly or indirectly connected to a decision-maker. What is crucial, therefore, is the a priori bias. The more biased a decision-maker is, the less attractive of a lobbying target he is, whereas the most neutral ones become more attractive. The lobbyists can gain most if they focus on the most neutral decision-maker(s) as there, the return at the margin is greatest. This is why only the least biased decision-makers are lobbied in equilibrium, which implies that both lobbyists follow counteractive lobbying strategies.

The results for this set up so far are similar to Austen-Smith and Wright (1994) who show that interest groups lobby weak allies, unbiased decision-makers, or weak opponents rather than strong allies or opponents when they face a single decision-maker. However, the results change if resources are not flowing well through the political network. Then the lobbying strategies differ and depend on the constellation of biases within the network.

Proposition 5. *If there is a ring with three decision-makers and δ is sufficiently small, then all decision-makers – including very biased ones – are lobbied with positive probability.*

We have shown that unbiased decision-makers are more favorable lobbying targets. However, our results imply that not necessarily every biased decision-maker is lobbied, but that in every equilibrium a biased decision-maker is lobbied with positive probability. We illustrate the intuition with two of those cases. First, suppose there is only one biased decision-maker and two neutral ones – i.e., $\varphi_i > \frac{1}{2} = \varphi_j = \varphi_k$. The initially favored lobbyist prefers to lobby the same decision-maker as the opposing lobbyist, who in turn prefers to lobby a different one. The equilibrium strategies are similar to what happens when all decision-makers are in favor of one lobbyist, see the second case of Proposition 2. In other words, L_1 has an incentive to target the same decision-maker as the initially disadvantaged L_2 and to counteract L_2's lobbying effort; the initially disadvantaged L_2 will prefer to lobby a different one than L_1. This implies that the favored lobbyist engages in counteractive lobbying but the disadvantaged one follows again a non-competitive strategy.

For the second case suppose all three decision-makers are biased but that the minority decision-maker is less biased than both majority decision-makers – i.e., $\varphi_i > \varphi_j > \frac{1}{2}$ and $1 - \varphi_j < \varphi_k$. In this case, the initially more favored L_1 chooses between the most biased decision-maker in his favor as well as the opposing decision-maker with positive probability. L_2 assigns positive probability to both opposing decision-makers; L_1 again prefers to lobby the same one as L_2, whereas L_2 always prefers to target a different decision-maker. But for L_2 it is never a best response to choose the decision-maker in his favor. Therefore he never assigns positive probability to this decision-maker, but instead lobbies only decision-makers i and j. The lobbying strategies imply that the initially favored lobbyist mixes between expanding his support in the network and



(a) Equilibrium Strategy of L_1. (b) Equilibrium Strategy of L_2.

Figure 3: Equilibrium Strategies for $\varphi_1 = 0.8$, $\varphi_2 = .6$,$\varphi_3 = .5$.

defending the most biased decision-maker as a counteractive effort to L_2's lobbying strategy of lobbying his opposing decision-makers.

Finally, let us analyze what happens for an intermediate level of flow. Suppose there are two biased decision-makers in favor of L_1 and one unbiased decision-maker, with the biases differing. An example of this is given in Figure 3, where decision-maker 1 is the most biased and decision-maker 3 is neutral. The Nash equilibrium that emerges depends on the magnitude of the flow of resources δ. For a low resource flow, L_1 assigns positive probability to the neutral and the most biased decision-maker, in our example decision-maker 1. There is a trade off for L_1 in defending his most valuable supporter and attaching himself to the neutral decision-maker where he can realize the greatest gain at the margin. L_2 tries to neutralize one of the supporters of his opponent and attaches positive probability to both of them. This leads to the mixing we observe. As the sharing of resources increases, the necessity to defend the most valuable supporter does not arise anymore as even from a distance, L_1 will always realize a greater payoff at node 1 than L_2. L_2 still assigns positive probability to the two most biased decision-makers for this level of the resources flow. Eventually, both L_1 and L_2 assign positive probability to the two least biased decision-makers, until for a high resource flow only the unbiased decision-maker is targeted with positive probability. This highlights the role of the flow of resources. For a low flow the more biased decision-makers are still attractive lobbying targets. But as the flow of resources increases, the more likely it is that the less biased decision-makers are lobbied and eventually only neutral decision-makers will be targeted, see Proposition 4.

5 Centrality

Whereas the previous discussion was meant to give a general insight into the dynamics at play in a network where individuals are the same in terms of centrality, we focus now on what happens when there are differences in centrality. Therefore, we consider a line with three decision-makers.[30] Accordingly, we have one decision-maker, who is more central to the network and through whom the other two decision-makers share resources. In our setup this decision-maker is directly connected to all decision-makers and holds the largest number of direct political contacts. An example for such a structure would be heterogeneous networking abilities among decision-makers or the traditional left-right spectrum with median decision-makers between the extreme ones. To incorporate ideology into our analysis, we denote decision-maker D_1 as the decision-maker on the left, D_2 as the decision-maker in the center and D_3 as the right decision-maker, which illustrates the common left-right ordering in politics, in which D_2 is the connection median that is also the center decision-maker. This is denoted formally in Assumption 1.

Assumption 1. *For a line with three decision-makers let $\varphi_1 \geq \varphi_2 \geq \varphi_3$.*

We show that generally the most central decision-maker will be targeted. However, there can be a trade-off between choosing the unbiased decision-maker and the central one. In other words, there are cases in which the a less well-connected but less biased decision-maker will be targeted.

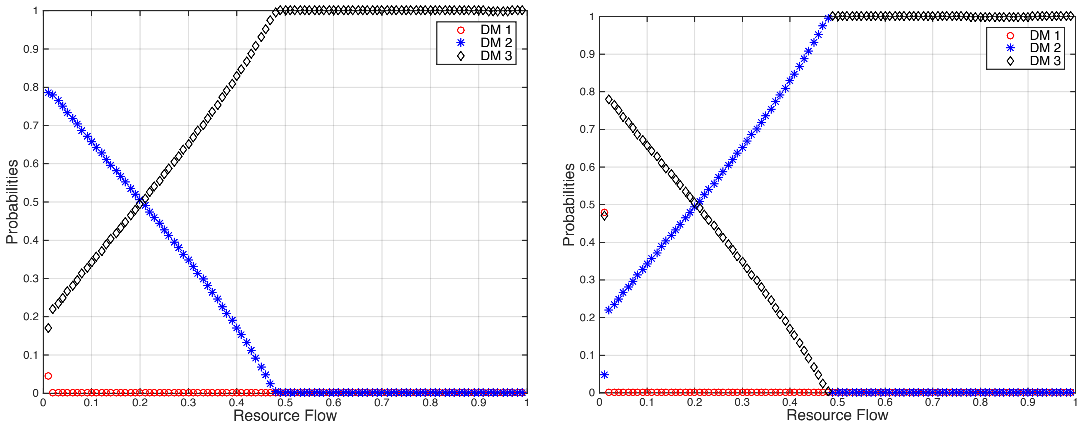
Proposition 6. *For a line with three decision-makers and any $\delta \in (0, 1)$, both lobbyists lobby the central decision-maker in the unique Nash equilibrium if*

1. *at least two decision-makers are unbiased; or*
2. *each lobby is initially supported by at least one decision-maker such that $\varphi_1 = 1 - \varphi_3$.*

The first case of Proposition 6 is little surprising. If all decision-makers are neutral, $\varphi_1 = \varphi_2 = \varphi_3 = \frac{1}{2}$, it is clearly the best choice to pick the most central decision-maker, as the resources diffuse directly with higher probability to the other decision-makers – i.e., each peripheral decision-maker receives the resources with probability δ rather than one of them with probability δ^2. If one peripheral decision-maker is biased, e.g. $\varphi_3 < \frac{1}{2}$, then choosing the most central decision-maker is still the best option as a neutral decision-maker is a good target anyways and, moreover, he offers access to the other decision-makers indirectly. In each of the two settings both lobbyists follow the same lobbying strategy and target the same decision-maker as expansive strategy.

The second case of Proposition 6 implies that decision-maker D_1 is either neutral or biased in favor of one of the lobby groups. If D_2 is unbiased, then it is straightforward to show that both lobbies compete for the unbiased and central rather than their ally or opponent. If D_2 is biased, then it is still optimal to choose the most central decision-maker. Suppose that $\varphi_2 > \frac{1}{2}$. Recall that in Proposition 3 L_1 chooses the opposing decision-maker and L_2 assigns positive probability

[30] We consider the simplest case of a line with three nodes. However, the intuition of the line carries over to a star with n nodes.



(a) Equilibrium Strategy of L_1 for $\varphi_1 = \varphi_2 = .9$. (b) Equilibrium Strategy of L_2 for $\varphi_1 = \varphi_2 = .9$.

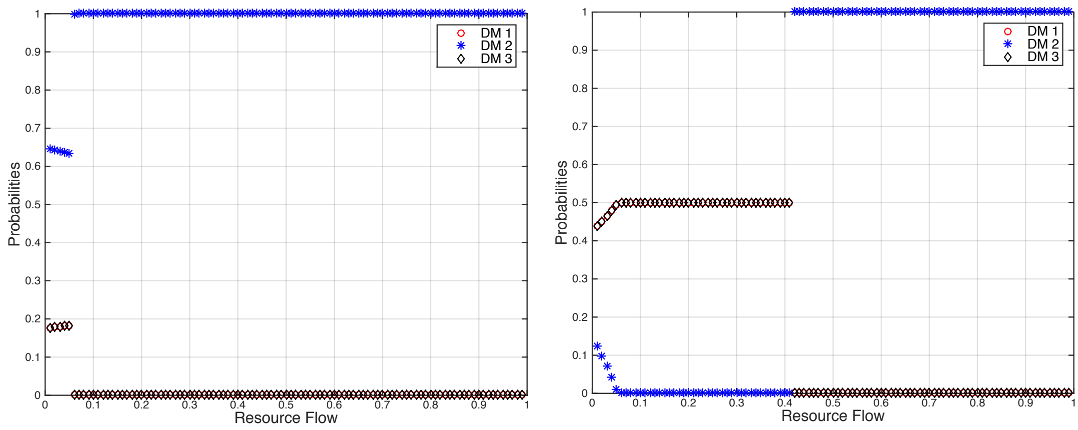
Figure 4: Equilibrium Strategies for 2 Biased and 1 Unbiased Decision-Maker.

to both decision-makers in favor of L_1. So, L_1 has an interest in lobbying the opposing decision-maker. If the political network is a line, though, then it is more valuable for him to lobby the central decision-maker than the opposing one.

In all cases so far the central decision-maker is the most attractive lobbying target and both lobbyists follow counteractive lobbying strategies. But there are cases in which a peripheral decision-maker is lobbied with positive probability. When this occurs is shown in the following two examples.

Example 1: $\varphi_1 = \varphi_2 = \frac{9}{10}$, $\varphi_3 = \frac{1}{2}$. First, consider the case when one lobbyist is favored by two decision-makers with $\varphi_1 = \varphi_2 = \frac{9}{10}$ and the third decision-maker is neutral. In other words, we focus on the trade-off between lobbying a biased central decision-maker and an unbiased decision-maker with fewer direct connections. Figure 4 illustrates that when the sharing of resources is high, then we find that both lobbyists choose the unbiased peripheral decision-maker. The reason is that for L_2 choosing the unbiased node is now a strictly dominant strategy. As L_1 prefers to lobby the same decision-maker as L_2, we obtain the described equilibrium.

Example 2: $\varphi_1 = \varphi_2 = \varphi_3 = \frac{9}{10}$. Next consider the case when one lobbyist is similarly favored by three decision-makers with $\varphi_1 = \varphi_2 = \varphi_3 = \frac{9}{10}$ as illustrated in Figure 5. If the flow of resources is very low, then both lobbyists assign positive probability to all decision-makers. For an intermediate flow of resources, L_1 chooses the central decision-maker, L_2 randomizes between the peripheral decision-makers. The strategic considerations are again that L_1 prefers to target the same decision-maker as L_2, whereas L_2 prefers to lobby a different one. However, for an intermediate flow of resources, L_1 prefers to be at the center instead of trying to end up at the same node. In expectation it leads to a higher payoff if L_1 always has a distance of one to the the decision-maker, L_2 chose instead of being with probability $\frac{1}{2}$ at the same node and with proba-



(a) Equilibrium Strategy of L_1.

(b) Equilibrium Strategy of L_2.

Figure 5: Equilibrium Strategies for $\varphi_1 = \varphi_2 = \varphi_3 = .9$.

bility $\frac{1}{2}$ at a node two links away. When the flow is high, then both L_1 and L_2 target the central decision-maker.

These two examples highlight that there are two very different settings in which a peripheral decision-maker is targeted. The first case occurs if decision-makers 1 and 2 are very biased and the resource flow is sufficiently high. Then, the biased decision-makers cannot be sufficiently influenced anymore and this implies that for both lobbyists it is better to focus on the unbiased one. The second case occurs when all decision-makers are very biased and the resource flow is low. Then the goal of L_2 is to target a different node than L_1 even at the cost of foregoing a connection to the most central decision-maker.

6 Decision-Makers' Contacts

Whereas the previous section focused on the interplay of ideology and centrality, we focus finally on the role of direct and indirect connections between decision-makers. We illustrate the implications of direct and indirect contacts with a ring in which all decision-makers have the same centrality but different neighbors. In a first step we consider the case of one biased decision-maker and $n-1$ unbiased ones to illustrate the effects of the decision-maker's own ideology and the ideology of his direct connections. In a second step we consider two biased decision-makers and two unbiased ones, who are either directly or indirectly with each other connected. The comparison of these two settings allows us to illustrate the role of homphily in networks. Homophily expresses the phenomenon that like minded individuals are more likely to be connected in social networks, which was first discussed by McPherson et al. (2001). Figure 6 illustrates the three network structures of interest if lobbyist L_1 is initially favored by at least one decision-maker and where (b) illustrates homophily with similarly biased decision-makers directly connected,

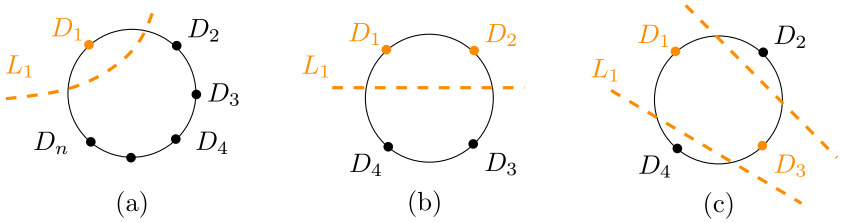


Figure 6: Initial Preferences of Interest.

and with (c) illustrating the opposite of homophily.

We can show that not only the ideology of the individual decision-makers matters, but also the ideology of his neighbors. If the sharing of resources is sufficiently high, then the unbiased decision-maker with the greatest distance to the biased decision-maker will be chosen. Additionally, if there is homophily in networks, then this increases the advantage of an already favored lobbyist compared to a network structure in which individuals do not exhibit homophily.

6.1 The Role of Contacts' Ideology

We have seen that biased decision-makers are less attractive lobbying targets. Now we show that this extends to their first degree contacts, to their second degree contacts and so forth, at a decreasing rate. Depending on the flow of resources there is a ranking in the attractiveness of decision-makers for lobbyists, with the most attractive lobbying target being the one that has the greatest distance to the biased decision-maker.

Suppose that decision-maker D_1 is initially in favor of L_1 and that all other decision-makers are unbiased, which is illustrated by (a) in Figure 6.[31]

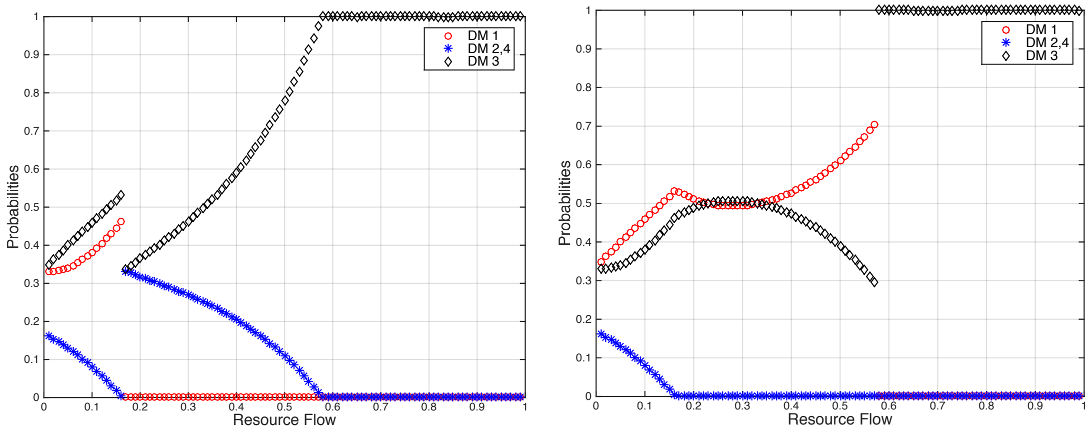
Proposition 7. *Let there be n decision-makers who are identical in their centrality but only one of them is biased. Then, if*

1. *the flow of resources is sufficiently low, both lobbyists assign positive probability to all decision-makers; and if*
2. *the flow of resources is sufficiently high, both lobbyists connect to the decision-maker with the greatest distance to the biased decision-maker.*

The implication is that both lobbyists' strategies depend on the decision-makers' contacts and the flow of resources in their network. For illustration, suppose there are four decision-makers and D_1's initial bias is $\varphi_1 = \frac{3}{4}$. The arising lobbying strategies for four decision-makers are then illustrated by Figure 7.

First, consider a low sharing of resources, then it is most important for L_1 to target the same decision-maker as L_2 and to engage in counteractive lobbying, whereas it is most important

[31] We consider the simplest case of n decision-makers with only one bias. However, it could be shown that the intuition carries over to n decision-makers and two arbitrary biases.



(a) Equilibrium Strategy of L_1.

(b) Equilibrium Strategy of L_2.

Figure 7: Equilibrium Strategies for $\varphi_1 = \frac{3}{4}$.

for L_2 to target a different decision-maker and follow a non-competitive lobbying strategy. As a result, both lobbyists assign a positive probability to all decision-makers. Now consider an intermediate flow of resources. $L1$ assigns positive probability to all unbiased nodes, but he never chooses the decision-maker biased in his favor. As D_2 and D_4 are equivalent both in terms of their ideology and their position in the network, the lobbyists assign the same probability to them. L_2 now puts positive probability on D_1 and D_3, but will never lobby D_2 or D_4. Finally, if there is a higher sharing of resources, then lobbying the same or different decision-maker is of less importance.[32] The biased decision-maker is less likely to be convinced and the lobbyists compete for the unbiased decision-maker, who is not directly connected to the biased decision-maker but is the connector to both unbiased decision-makers.

The neighborhood effects of this illustration with four decision-makers carry over to a ring with any arbitrarily large number of decision-makers and imply a ranking for the attractiveness of lobbying targets that illustrates that personal distance to a biased decision-maker increases a decision-maker's attractiveness. In other words, both lobbyists take the ideology of decision-makers and their direct and indirect contacts into account.

6.2 The Role of Homophily

Now suppose that there are two biased decision-makers and two unbiased ones who are either directly or indirectly with each other connected. In other words, we consider homphily in networks and discuss the different lobbying strategies that arise. The analysis shows that the initially favored lobbyist has a greater advantage over the other lobbyist to influence the network if decision-makers exhibit homophily. Due to the complexity of the network interactions at play

[32]We illustrate the interplay of the decision-maker's bias and the flow of resources in the Supplemental Appendix. There we show that the magnitude of the flow of resources has a greater influence on the type of equilibrium to emerge than the level of the bias.

we discuss the implications of homophily and illustrate our arguments numerically.[33] We set the ideological bias of the decision-makers in favor of L_1 to $\frac{3}{4}$.

Targeting Under Homophily First consider homophily where the two biased decision-makers are directly connected – i.e., let decision-makers D_1 and D_2 be the biased decision-makers as illustrated by (b) of Figure 6. In this case there are two types of Nash Equilibria. In the first type of equilibrium, both lobbyists assign positive probability to all decision-makers, whereas in the second type, only the unbiased decision-makers are lobbied.[34] Whenever the flow of resources is low, L_1 prefers to target the same decision-maker as L_2, whereas L_2 prefers to lobby a different one. Due to these opposing goals, both lobbyists end up mixing between all their strategies. However, as flow of resources increases, L_1 can defend his allies without being directly attached to them. This implies he only targets the unbiased decision-makers. But he prefers to be as close as possible to the decision-maker L_2 chooses. That is, if L_2 connects to a biased decision-maker, L_1 chooses the directly connected unbiased one and if L_2 chooses one of the unbiased decision-makers, L_1 wants to target the same one. L_2 still prefers to lobby a different one than L_1 and prefers the unbiased ones to the biased decision-makers. This results in both lobbyists targeting the unbiased decision-makers with equal probability.

Targeting Without Homophily Now suppose that the biased decision-makers are not directly connected as illustrated by (c) in Figure 6. Assume that decision-makers D_1 and D_3 are the biased ones such that D_2 and D_4 are equivalent.[35] There are again two kinds of equilibria. In the first type of equilibrium, there is mixing between all decision-makers; in the second type of equilibrium, L_1 mixes between the unbiased decision-makers, whereas lobbyist L_2 mixes between the two biased ones. This is different to the previous case with two directly connected biased decision-makers. For L_1 it is always best to lobby the same decision-maker as L_2, L_2 prefers to target a different one. Initially, L_1 still tries to counteract L_2's effort, eventually, as the flow of resources increases, L_1 is better off always being linked to the decision-maker who is directly connected with L_2's target. L_2 always chooses one of the biased decision-makers and is indifferent between them, when L_1 chooses an unbiased one, leading to the observed equilibria.[36]

Comparing the two lobbying outcomes under and without homophily, we can show that

[33]The complexity arises from the various interplays between the flow of resources, ideology and neighbor effects as well as the comparison of two network structures. An analytical solution comes at significant cost in terms of algebra and complexity but in terms of economics only adds little.

[34]The equilibrium strategies can be seen in the Appendix and are unique.

[35]This implies that the equilibrium strategies presented in the Appendix are not unique strategies.

[36]In the Appendix we show that there is a difference between the likelihood of lobbying strategies when the biased decision-makers are directly connected and when they are not. If the biased decision-makers are directly connected, then it is more likely to have mixing between all agents, than when they are not directly connected.

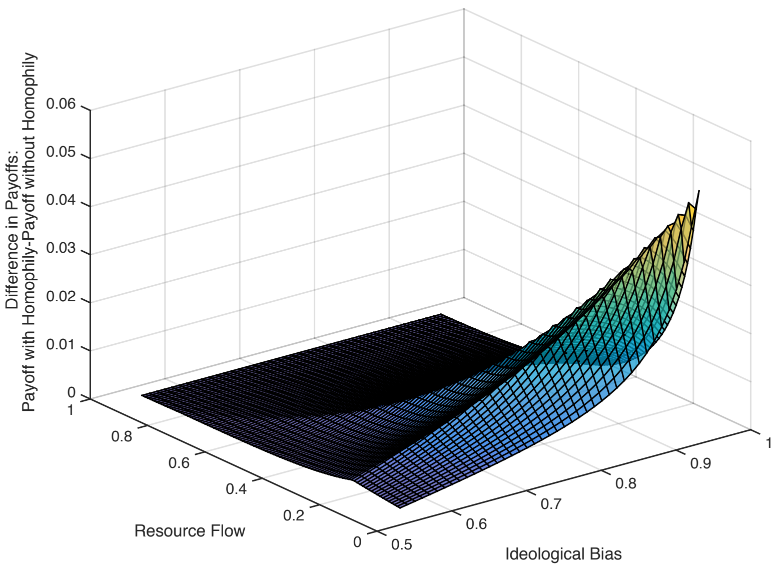


Figure 8: Difference in Value between Biased are Neighbors vs. Biased are Opposite.

homophily implies a greater advantage to the already favored lobbyist. Figure 8 shows the differences in payoffs for the initially favored lobbyist L_1, when the biased decision-makers are directly connected and when not. The difference between L_1's payoffs is largest when the sharing of resources is low and the ideological bias is large. As the equilibrium is in mixed strategies, the overall payoff is a linear combination of the different payoffs obtained at the different combinations of connections formed. The payoff of the combinations for L_1 is on average lower when the biased decision-makers are not directly connected, the overall payoff is as well.

To see why this difference in payoffs occurs, suppose that L_2 ends up being connected to an opposing decision-maker. Recall first that whenever the resource flow is low, the unfavored lobby group can almost neutralize the bias, when it is directly connected to a biased decision-maker and L_1 is not. In the case of no homophily, the biased decision-makers are opposite of each other and so, if L_2 is connected to a biased one, he is also in a neighborhood of unbiased decision-makers. Then, depending on the decision-maker L_1 picks, he is either closer to one of the unbiased decision-makers and further away from the other one or he has equal distance to both of them relative to L_1. So, as long as L_1 and L_2 end up choosing two different decision-makers L_2 can neutralize the bias of one decision-maker and his payoff from the unbiased decision-makers are equal to those of L_1.

If on the other hand, the biased decision-makers are directly connected, then if L_2 is connected to a biased one, he has one unbiased neighbor, but also one biased. This implies that, again depending on L_1's pick, it is either as if L_2 had equal distance to both unbiased decision-makers or that he has same distance to one of them and a greater distance to the other one, again relative to L_1. Thus, if L_1 and L_2 choose different decision-makers, L_2 can still neutralize the bias of the decision-maker he is directly connected to, but he incurs a loss at the unbiased decision-makers.[37]

Therefore, the overall payoff is higher for L_2 without homophily and this in turn results in a lower payoff for L_1. On the other hand, if the biased decision-makers are connected, then the payoff of L_1 is higher. As this scenario is more likely, due to the pervasiveness of homophily in any social relations, we would expect the network structure to prevail that gives a greater advantage to the already favored lobbyist.

7 Lobbying Patterns

In this section we relate the results of the model to various empirical facts of lobbying activities.[38] Our following discussion illustrates that several of our predictions are consistent with established empirical lobbying facts.

1. Unbiased decision-makers are more attractive lobbying targets.

Our analysis has highlighted consistently that unbiased, or undecided, decision-makers are the more attractive lobbying targets as they offer greater leverage effects in political networks. This result is surprising as one may expect that lobbyists mobilize either their initial allies among decision-makers or their opponents to weaken resistance. In our model this happens generally if the flow of resources is high as then lobbyists' resources reach all decision-makers, even biased ones. However, it is impossible to sway biased decision-makers independent of whom is directly targeted, and therefore it is more crucial to target the decision-maker who is neutral directly. This prediction is generally consistent with de Figueiredo and Richter (2014)'s review of the empirical lobbying literature. There is a common agreement in the literature that too biased decision-makers are never lobbied. Or, to quote Larry Whitt, vice president of Pizza Hut, from Baron (2013)'s textbook: "We focus on those on the fence". The attractiveness of undecided decision-makers is also documented by Hojnacki and Kimball (1999, 2001). Further, Mian et al. (2010) document that there is only a low probability of influencing and winning over biased decision-makers. In their analysis they show that conservative politicians were less likely to respond to lobbying pressure by financial interests and more likely to refuse financial government support of the Emergency Economic Stabilization Act of 2008 for companies even during a severe financial crisis. The financial crisis of 2008 and the aftermath received considerable attention of politicians and committee meetings at the time, which imply a greater flow of resources among politicians and illustrate our predictions.

[37] Along the same lines, one can show that the payoff is higher under no homophily if L_2 chooses an unbiased decision-maker and L_1 chooses a different decision-maker.

[38] de Figueiredo and Richter (2014) provide a recent review for the empirical research on lobbying activities and Ludema et al. (2013) as well as Kerr et al. (2014) discuss various empirical challenges.

2. Lobbyists target both opposing and friendly decision-makers.

Our results have shown that lobbyists also target friendly and opposing decision-makers depending on the network structure, the flow of resources and the other lobbyist's strategy, which is generally consistent with de Figueiredo and Richter (2014)'s review of empirical studies. Despite the evidence for lobbying unbiased decision-makers, there are also empirical studies that find that opposing decision-makers are lobbied (e.g., Austen-Smith and Wright (1994, 1996), de Figueiredo and Cameron (2008)) or like-minded decision-makers are preferred targets (e.g., Kollman (1997), Hojnacki and Kimball (1998), Eggers and Hainmueller (2009), Igan and Mishra (2014), Mian et al. (2013)). Our model explains these contrary observations with differences in the flow of resources and the centrality of decision-makers.

For example, Austen-Smith and Wright (1994, 1996) look at the lobbying efforts in the confirmation battle over Robert Bork's nomination to the U.S. Supreme Court in 1987. They find that lobbyists choose a priori opposing politicians and only lobby supporting legislators to counteract the influence of the opposing lobby group. The confirmation battle did not focus on drafting a bill but on getting votes, which implies a lower degree of resource sharing among legislators. This is in line with our model that predicts that if there is a low flow of resources lobbyists target both opponents and supporters. de Figueiredo and Cameron (2008) use state lobbying data that exclude contributions and they find that lobbyists tend to target opponents with costly lobbying signals in order to neutralize their resistance. They stress that lobbying expenditures do not consist of much "thinly disguised bribes" or "walking around money", i.e., resources that can be easily shared among decision-makers. This is in line with our predictions when we consider a low flow of resources.

Our predictions are also consistent with other studies which find empirical evidence that lobbyists target their political allies. For example, Hojnacki and Kimball (1998) surveyed interest groups and found that lobby groups choose politicians from districts to which the interest groups have ties as well as politicians that are allies. They find no evidence of counteractive lobbying. In their survey, 77% of the interest groups enlisted committee members to lobby other legislators on the group's behalf. This implies that although lobbyists addressed politicians in their favor, they ultimately did so to lobby undecided and opposing legislators indirectly. This is in line with our model when lobbyists face a political network in which their initial allies are well-connected and spread lobbyists' resources and influence in political networks. Eggers and Hainmueller (2009) argue that labor unions targeted Labor MP's in the United Kingdom and that business interest groups, as a response, targeted Conservatives. In light of our work this implies that Labor MP's are more central regarding labor and education issues and are therefore attractive lobbying targets for unions, whereas businesses target conservative decision-makers who are more central in competition and trade policies. Similarly, Igan and Mishra (2014) show that financial interests could expand their support in financial deregulation by targeting conservatives who are more likely to support policies of business deregulation. Again, our model is in line with the notion that conservative politicians are more central in competition policy and therefore more attractive lobbying targets.

Mian et al. (2013) provide an empirical analysis of the subprime mortgage crisis and politicians' responses to pressure by special interests and constituents. Looking at both campaign contributions and lobbying expenditures by mortgage lenders, they show that lenders' efforts increased during the early 2000s and targeted representatives from high subprime share congressional districts. Further, they document how the fraction of constituents with low credit scores and mortgage lenders' lobbying efforts explain representatives' voting patterns on legislation related to housing and mortgages. Their analysis illustrates that special interests targeted their allies. However, it also provides the argument that representatives from those districts had stronger interests in housing and finance and had therefore stronger reasons to become more central in committees related to housing, mortgages and financial regulation, which would make them more attractive lobbying targets.

3. Well-connected decision-makers are the most attractive lobbying targets.

Our analysis of centrality has illustrated the important role of decision-makers' direct and indirect contacts in a political network that shapes lobbyists' targeting strategies. Well-connected decision-makers offer lobbyists to expand their influence in the political network by offering those central players their resources. The importance of connections in lobbying is documented by Blanes i Vidal et al. (2012) who state that "connections to people in power represent a critical asset for the actors who serve as intermediaries in the lobbying process." Though their analysis focuses on lobbyists and their personal contacts to legislators, the importance of scarce, valuable connections that could be traded should carry over to politicians who act as de facto lobbyists in their political networks and spread the resources of their lobbying contacts.

Other studies have focused on the characteristics of lobbied decision-makers. Evans (1996) provides evidence that legislators with greater seniority are more attractive targets. Further, Hojnacki and Kimball (1998) surveyed interest groups and found that they were more likely to lobby the chairs of the committee. Similar evidence is provided by Krozner and Stratmann (1998) who show that Congress members with seniority, important committee memberships and chairs are attracting more campaign contributions. Seniority and chair assignments can imply various important political characteristics such as a greater formal power or agenda influence. However, these positions can also imply that those legislators could establish more political connections over time and could become more central players because of their relationship building abilities, or as discussed above, their relationship building incentives. In other words, we provide a new channel why seniority and chair assignments matter besides formal power and agenda influence that received attention in the literature. Our predictions are consistent with observed lobbying patterns but the role of relationship building has not been yet subject of a more detailed empirical analysis to distinguish between the various implications of seniority.

4. Decision-makers' network position matters.

Our analysis highlights that decision-makers' centrality and connections in the network matter for lobbyists' optimal targeting strategies. In a first step we have shown that there emerges a trade-off between choosing the unbiased and the central decision-maker. The decision-maker with most direct contacts to other decision-makers will in general be chosen if the bias is not too large in absolute terms for any flow of resources. However, the least biased decision-maker will be chosen if the biases of the more biased decision-makers are much higher and the flow of resources is sufficiently high.

A series of articles by Loewenberg (2003a,b) provides a case study of American lobbying activities in the European Union, which illustrates the trade-off between centrality and bias.[39] When the EU Commission wanted to increase the safety standards for chemicals due to pressure by environmental interest groups, American firms initiated a lobbying campaign against the proposal. The campaign did not target the EU Commission or the Commissioner in charge directly but focused on American policymakers who then lobbied on their behalf to other European member states. The lobbyists of American and European companies identified Greece as the most receptive lobbying target as well as France, Germany and even Japan and China for their orchestrated lobbying efforts. By the end of 2005 lobbyists could reduce the testing requirements to one-third of the initial proposal.

The results of our model suggest that the EU Commission was central but too biased, and therefore American and European lobbyists chose less biased lobbying targets in order to influence the overall political EU network, especially if the flow of resources is high. The importance of industrial EU politics on individual member states is significant and the Council of the European Union meets regularly to coordinate legislation and directives by the European Commission.[40] The frequency of exchanges among national ministers and the equality among them would imply a greater flow of resources and therefore predict a greater likelihood of peripheral lobbying if the central player is too biased, which is consistent with our model.

Lobbyists do not only take the centrality and bias of individual decision-makers into account but also decision-makers' chain of direct and indirect connections. Our model has shown that not only the ideology of the individual decision-maker matters, but also the ideology of the

[39]For a formal analysis of the costs and benefits of foreign lobbying and potential regulatory responses see Aidt and Hwang (2014).

[40]The Council is the representation of the ministers of states by issues and its chairs change every six months. For a review of EU institutions see Jorgensen et al. (2006).

decision-maker's first-degree, second-degree and so forth contacts. This is consistent with Hojnacki and Kimball (1998)'s survey, in which a large majority of interest groups enlisted committee members to lobby other legislators on the group's behalf. This implies that although lobbyists addressed specific politicians in their favor, they also took the targeted legislators' direct and indirect contacts of the political network into account. Our analysis suggests that lobbyists target decision-makers with the greatest distance to biased decision-makers. However, if the centrality of the decision-makers is the dominant lobbying factor, then lobbyists target central supporters.

5. Homophily in political networks affects lobbyists' strategies and payoffs.

Our analysis has shown that already favored lobbyists can expect greater advantages in lobbying from the network structures in which homophily among decision-makers is present. These differences in payoffs due to homophily are also increasing in the size of the network, and we would expect them to be significant in environments such as the politically polarized U.S. Congress or the gradually founded and expanded European Union. Recent studies have emphasized the role of homophily among decision-makers in party and coalition networks (Grossman and Dominguez (2009); Koger et al. (2009)), among interest groups of the revolving door or in parties (Lorenz and Hall (2013)) as well as among an interest group and a decision-maker (de Figueiredo and Silverman (2006)). McCarty et al. (2006) document the growing polarization of American politics and illustrate a positive correlation between this polarization and lobbying efforts. Our model provides a novel explanation for this observed co-movement of polarization and lobbying. As it is more likely that like-minded decision-makers are connected, due to homophily and polarization in politics, our model predicts such growing lobbying activities. We also believe the empirical analysis how lobbyists take homophily among decision-makers, e.g., among Democrats and Republicans, into account and influence political networks is intriguing and deserves further attention.

8 Conclusion

Our research question has centered on the behavior of competing lobbyists and their search of their most attractive lobbying target when they face heterogeneous but equal decision-makers who share resources such as information or financial resources in their political networks. We have shown that lobbying efforts do not neutralize each other and highlighted different lobbying strategies, depending on the network structure and decision-makers' ideologies. Our model therefore allows to reconcile seemingly contrary observations of the empirical lobbying literature.

The predictions of our model of lobbying with the consideration of decision-makers' networks are in line with observed lobbying patterns in the United States and Europe. For future research one may ask the interesting question how networks of decision-makers arise and how the formation relates to lobbying activities.[41] The argument could be that decision-makers, who anticipate the behavior of lobbyists, may have an incentive to become that optimal lobbying target and organize their networks as a best response if the interest groups' resources, or contributions, were beneficial to them. Furthermore, our analysis abstracted from the observations of costly access to decision-makers and differences in lobbies' endowments. Such differences in privileged access and resource endowments may expand the strategies of lobbies when they target decision-makers who are connected through a network.

Our analysis of targeting is essentially a Colonel Blotto game with the additional difficulty of a network with spillovers and heterogeneous decision-makers. The complexities of Colonel Blotto games are well understood and the additional features of our set up add two technical challenges. Despite those externality effects and heterogeneities we provided novel analytical solutions that are consistent with the empirical literature. Adding specific voting rules to our current setup, such as a majority rule, increase the complexity of the game further as it introduces another discontinuity and makes an analytical solution unattainable. Finally, our analysis opens new venues for empirical research on lobbying connected policymakers that would consider the explicit structure of networks, would allow to identify our analyzed mechanisms once the current data limitations of recent studies could be solved. We leave these questions for future research.

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[41]This would combine Krozner and Stratmann (1998)'s analysis of the formation of Congress committees with our current setup. In their analysis they focus on the facilitation of legislative support by decision-makers in exchange for financial contributions by special interests in the absence of legally binding contracts and presence of repeated interactions and reputation building.

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A Appendix

A.1 Proofs

Proof Proposition 1: Lobbying Payoffs

If no lobby group lobbies, then the payoff is simply the average of the initial biases,

$$\pi^1(0,0) = \frac{1}{n}\sum_{k=1}^{n} \varphi_i$$

Now let distance of the lobby group to nodes 1 through $\overline{k}$ differ, where $\overline{k} \in N$. Define $q_k = l(i,k) - l(j,k)$ and note that for any $k \leq \overline{k}$, $q_k > 0$ and for any $k > \overline{k}$, $q_k = 0$. Then the payoffs are given by

$$\pi^1(i,j) = \frac{1}{n}\left(\sum_{k=1}^{\overline{k}} \frac{\varphi_k}{\varphi_k + \delta^{q_k}(1-\varphi_k)} + \sum_{k=\overline{k}+1}^{n} \varphi_k\right),$$

where $i \neq j$. Then, $\frac{\varphi_k}{\varphi_k+\delta^{q_k}(1-\varphi_k)} \neq \varphi_k$ for $k \leq \overline{k}$. Additionally, for $\varphi_k \neq \frac{1}{2}$, $\frac{\varphi_k}{\varphi_k+\delta^{q_k}(1-\varphi_k)} + \frac{\delta^{q_k}\varphi_k}{\delta^{q_k}\varphi_k+(1-\varphi_k)} \neq 2\varphi_k$ as previously established. But it holds that

$$\frac{\varphi_k}{\varphi_k + \delta^{q_k}(1-\varphi_k)} + \frac{\delta^{q_k}(1-\varphi_k)}{\delta^{q_k}(1-\varphi_k) + \varphi_k} = 1 = \varphi_k + (1-\varphi_k) \tag{4}$$

This implies that if there exists a node which has a bias of $(1-\varphi_k)$ and the relative distance compared to node k is reversed, that is $-l(i,k) + l(j,k)$, then lobbying payoffs are the same as under the benchmark of no lobbying. It follows that generically, in biases, $\pi^1(0,0) \neq \pi^1(i,j)$.

Proof Proposition 2: Ring, n, $\varphi_i = \varphi\ \forall i \in N$

Case 1: All decision-makers are neutral If all decision-makers are neutral, $\varphi = \frac{1}{2}\ \forall i$, then the payoff no matter the strategy is equal to $\frac{1}{2}$. To see this, suppose first that n is even. Fix the node that one of the lobbyists chooses, and suppose without loss of generality that L_1 chooses decision-maker 1. Denote by s_k^+ the node that lies to the right of node 1 and is $k-1$ links away from node 1. The nodes to the left are denoted by s_k^-. Note that $k \in \{2,\ldots,\frac{n}{2}-1\}$. Let L_2 choose node s_k^+. Then any node between s_k^+ and $s_{\frac{n}{2}+1}$ yields an advantage for L_2. Due to the symmetry of the ring there exist a set of decision-makers at which L_2 is disadvantaged. This yields a payoff of

$$\left(\frac{n}{2} - k + 2\right)\left(\frac{\delta^{k-1}}{1+\delta^{k-1}} + \frac{1}{1+\delta^{k-1}}\right) = \left(\frac{n}{2} - k + 2\right) \tag{5}$$

Consider now the remaining nodes for which the payoff has not been specified. These are nodes that lie between nodes 1 and k as well as between $\frac{n}{2}+1$ and $\frac{n}{2}+k-1$. Consider first the nodes that lies between 1 and k. Then for each node either L_1 and L_2 have the same distance or if L_1 has a relative distance to a node that is lower than that of L_2 there exists another node where the relative distances are reversed. By symmetry, this also follows for the remaining nodes between $\frac{n}{2}+1$ and $\frac{n}{2}+k-1$ and thus for the remaining nodes the payoff is given by $(k-2)$. Therefore, the overall payoff is given by $\frac{1}{n}\frac{n}{2}=\frac{1}{2}$. This establishes that for any pure strategy combination, the payoff is $\frac{1}{2}$ and therefore, the payoff from any mixed strategy is also $\frac{1}{2}$. Then, it follows that any strategy can be part of an equilibrium. Next suppose that n is odd. The reasoning is similar to before. There are $\frac{n+1}{2}-k+1$ nodes at which the relative distance is k. Additionally, there are $\frac{n+1}{2}-\left(\frac{n+1}{2}-k+1\right)-1$ nodes to the right of node 1 at which the average payoff is $\frac{1}{2}$ and $\frac{n+1}{2}-\left(\frac{n+1}{2}-k+1\right)$ nodes to the left of node one, which again have an average payoff of $\frac{1}{2}$. This implies that the overall payoff is again $\frac{1}{2}$ and thus every strategy can be part of an equilibrium.

Case 2: All decision-makers are identically biased Suppose without loss of generality that $\varphi>\frac{1}{2}$. There cannot be an equilibrium in pure strategies. To see this note that

$$2\varphi>\frac{\delta\varphi}{\delta\varphi+(1-\varphi)}+\frac{\varphi}{\varphi+\delta(1-\varphi)} \tag{6}$$

This holds for any δ and due to the symmetry of the ring and thus a node at which L_1 has an advantage is always matched by one at which he has a disadvantage. It implies that L_1 always prefers to be at the same node as L_2 and as we have a zero sum game, L_2 prefers to be at a different node.

Suppose first that L_1 chooses a pure strategy. Then for an arbitrary strategy of L_2 his payoff is given by

$$\begin{aligned}
\Pi^1(1,\sigma^2)&=\sum_{i=1}^{n}\sigma^2(i)\pi^1(1,i)\\
\Pi^1(2,\sigma^2)&=\sum_{i=1}^{n}\sigma^2(i)\pi^1(2,i)=\sigma^2(1)\pi^1(1,n)+\sigma^2(2)\pi^1(1,1)+\cdots+\sigma^2(n)\pi^1(1,n-1)\\
\Pi^1(3,\sigma^2)&=\sum_{i=1}^{n}\sigma^2(i)\pi^1(3,i)=\sigma^2(1)\pi^1(1,n-1)+\sigma^2(2)\pi^1(1,n)+\cdots+\sigma^2(n)\pi^1(1,n-2)\\
&\;\;\vdots\\
\Pi^1(n,\sigma^2)&=\sum_{i=1}^{n}\sigma^2(i)\pi^1(n,i)=\sigma^2(1)\pi^1(1,2)+\sigma^2(2)\pi^1(1,3)+\cdots+\sigma^2(n)\pi^1(1,1)
\end{aligned}$$

This implies that if each node is chosen with equal probability, then the average payoff is independent of L_2's strategy as

$$\frac{1}{n}\sum_{i=1}^{n}\Pi^1(i,\sigma^2)=\frac{1}{n}\sum_{i=1}^{n}\pi^1(1,i) \tag{7}$$

This is equivalent to L_1's minimax payoff as

$$\max\{\sum_{i=1}^{n}\sigma^2(i)\pi^1(1,i),\sum_{i=1}^{n}\sigma^2(i)\pi^1(2,i),\dots,\sum_{i=1}^{n}\sigma^2(i)\pi^1(n,i)\}\geq\frac{1}{n}\sum_{i=1}^{n}\pi^1(1,i) \tag{8}$$

We can show the same for L_2, which then establishes that it is a Nash equilibrium for each lobbyists to assign probability $\frac{1}{n}$ to each node. To show uniqueness, note that if strategies (σ^1,σ^2) and $(\sigma^{1\prime},\sigma^{2\prime})$, then $(\sigma^1,\sigma^{2\prime})$ has to be a Nash equilibrium as well. $\sigma^{2\prime}$ has to assign a probability of greater than $\frac{1}{n}$ to at least one node and a probability of smaller than $\frac{1}{n}$ to another node. Suppose without loss of generality that $\sigma^2(1)>\frac{1}{n}$ and $\sigma^2(1)<\frac{1}{n}$. Then, $\Pi^1(1,\sigma^2)>\Pi^1(2,\sigma^2)$, and thus assigning equal probability to each node is not a best response. This shows uniqueness.

Proof Proposition 3: Ring, $n=3$**,** $\varphi_i=\varphi_j=1-\varphi_k\neq\frac{1}{2}$

Let L_1 be the lobbyist favored by the majority of the decision-makers. Note that

$$\pi^1(k,i)=\pi^1(k,j)=\frac{1}{3}(\frac{\delta\varphi}{\delta\varphi+1-\varphi}+\varphi+\frac{1-\varphi}{1-\varphi+\delta\varphi})=\frac{1}{3}(1+\varphi)=\pi^1(k,k)$$

As nodes i and j have the same network position and ideology, it is sufficient to consider only the payoffs when L_2 chooses node i to see whether there are profitable deviations.

$$\begin{aligned}\pi^1(i,i)&=\pi^1(i,k)=1+\varphi\\ \pi^1(j,i)&=\frac{\delta\varphi}{\delta\varphi+1-\varphi}+\frac{\varphi}{\varphi+\delta(1-\varphi)}+1-\varphi\end{aligned}$$

It is straightforward to establish that choosing the opposing decision-maker is s a weakly dominant strategy for L_1. Therefore, L_1 never has an incentive to deviate. But given L_1 chooses the opposing decision-maker, L_2 is indifferent between all decision-makers. It remains to be shown that there cannot be any other Nash equilibria. It can never be optimal for L_1 to choose a decision-maker who favors him with probability one as L_2 would then have an incentive to choose the other decision-maker in favor of L_1 as $\pi^2(i,j)>\pi^2(i,i)$ and $\pi^2(i,j)>\pi^2(i,k)$. So, in any other possible Nash equilibrium, L_1 would assign positive probability to at least two decision-makers. But then L_2 will assign positive probability to at least one of the decision-makers in favor of L_1. This implies that with positive probability L_1 will lobby i, when L_2 lobbies j. Therefore, L_1 has an incentive to deviate to assigning all probability to the opposing decision-maker and thus, L_1

will never mix in any Nash equilibrium.

Proof Proposition 4: Ring, $n = 3$, high flow of resources

We already established what happens if all decision makers have the same bias, so we restrict attention to the case where at most two decision-makers have the same bias. We show that for δ sufficiently high, choosing the least biased decision-makers strictly dominates selecting a strictly more biased one.

We can order the biases such that

$$\forall j \quad |\varphi_i - \frac{1}{2}| \leq |\varphi_j - \frac{1}{2}|, \quad i \neq j, \quad i, j \in \{1, 2, 3\}$$

To see that choosing i strictly dominates j, with $|\varphi_i - \frac{1}{2}| < |\varphi_j - \frac{1}{2}|$, $i \neq j$, we compare the payoff combinations of all actions, that is we show that $\pi^1(i, i) > \pi^1(j, i)$, $\pi^1(i, j) > \pi^1(j, j)$ and $\pi^1(i, k) > \pi^1(j, k)$. From this it follows that choosing i is a strictly dominant strategy compared to j iff

$$\varphi_i + \varphi_j > \frac{\delta\varphi_i}{\delta\varphi_i + (1 - \varphi_i)} + \frac{\varphi_j}{\varphi_j + \delta(1 - \varphi_j)} \tag{9}$$

$$\frac{\varphi_i}{\varphi_i + \delta(1 - \varphi_i)} + \frac{\delta\varphi_j}{\delta\varphi_j + (1 - \varphi_j)} > \varphi_i + \varphi_j \tag{10}$$

$$\frac{\varphi_i}{\varphi_i + \delta(1 - \varphi_i)} + \varphi_j > \varphi_i + \frac{\varphi_j}{\varphi_j + \delta(1 - \varphi_j)} \tag{11}$$

We do the same payoff comparison for L_2 and we find that for $\delta > \frac{\frac{\varphi_i}{1-\varphi_i}}{\frac{\varphi_j}{1-\varphi_j}}$. To see this consider the case where $\varphi_j > \varphi_i > \frac{1}{2}$. Then note that (9) and (11) always hold, for any δ. Inserting $\frac{\frac{\varphi_i}{1-\varphi_i}}{\frac{\varphi_j}{1-\varphi_j}}$ into (10) establishes the result. Going through the different bias combinations shows that for $\delta > \frac{\frac{\varphi_i}{1-\varphi_i}}{\frac{\varphi_j}{1-\varphi_j}}$ it is always a strictly dominant strategy to choose i.

This establishes that only the most unbiased nodes are chosen in equilibrium, for δ sufficiently high.

Proof of Proposition 7: Biased decision-makers and their neighbors are unattractive

δ high Suppose n is even. Note that independently of the chosen strategies every lobbyist has a share of $\frac{1}{2}(n - 2)$ for sure. The payoff thus only depends on two nodes. All possible payoffs for L_1 are given by:

$$\{\frac{\delta^q\varphi_1}{\delta^q\varphi_1 + 1 - \varphi_1} + \frac{1}{1 + \delta^q}, \ldots, \varphi_1 + \frac{1}{2}, \ldots, \frac{\varphi_1}{\varphi_1 + \delta^q(1 - \varphi_1)} + \frac{\delta^q}{1 + \delta^q}\}$$

where q is the the largest relative distance in the ring. It holds that

$$\frac{\delta^{q_i}\varphi_1}{\delta^{q_i}\varphi_1+1-\varphi_1}+\frac{1}{1+\delta^{q_i}}>\varphi_1+\frac{1}{2}$$
$$>\frac{\varphi_1}{\varphi_1+\delta^{j}(1-\varphi_1)}+\frac{\delta^{q_j}}{1+\delta^{q_j}}\quad\forall q_i,q_j\in\{1,\dots,q\},$$

for $\delta>(\frac{1-\varphi_1}{\varphi_1})^{\frac{1}{q}}$. This implies that L_1 prefers to be as far away from node the biased node as possible. The argument is the same for L_2. For this lobbyist, the different possible payoffs are

$$\{\frac{\delta^{q}(1-\varphi_1)}{\varphi_1+\delta^{q}(1-\varphi_1)}+\frac{1}{1+\delta^{q}},\dots,(1-\varphi_1)+\frac{1}{2},\dots,\frac{1-\varphi_1}{\delta^{q}\varphi_1+1-\varphi_1}+\frac{\delta^{q}}{1+\delta^{q}}\}.$$

Now,

$$\frac{\delta^{q_i}(1-\varphi_1)}{\varphi_1+\delta^{q_i}(1-\varphi_1)}+\frac{1}{1+\delta^{q_i}}>(1-\varphi_1)+\frac{1}{2}$$
$$>\frac{1-\varphi_1}{\delta^{q_j}\varphi_1+1-\varphi_1}+\frac{\delta^{q_j}}{1+\delta^{q_j}}\quad\forall q_i,q_j\in\{1,\dots,q\},$$

when $\delta>(\frac{1-\varphi_1}{\varphi_1})^{\frac{1}{q}}$. This implies that L_2 would like to be further away from the biased node than L_1. Therefore in equilibrium, both choose to connect to node $1+\frac{n}{2}$. The argument is similar when n is odd and is therefore omitted.

δ low, no NE in pure strategies As before we can use the fact that for every strategy combination the payoff $\frac{1}{2}(n-2)$ is certain. Thus we can again restrict attention to the following sets of payoffs for L_1 and L_2 :

$$\{\frac{\delta^{q}\varphi_1}{\delta^{q}\varphi_1+1-\varphi_1}+\frac{1}{1+\delta^{q}},\dots,\varphi_1+\frac{1}{2},\dots,\frac{\varphi_1}{\varphi_1+\delta^{q}(1-\varphi_1)}+\frac{\delta^{q}}{1+\delta^{q}}\},$$
$$\{\frac{\delta^{q}(1-\varphi_1)}{\varphi_1+\delta^{q}(1-\varphi_1)}+\frac{1}{1+\delta^{q}},\dots,(1-\varphi_1)+\frac{1}{2},\dots,\frac{1-\varphi_1}{\delta^{q}\varphi_1+1-\varphi_1}+\frac{\delta^{q}}{1+\delta^{q}}\}.$$

We find that

$$\varphi_1+\frac{1}{2}>\frac{\varphi_1}{\varphi_1+\delta^{q_i}(1-\varphi_1)}+\frac{\delta^{q_i}}{1+\delta^{q_i}}\quad\forall q_i\in\{-q,\dots,q\}$$
$$(1-\varphi_1)+\frac{1}{2}<\frac{\delta^{q_i}(1-\varphi_1)}{\varphi_1+\delta^{q_i}(1-\varphi_1)}+\frac{1}{1+\delta^{q_i}}\quad\forall q_i\in\{-q,\dots,q\}$$

This implies that L_1 wants to be at the same node as L_2, whereas L_2 wants to be at a different node and so there cannot be a Nash equilibrium in pure strategies. [42]

We proceed to establish that for a low enough flow of resources, both lobbyists assign positive

[42]It also cannot be the case that L_2 plays a pure strategy in a Nash equilibrium, but it might be that L_1 plays a pure strategy and L_2 mixes between s_1 and s_{1+d}.

probability to all nodes. Suppose L_1 mixes between all strategies. For this to be the case it has to hold that

$$\underbrace{\begin{pmatrix} \varphi_1 + \frac{1}{2} & \frac{\varphi_1}{\varphi_1+\delta(1-\varphi_1)} + \frac{\delta}{1+\delta} & \cdots & \frac{\varphi_1}{\varphi_1+\delta^q(1-\varphi_1)} + \frac{\delta^q}{1+\delta^q} \\ \frac{\delta\varphi_1}{\delta\varphi_1+(1-\varphi_1)} + \frac{1}{1+\delta} & \varphi_1 + \frac{1}{2} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\delta^q\varphi_1}{\delta^q\varphi_1+(1-\varphi_1)} + \frac{1}{1+\delta} & \cdots & \cdots & \varphi_1 + \frac{1}{2} \end{pmatrix}}_{=A} \begin{pmatrix} \sigma^2(s_1) \\ \sigma^2(s_2) \\ \vdots \\ \sigma^2(s_{1+q}) \end{pmatrix} = \underbrace{\begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_q \end{pmatrix}}_{=c},$$

where c denotes the vector of payoffs. Now, let $\delta \to 0$.

$$\lim_{\delta\to 0} A = \begin{pmatrix} \varphi_1 + \frac{1}{2} & 1 & \cdots & 1 \\ 1 & \varphi_1 + \frac{1}{2} & \cdots & 1 \\ \vdots & \vdots & \ddots & 1 \\ 1 & 1 & \cdots & \varphi_1 + \frac{1}{2} \end{pmatrix}$$

Therefore, at the limit, to make L_1 indifferent between all the strategies, L_2 assigns equal probability to all the decision-makers. As the payoffs of L_2 in the limit are given by $B = 2 - A$, for L_2 to be indifferent, L_1 also has to mix between all his strategies with the same probabilities. Therefore,

$$\sigma^1(s_1) = \sigma^1(s_2) = \cdots = \sigma^1(s_d) = \sigma^2(s_1) = \sigma^2(s_2) = \cdots = \sigma^2(s_d) \equiv \sigma$$

The payoff of L_1 when mixing between all strategies is

$$q\sigma^2(\varphi_1 + \frac{1}{2}) + (1 - q\sigma^2).$$

Due to the structure of the payoff matrix, any strategy yields the same payoff. Therefore L_1 does not have an incentive to deviate. Due to the fact that we have a constant sum game, L_2 then also has no incentive to deviate. This shows that mixing between all strategies is indeed an equilibrium in the limit.

As

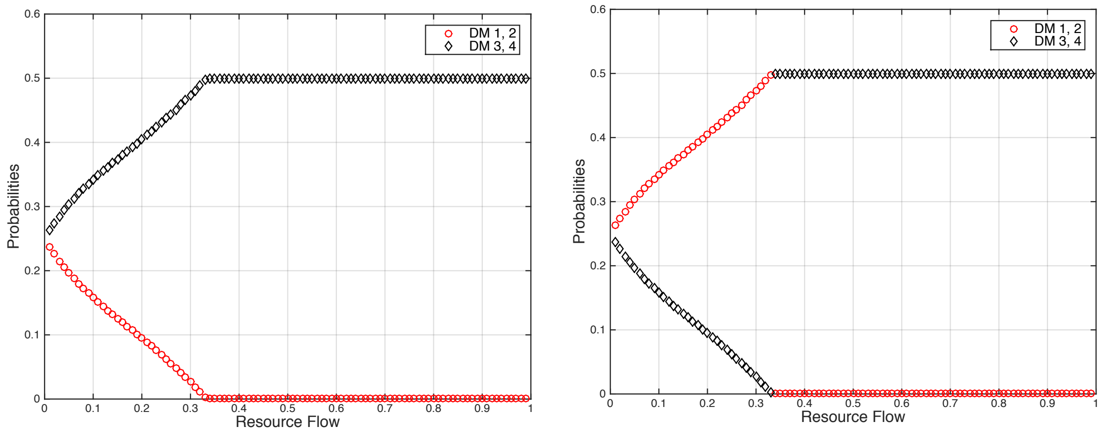
$$\frac{\delta^{q_i}\varphi_1}{\delta^{q_i}\varphi_1 + (1-\varphi_1)} + \frac{1}{1+\delta^{q_i}} \qquad \forall q_i \in \{-q, \ldots, q\}$$

is continuous in δ and the overall payoff is a sum of continuous functions, there exists a $\overline{\delta}(\varphi_1)$ such that for $\delta < \overline{\delta}(\varphi_1)$ both lobbyists mix between all strategies.

A.2 Simulations

Ring: Role of Homophily for Four Decision-Makers

Homphily Suppose first that the biased decision-makers are directly connected, which illustrates homophily amongst biased decision-makers. Let decision-makers D_1 and D_2 be the biased decision-makers. Here we illustrate again the interplay of the biased decision-makers' biases and the flow of resources. The equilibrium strategies can be seen in Figure 9.[43]



(a) Equilibrium Strategy of L_1. (b) Equilibrium Strategy of L_2.

Figure 9: Equilibrium Strategies for 2 Biased and 2 Unbiased Decision-Makers (Neighbors) with $\varphi_1 = \varphi_2 = \frac{3}{4}$.

Figure 10 illustrates when which type of described equilibrium occurs. Here the biases of decision-makers D_1 and D_2 are of the same magnitude. Again, the flow of resources reinforces the bias. If both the bias and the flow of resources are relatively low, then we have mixing between all decision-makers and if the bias and the flow are high, only the unbiased decision-makers are lobbied with positive probability.

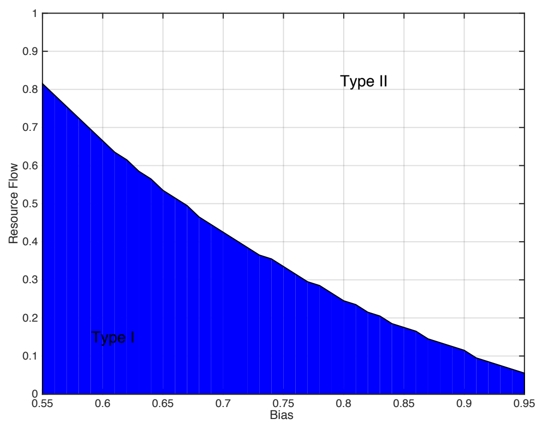
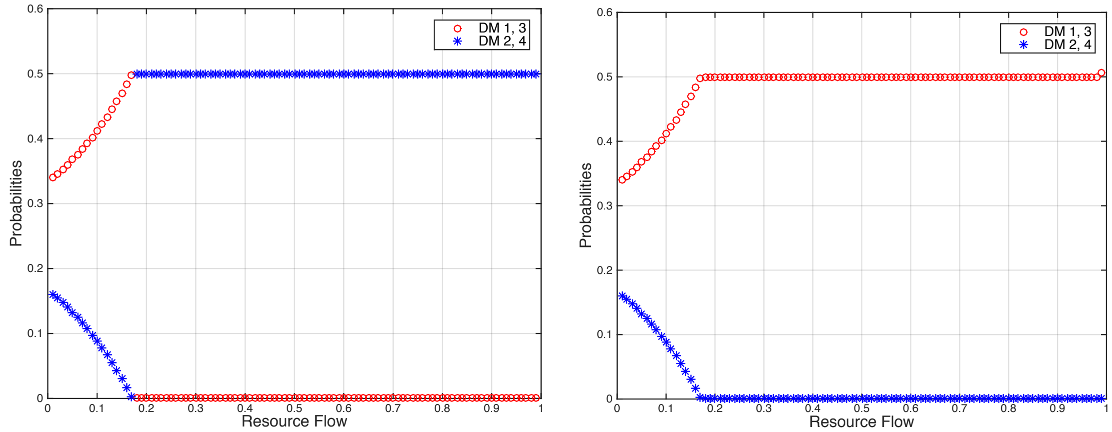


Figure 10: Thresholds for 2 Biased and 2 Unbiased Decision-Makers (Neighbors).

[43]The bias is fixed at $\varphi_1 = \varphi_2 = \frac{3}{4}$ and note that the equilibria depicted here are unique.

No Homphily Now suppose that the biased decision-makers are not directly connected. Let decision-makers D_1 and D_3 be the biased decision-makers. We illustrate again the interplay of the biased decision-makers' biases and the flow of resources.

The equilibrium strategies can be seen in Figure 11 and Figure 12 illustrates the interactions between both.



(a) Equilibrium Strategy of L_1.

(b) Equilibrium Strategy of L_2.

Figure 11: Equilibrium Strategies for 2 Biased and 2 Unbiased Decision-Makers (Opposite) with $\varphi_1 = \varphi_3 = \frac{3}{4}$.

If the flow of resources is low, then mixing between all decision-makers is the equilibrium outcome. If the flow is high, L_1 choosing the unbiased decision-makers and L_2 choosing the biased decision-makers is the equilibrium that emerges. Note here the difference between the size of the regions when the biased decision-makers are next to each other and opposite of each other. If the biased decision-makers are direct neighbors, then it is more likely to have mixing between all agents, than when they are opposite of each other.

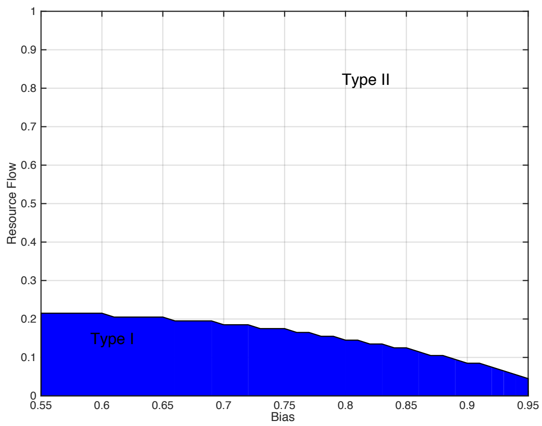


Figure 12: Thresholds for 2 Biased and 2 Unbiased Decision-Makers (Opposite).

B Supplemental Appendix: Online

Proof Proposition 5 : Even very biased decision-makers are lobbied

Case 1: Decision-makers i and j are unbiased, k is in favor of L_1 Note that i and j are payoff equivalent. L_1 is indifferent between his strategies if

$$\sigma^2(i) + \sigma^2(j) = \frac{(\delta(1-\varphi_k) - \varphi_k)(1-\varphi_k - \delta\varphi_k)}{2(1-\delta^2)(1-\varphi_k)\varphi_k},$$

whereas L_2 is indifferent if

$$\sigma^1(i) + \sigma^1(j) = 1 - \frac{(\delta(1-\varphi_k) - \varphi_k)(1-\varphi_k - \delta\varphi_k)}{2(1-\delta^2)(1-\varphi_k)\varphi_k}.$$

Last, note that $\frac{(\delta(1-\varphi_k)-\varphi_k)(1-\varphi_k-\delta\varphi_k)}{2(1-\delta^2)(1-\varphi_k)\varphi_k} \in (0,1)$ if and only if $\delta < \min\{\frac{\varphi_k}{1-\varphi_k}, \frac{1-\varphi_k}{\varphi_k}\}$. There cannot be a pure strategy equilibrium as again, the lobbyist k favors prefers to be at the same node as the lobbyist k opposes, whereas the lobbyist k dislikes prefers to be at a different node. Thus, the set of Nash equilibria given is unique.

Case 2: Decision-makers i and j are biased, k is unbiased We define $\underline{\delta}_i = \min\{\frac{1-\varphi_i}{\varphi_i}, \frac{\varphi_i}{1-\varphi_i}\}$ and $\overline{\delta} = \max\{\underline{\delta}_i, \underline{\delta}_j, \underline{\delta}_k\}$ and let $\delta < \overline{\delta}$. We show case by case that the following are the unique Nash equilibria.

(a) $\varphi_j = \varphi_k \equiv \varphi > \frac{1}{2}$: L_1 chooses the unbiased decision-maker, L_2 mixes between the biased decision-makers.

(b) $\varphi_k > \varphi_j > \frac{1}{2}$:

(i) $\frac{1-\varphi_k}{\varphi_k} < \delta < \frac{1-\varphi_j}{\varphi_j} = \overline{\delta}$: Both lobbyists assign positive probability to decision-makers i and j.

(ii) $\frac{1-\varphi_j-\varphi_k+\varphi_j\varphi_k}{\varphi_j\varphi_k} < \delta < \frac{1-\varphi_k}{\varphi_k}$: L_1 assigns positive probability to decision-makers i and j, L_2 assigns positive probability to j and k.

(iii) $0 < \delta < \frac{1-\varphi_j-\varphi_k+\varphi_j\varphi_k}{\varphi_j\varphi_k}$: L_1 assigns positive probability to i and k, L_2 to j and k

(c) $\varphi_j = 1 - \varphi_k > \frac{1}{2}$: both lobbyists assign positive probability to the biased decision-makers.

(d) $\varphi_j > 1 - \varphi_k > \frac{1}{2}$

(i) $\frac{1-\varphi_j}{\varphi_j} < \delta < \frac{\varphi_k}{1-\varphi_k}$: both lobbyists assign positive probability to i and k

(ii) $\frac{\varphi_k}{1-\varphi_k}\frac{1-\varphi_j}{\varphi_j} < \delta < \frac{1-\varphi_j}{\varphi_j}$: L_1 assigns positive probability to i and k, L_2 to j and k.

(iii) $0 < \delta < \frac{\varphi_k}{1-\varphi_k}\frac{1-\varphi_j}{\varphi_j}$: both lobbyists assign positive probability to j and k.

(a) $\varphi_j = \varphi_k \equiv \varphi > \frac{1}{2}$

Choosing the unbiased decision-maker is indeed a best response for L_1 to $L_2's$ strategy if

$$
\begin{aligned}
&\frac{1}{1+\delta}+\varphi+\frac{\delta\varphi}{\delta\varphi+(1-\varphi)}\\
&\quad(\sigma^1(i)\sigma^2(j)+\sigma^1(i)(1-\sigma^2(j)))(\frac{1}{1+\delta}+\varphi+\frac{\delta\varphi}{\delta\varphi+(1-\varphi)})\\
&\quad+(\sigma^1(j)\sigma^2(j)+(1-\sigma^1(i)-\sigma^1(j))(1-\sigma^2(j)))(\frac{1}{2}+2\varphi)\\
&\quad+(\sigma^1(j)(1-\sigma^2(j))+(1-\sigma^1(i)-\sigma^1(j))\sigma^2(j))(\frac{1}{2}+\frac{\varphi}{\varphi+\delta(1-\varphi)}+\frac{\delta\varphi}{\delta\varphi+1-\varphi})
\end{aligned}
$$

Simplifying yields

$$
\begin{aligned}
&(1-\sigma^1(i))(\frac{1}{1+\delta}+\varphi+\frac{\delta\varphi}{\delta\varphi+(1-\varphi)})\\
&\quad(\sigma^1(j)\sigma^2(j)+(1-\sigma^1(i)-\sigma^1(j))(1-\sigma^2(j)))(\frac{1}{2}+2\varphi)\\
&\quad+(\sigma^1(j)(1-\sigma^2(j))+(1-\sigma^1(i)-\sigma^1(j))\sigma^2(j))(\frac{1}{2}+\frac{\varphi}{\varphi+\delta(1-\varphi)}+\frac{\delta\varphi}{\delta\varphi+1-\varphi})\\
&\Leftrightarrow\\
&\frac{1}{1+\delta}+\varphi+\frac{\delta\varphi}{\delta\varphi+(1-\varphi)}\\
&\quad(x\sigma^2(j)+(1-x)(1-\sigma^2(j)))(\frac{1}{2}+2\varphi)\\
&\quad+(x(1-\sigma^2(j))+(1-x)\sigma^2(j))(\frac{1}{2}+\frac{\varphi}{\varphi+\delta(1-\varphi)}+\frac{\delta\varphi}{\delta\varphi+1-\varphi}),
\end{aligned}
$$

where $x=\frac{\sigma^1(j)}{1-\sigma^1(i)}$ and $1-x=\frac{1-\sigma^1(i)-\sigma^1(j)}{1-\sigma^1(i)}$. As $2\varphi>\frac{\varphi}{\varphi+\delta(1-\varphi)}+\frac{\delta\varphi}{\delta\varphi+1-\varphi}$, $L_1's$ problem is

$$\max_x \qquad \big(x(1-\sigma^2(j))+(1-x)\sigma^2(j)\big)$$

If $\sigma^2(j)=\frac{1}{2}$, any value of x is a solution – i.e., L_1 is indifferent between choosing decision-makers 2 and 3. But in this case, choosing $\sigma^1(i)=1$ is the unique best response as

$$\frac{1}{1+\delta}+\varphi+\frac{\delta\varphi}{\delta\varphi+(1-\varphi)}>\frac{1}{2}(\frac{1}{2}+2\varphi)+\frac{1}{2}(\frac{1}{2}+\frac{\varphi}{\varphi+\delta(1-\varphi)}+\frac{\delta\varphi}{\delta\varphi+1-\varphi})$$

If $\sigma^2(j)<\frac{1}{2}$, L_1 chooses $\sigma^1(j)=0$ and if $\sigma^2(j)>\frac{1}{2}$, $\sigma^1(k)=0$. If $\sigma^2(j)<\frac{1}{2}$, L_1 prefers decision-maker i to decision-maker k if $\sigma^2(j)>\frac{1}{2}-\frac{\delta(2\varphi-1)}{2(1-\delta^2)\varphi(1-\varphi)}$. And if $\sigma^2(j)>\frac{1}{2}$, L_1 prefers decision-maker i to decision-maker j if $\sigma^2(j)<\frac{1}{2}+\frac{\delta(2\varphi-1)}{2(1-\delta^2)\varphi(1-\varphi)}$, which establishes the given boundaries. Given L_1 chooses $\sigma^1(i)=1$, L_2 is indifferent between the biased decision-makers and prefers the biased decision-makers to the neutral ones as

$$1-\frac{1}{3}(\frac{1}{1+\delta}+\varphi+\frac{\delta\varphi}{\delta\varphi+(1-\varphi)})>1-\frac{1}{3}(\frac{1}{2}+2\varphi)$$

for the specified levels of δ.

The question that remains is whether the set of Nash equilibria is unique. To check this consider the possible strategies of L_2. Suppose first that L_2 assigns positive probability to all decision-makers. L_1 is never indifferent between all decision-makers in this case as when he is indifferent between j and k, he strictly prefers the unbiased decision-maker i to the biased ones. For this same reason it can never be a best response of L_1 to mix between the biased decision-makers alone. It might be a best response for L_1 to mix between either i and j or i and k. But if L_1 mixes between i and j (k), L_2 chooses k (j) and does not mix anymore. So this can also not be a Nash equilibrium. If L_1 chooses a pure strategy, then L_2 also has an incentive to deviate. L_2 will then be better off choosing the decision-makers, L_1 assigns zero probability to. Therefore, there cannot be a Nash equilibrium that involves L_2 mixing between all his strategies. Next, suppose L_2 mixes between the unbiased and one of the biased decision-makers. Then, L_1 will never mix between all strategies as he will never be indifferent between the biased decision-makers. Therefore, he will also not randomize over the biased decision-makers only. It can be a best response to assign positive probability to the unbiased decision-maker and the same biased decision-maker L_2 chooses. But then, L_2 will deviate to the decision-maker, L_1 does not lobby with positive probability. Last, L_1 might choose a pure strategy. But the best response to a pure strategy is never mixing between one biased decision-maker and the unbiased one. Thus, it can also not be part of a Nash equilibrium that L_2 mixes between the unbiased and one biased decision-maker. We already checked what happened when L_2 mixes between the biased decision-makers. And last, playing a pure strategy can never be part of a Nash equilibrium as L_1 will always have an incentive to choose the same decision-maker, which then leads L_2 to choose a different one. This shows that there is indeed only the specified set of Nash equilibria.

(b) $\varphi_k > \varphi_j \equiv \varphi > \frac{1}{2}$

(i) $\frac{1-\varphi_k}{\varphi_k} < \delta < \frac{1-\varphi_j}{\varphi_j}$

If the discount factor lies in this range, the unique Nash equilibrium is given by

$$\sigma^1(i) = 1 - \sigma^1(j), \quad \sigma^1(j) = \frac{\delta + \varphi_j - 2\delta\varphi_j - \delta^2\varphi_j - \varphi_j^2 + \delta^2\varphi_j^2}{2\varphi_j - 2\delta^2\varphi_j - 2\varphi_j^2 + 2\delta^2\varphi_j^2}$$
$$\sigma^2(i) = 1 - \sigma^2(j), \quad \sigma^2(j) = \frac{-\delta + \varphi_j + 2\delta\varphi_j - \delta^2\varphi_j - \varphi_j^2 + \delta^2\varphi_j^2}{2\varphi_j - 2\delta^2\varphi_j - 2\varphi_j^2 + 2\delta^2\varphi_j^2}$$

It is straightforward to verify that the proposed strategies define a Nash equilibrium. It remains to show that the Nash equilibrium is unique. Suppose L_2 assigns positive probability to all decision-makers. For L_1 it is never a best response to mix between all nodes, as he is never indifferent between them. However, mixing between i and j can be a best response, but then L_2

does not have an incentive to mix between all three nodes. Mixing between i and k as well as between j and k cannot be a best response for the given range of δ. And choosing a pure strategy leads L_2 to prefer some pure strategy to mixing. Therefore, L_2 can assign positive probability to at most two decision-makers. Suppose next, L_2 assigns positive probability to i and k. In this case, L_1 is never indifferent between i and j and therefore, mixing between all nodes or mixing between i and j can never be a best response. Also, mixing between i and k is not a best response for the specified δ. Mixing between j and k is not a best response as choosing i is always better than choosing j. As before, if $L1$ chooses a pure strategy, L_2 will not mix. Now suppose, L_2 chooses to mix between j and k. As before, mixing between all nodes will not be a best response for L_1. It is a best response for L_1 to mix between i and j. But then mixing between j and k is not a best response for L_2. Mixing between i and k or between j and k is not best response for the given range of δ. So, what remains is that L_2 chooses a pure strategy. If L_2 chooses i (j), L_1 chooses i (j) as well. But given L_1 chooses i (j), choosing i (j) is strictly dominated for L_2. For $\frac{1-\varphi_k}{\varphi_k} < \delta < \frac{1-\varphi_j}{\varphi_j}$, if L_2 chooses k, L_1 chooses i. But L_2's best response is then choosing j. This establishes uniqueness. For all additional cases uniqueness can be establishes the same way and is therefore omitted.

(ii) $\frac{1-\varphi_j-\varphi_k+\varphi_j\varphi_k}{\varphi_j\varphi_k} < \delta < \frac{1-\varphi_k}{\varphi_k}$

In this case the unique Nash equilibrium is given by

$$\sigma^1(i) = 1 - \sigma^1(j), \quad \sigma^1(j) = \frac{(\delta(1-\varphi_j)+\varphi_j)(\varphi_k-\varphi_j)(-1+\varphi_k+\varphi_j(1-(1-\delta)\varphi_k))}{(1-\delta)\varphi_j(3\varphi_j-1-2\varphi_j^2)(1-(1-\delta)\varphi_k)}$$
$$\sigma^2(j) = 1 - \sigma^2(k), \quad \sigma^2(k) = \frac{\delta+\varphi_j-2\delta\varphi_j-\delta^2\varphi_j-\varphi_j^2+\delta^2\varphi_j^2}{2\varphi_j-2\delta^2\varphi_j-2\varphi_j^2+2\delta^2\varphi_j^2}$$

(iii) $0 < \delta < \frac{1-\varphi_j-\varphi_k+\varphi_j\varphi_k}{\varphi_j\varphi_k}$

The unique Nash equilibrium is given by

$$\sigma^1(i) = 1 - \sigma^1(k), \quad \sigma^1(k) = \frac{(\varphi_k-\varphi_j)(\delta(1-\varphi_k)+\varphi_k)(1-\varphi_k-\varphi_j(1-(1-\delta)\varphi_k))}{(1-\delta)(1-(1-\delta)\varphi_j)\varphi_k(3\varphi_k-1-2\varphi_k^2)}$$
$$\sigma^2(j) = 1 - \sigma^2(k), \quad \sigma^2(k) = \frac{-\delta+\varphi_k+2\delta\varphi_k-\delta^2\varphi_k-\varphi_k^2+\delta^2\varphi_k^2}{2\varphi_k-2\delta^2\varphi_k-2\varphi_k^2+2\delta^2\varphi_k^2}$$

It is straightforward to verify that this is indeed a Nash equilibrium. Uniqueness can be shown along the same lines as previously and is therefore omitted.

(c) $\varphi_j = 1 - \varphi_k > \frac{1}{2}$ The set of Nash equilibria is given by

$$\sigma^1(j) \in [0, \frac{\delta+\varphi_j-2\delta\varphi_j-\delta^2\varphi_j-\varphi_j^2+\delta^2\varphi_j^2}{2\varphi_j-2\delta^2\varphi_j-2\varphi_j^2+2\delta^2\varphi_j^2}], \quad \sigma^1(k) = 1 - \sigma^1(k)$$
$$\sigma^2(j) \in [\frac{\delta+\varphi_k-2\delta\varphi_k-\delta^2\varphi_k-\varphi_k^2+\delta^2\varphi_k^2}{2\varphi_k-2\delta^2\varphi_k-2\varphi_k^2+2\delta^2\varphi_k^2}, 1], \quad \sigma^2(k) = 1 - \sigma^2(j)$$

It is again easy to verify that these are Nash equilibria as well as that these are the only Nash equilibria and is therefore omitted.

(d)$\varphi_j > 1-\varphi_k > \frac{1}{2}$

(i) $\frac{1-\varphi_j}{\varphi_j} < \delta < \frac{\varphi_k}{1-\varphi_k}$

The unique Nash equilibrium is given by

$$\sigma^1(i) = \frac{\delta+\varphi_k-2\delta\varphi_k-\delta^2\varphi_k-\varphi_k^2+\delta^2\varphi_k^2}{2\varphi_k-2\delta^2\varphi_k-2\varphi_k^2+2\delta^2\varphi_k^2}, \quad \sigma^1(k) = 1-\sigma^1(i)$$
$$\sigma^2(i) = \frac{-\delta+\varphi_k+2\delta\varphi_k-\delta^2\varphi_k-\varphi_k^2+\delta^2\varphi_k^2}{2\varphi_k-2\delta^2\varphi_k-2\varphi_k^2+2\delta^2\varphi_k^2}, \quad \sigma^2(k) = 1-\sigma^2(i)$$

(ii) $\frac{\varphi_k}{1-\varphi_k}\frac{1-\varphi_j}{\varphi_j} < \delta < \frac{1-\varphi_j}{\varphi_j}$

The unique Nash equilibrium is given by

$$\sigma^1(i) = \frac{(-1+\varphi_j+\varphi_k)(1+(-1+\delta)\varphi_k)(\delta\varphi_j(-1+\varphi_k)+\varphi_k-\varphi_j\varphi_k)}{(-1+\delta)(1+(-1+\delta)\varphi_j)\varphi_k(1-3\varphi_k+2\varphi_k^2)}, \quad \sigma^1(k) = 1-\sigma^1(i)$$
$$\sigma^2(j) = \frac{\delta+\varphi_k-2\delta\varphi_k-\delta^2\varphi_k-\varphi_k^2+\delta^2\varphi_k^2}{2\varphi_k-2\delta^2\varphi_k-2\varphi_k^2+2\delta^2\varphi_k^2}, \quad \sigma^2(k) = 1-\sigma^2(j)$$

(iii) $0 < \delta < \frac{\varphi_k}{1-\varphi_k}\frac{1-\varphi_j}{\varphi_j}$

The unique Nash equilibrium is given by

$$\sigma^1(j) = \frac{((\delta(-1+\varphi_j)-\varphi_j)(1+(-1+\delta)\varphi_k)(\delta\varphi_j(-1+\varphi_k)+\varphi_k-\varphi_j\varphi_k))}{C}, \sigma^1(k) = 1-\sigma^1(j)$$
$$\sigma^2(j) = \frac{((1+(-1+\delta)\varphi_j)(\delta(-1+\varphi_k)-\varphi_k)(-\delta\varphi_k+\varphi_j(1+(-1+\delta)\varphi_k)))}{C}, \quad \sigma^2(k) = 1-\sigma^2(j)$$

with

$$C = ((-1+\delta)(2(-1+\varphi_j)\varphi_j(-1+\varphi_k)\varphi_k+2\delta^2(-1+\varphi_j)\varphi_j(-1+\varphi_k)\varphi_k$$
$$+\delta(\varphi_k-2\varphi_k^2+\varphi_j^2(-2+4\varphi_k-4\varphi_k^2)+\varphi_j(1-2\varphi_k+4\varphi_k^2))))$$

Case 2: All decision-makers i, j, k are biased Let $\delta \to 0$ and let all decision-makers be biased – i.e., $\varphi_i \neq \frac{1}{2}, \forall i \in N$. If

(a) $\varphi_i > \frac{1}{2}, \forall i$: in the unique Nash equilibrium, both lobbyists assign positive probability to all decision-makers.

(b) $\varphi_i \geq \varphi_j > \frac{1}{2}, 1-\varphi_i > \varphi_k > 0$: in the unique Nash equilibrium, both lobbyists assign positive probability to all decision-makers.

(c) $\varphi_i > \varphi_j > \frac{1}{2}, 1-\varphi_j < \varphi_k$: in the unique Nash equilibrium, L_1 assigns positive probability to i and k and L_2 mixes between i and j.

(d) $\varphi_i = \varphi_j > \frac{1}{2}, 1-\varphi_j < \varphi_k$: in the set of Nash equilibria, L_1 chooses k, L_2 randomizes between i and j.

(e) $1 > \varphi_i > 1-\varphi_k \geq \varphi_j > \frac{1}{2}$: in the unique Nash equilibrium, L_1 chooses i and k and L_2

chooses between i and j.

(f) $1 > \varphi_i = 1 - \varphi_k > \varphi_j > \frac{1}{2}$: in the unique Nash equilibrium, both lobbyists mix between i and k.

We first take the limit of the payoff matrix:

$$\lim_{\delta \to 0} A = \begin{pmatrix} \varphi_1 + \varphi_2 + \varphi_3 & 1 + \varphi_3 & 1 + \varphi_2 \\ 1 + \varphi_3 & \varphi_1 + \varphi_2 + \varphi_3 & 1 + \varphi_1 \\ 1 + \varphi_2 & 1 + \varphi_1 & \varphi_1 + \varphi_2 + \varphi_3 \end{pmatrix}$$

Based on this, we find the Nash equilibria.

(a) $\varphi_i > \frac{1}{2}, \forall i$:

The unique Nash equilibrium $\forall x \in \{1, 2\}, \forall i \in \{1, 2, 3\}$, is given by

$$\begin{aligned} \sigma^x(i) &= \frac{(-1 + 2\varphi_i)(-1 + \varphi_j + \varphi_k)}{3 + 4\varphi_j(-1 + \varphi_k) - 4\varphi_k + 4\varphi_i(-1 + \varphi_j + \varphi_k)} \\ \sigma^x(j) &= \frac{(-1 + 2\varphi_j)(-1 + \varphi_i + \varphi_k)}{3 + 4\varphi_j(-1 + \varphi_k) - 4\varphi_k + 4\varphi_i(-1 + \varphi_j + \varphi_k)} \\ \sigma^x(k) &= 1 - \sigma^x(i) - \sigma^x(j) \end{aligned}$$

There cannot be another Nash equilibrium where one lobbyist assigns positive probability to all three decision-makers and the other one does not. A lobbyist can only be indifferent between all his pure strategies if the other one assigns positive probability to all three decision-makers. If L_1 assigns positive probability to two decision-makers, then L_2 will choose the decision-maker that L_1 does certainly not lobby. But then L_1 has an incentive to deviate and to assign positive probability to the decision-maker chosen by L_2. Last, a pure strategy cannot be a part of an equilibrium, as L_1 prefers to be at the same node as L_2, but L_2 at a different one.

(b) $\varphi_i \geq \varphi_j > \frac{1}{2}, 1 - \varphi_i > \varphi_k > 0$

The unique Nash equilibrium is the same as given in the previous subcase. Again, there cannot be another Nash equilibrium, which can be shown along the same lines as for the previous case.

(c) $\varphi_i > \varphi_j > \frac{1}{2}, 1 - \varphi_j < \varphi_k$

Here, the Nash equilibrium is

$$\begin{aligned} \sigma^1(i) &= \frac{\varphi_i - \varphi_j}{-1 + 2\varphi_i}, \quad \sigma^1(k) = 1 - \sigma^1(i) \\ \sigma^2(i) &= \frac{\varphi_i - \varphi_k}{-1 + 2\varphi_i}, \quad \sigma^2(j) = 1 - \sigma^2(i) \end{aligned}$$

(d) $\varphi_i = \varphi_j > \frac{1}{2}, 1 - \varphi_j < \varphi_k$:

The Nash equilibria are given by

$$
\begin{aligned}
&\sigma^1(k) = 1 \\
&\sigma^2(i) \in (\frac{-1+\varphi_j+\varphi_k}{-1+2\varphi_j}, \frac{\varphi_j-\varphi_k}{-1+2\varphi_j}), \quad \sigma^2(j) = 1-\sigma^2(i)
\end{aligned}
$$

(e) $1 > \varphi_i > 1-\varphi_k \geq \varphi_j > \frac{1}{2}$:

The unique Nash equilibrium is

$$
\begin{aligned}
\sigma^1(i) &= \frac{\varphi_i-\varphi_j}{-1+2\varphi_i}, \quad \sigma^1(k) = 1-\sigma^1(i) \\
\sigma^2(i) &= \frac{\varphi_i-\varphi_k}{-1+2\varphi_i} \quad \sigma^2(j) = 1-\sigma^2(i)
\end{aligned}
$$

(f) $1 > \varphi_i = 1-\varphi_k > \varphi_j > \frac{1}{2}$:

In the Nash equilibria both lobbyists mix between i and k

$$
\begin{aligned}
\sigma^1(i) &\in (0, \frac{\varphi_i-\varphi_j}{-1+2\varphi_i}), \quad \sigma^1(k) = 1-\sigma^1(i) \\
\sigma^2(i) &\in (\frac{\varphi_i-\varphi_j}{-1+2\varphi_i}, 1) \quad \sigma^2(k) = 1-\sigma^2(i)
\end{aligned}
$$

Proof of Proposition 6: Central decision-makers are lobbied

The payoff matrix is given by

$$
A = \frac{1}{3}\begin{pmatrix}
\varphi_1+\varphi_2+\varphi_3 & \frac{\varphi_1}{\varphi_1+\delta(1-\varphi_1)} + \frac{\delta\varphi_2}{\delta\varphi_2+(1-\varphi_2)} + \frac{\delta\varphi_3}{\delta\varphi_3+(1-\varphi_3)} & \frac{\varphi_1}{\varphi_1+\delta^2(1-\varphi_1)} + \varphi_2 + \frac{\delta^2\varphi_3}{\delta^2\varphi_3+(1-\varphi_3)} \\
\frac{\delta\varphi_1}{\delta\varphi_1+(1-\varphi_1)} + \frac{\varphi_2}{\varphi_2+\delta(1-\varphi_2)} + \frac{\varphi_3}{\varphi_3+\delta(1-\varphi_3)} & \varphi_1+\varphi_2+\varphi_3 & \frac{\varphi_1}{\varphi_1+\delta(1-\varphi_1)} + \frac{\varphi_2}{\varphi_2+\delta(1-\varphi_2)} + \frac{\delta\varphi_3}{\delta\varphi_3+(1-\varphi_3)} \\
\frac{\delta^2\varphi_1}{\delta^2\varphi_1+(1-\varphi_1)} + \varphi_2 + \frac{\varphi_3}{\varphi_3+\delta^2(1-\varphi_3)} & \frac{\delta\varphi_1}{\delta\varphi_1+(1-\varphi_1)} + \frac{\delta\varphi_2}{\delta\varphi_2+(1-\varphi_2)} + \frac{\varphi_3}{\varphi_3+\delta(1-\varphi_3)} & \varphi_1+\varphi_2+\varphi_3
\end{pmatrix}
$$

(a) $\varphi_1 = \varphi_2 = \frac{1}{2}, \varphi_3 \leq \frac{1}{2}$

Then choosing decision-maker 2 strictly dominates choosing decision-maker 1 for L_1 as

$$
\begin{aligned}
1 + \frac{\varphi_3}{\varphi_3+\delta(1-\varphi_3)} &> 1+\varphi_3 \\
\frac{2}{1+\delta} + \frac{\delta\varphi_3}{\delta\varphi_3+(1-\varphi_3)} &> 1 + \frac{\delta\varphi_3}{\delta\varphi_3+(1-\varphi_3)} \\
\frac{2}{1+\delta} + \frac{\delta\varphi_3}{\delta\varphi_3+(1-\varphi_3)} &> \frac{1}{1+\delta^2} + \frac{1}{2} + \frac{\delta^2\varphi_3}{\delta^2\varphi_3+(1-\varphi_3)}
\end{aligned}
$$

The same holds for L_2 as

$$
\begin{aligned}
1+\varphi_3 &> 1 + \frac{\delta\varphi_3}{\delta\varphi_3+(1-\varphi_3)} \\
1 + \frac{\varphi_3}{\varphi_3+\delta(1-\varphi_3)} &> 1+\varphi_3 \\
\frac{\delta^2}{1+\delta^2} + \frac{1}{2} + \frac{\varphi_3}{\varphi_3+\delta^2(1-\varphi_3)} &> \frac{2\delta}{1+\delta} + \frac{\varphi_3}{\varphi_3+\delta(1-\varphi_3)}
\end{aligned}
$$

Then, for L_1 choosing decision-maker two strictly dominates decision-maker three. But then, also for L_2 choosing decision-maker three is strictly dominated by choosing decision-maker two. Therefore, the unique Nash equilibrium is for both lobbyists to choose decision-maker two.

(b) $\varphi_1 = 1 - \varphi_3$

It is straightforward to verify that choosing the central decision-makers is a strictly dominant strategy, independently of his bias.