Online Appendix: Whom To Lobby? Targeting in Political Networks

Proof Proposition 5 : Even very biased decision-makers are lobbied

Case 1: Decision-makers i and j are unbiased, k is in favor of L_1 Note that i and j are payoff equivalent. L_1 is indifferent between his strategies if

$$\sigma^2(i) + \sigma^2(j) = \frac{(\delta(1-\varphi_k) - \varphi_k)(1 - \varphi_k - \delta\varphi_k)}{2(1-\delta^2)(1-\varphi_k)\varphi_k},$$

whereas L_2 is indifferent if

$$\sigma^1(i) + \sigma^1(j) = 1 - \frac{(\delta(1-\varphi_k) - \varphi_k)(1 - \varphi_k - \delta\varphi_k)}{2(1-\delta^2)(1-\varphi_k)\varphi_k}.$$

Last, note that $\frac{(\delta(1-\varphi_k)-\varphi_k)(1-\varphi_k-\delta\varphi_k)}{2(1-\delta^2)(1-\varphi_k)\varphi_k} \in (0,1)$ if and only if $\delta < \min\{\frac{\varphi_k}{1-\varphi_k}, \frac{1-\varphi_k}{\varphi_k}\}$. There cannot be a pure strategy equilibrium as again, the lobbyist k favors prefers to be at the same node as the lobbyist k opposes, whereas the lobbyist k dislikes prefers to be at a different node. Thus, the set of Nash equilibria given is unique.

Case 2: Decision-makers i and j are biased, k is unbiased We define $\underline{\delta}_i = \min\{\frac{1-\varphi_i}{\varphi_i}, \frac{\varphi_i}{1-\varphi_i}\}$ and $\overline{\delta} = \max\{\underline{\delta}_i, \underline{\delta}_j, \underline{\delta}_k\}$ and let $\delta < \overline{\delta}$. We show case by case that the following are the unique Nash equilibria.

(a)$\varphi_j = \varphi_k \equiv \varphi > \frac{1}{2}$: L_1 chooses the unbiased decision-maker, L_2 mixes between the biased decision-makers.

(b)$\varphi_k > \varphi_j > \frac{1}{2}$:

(i) $\frac{1-\varphi_k}{\varphi_k} < \delta < \frac{1-\varphi_j}{\varphi_j} = \overline{\delta}$: Both lobbyists assign positive probability to decision-makers i and j.

(ii) $\frac{1-\varphi_j-\varphi_k+\varphi_j\varphi_k}{\varphi_j\varphi_k} < \delta < \frac{1-\varphi_k}{\varphi_k}$: L_1 assigns positive probability to decision-makers i and j, L_2 assigns positive probability to j and k.

(iii) $0 < \delta < \frac{1-\varphi_j-\varphi_k+\varphi_j\varphi_k}{\varphi_j\varphi_k}$: L_1 assigns positive probability to i and k, L_2 to j and k

(c)$\varphi_j = 1 - \varphi_k > \frac{1}{2}$: both lobbyists assign positive probability to the biased decision-makers.

(d)$\varphi_j > 1 - \varphi_k > \frac{1}{2}$

(i) $\frac{1-\varphi_j}{\varphi_j} < \delta < \frac{\varphi_k}{1-\varphi_k}$: both lobbyists assign positive probability to i and k

(ii) $\frac{\varphi_k}{1-\varphi_k}\frac{1-\varphi_j}{\varphi_j} < \delta < \frac{1-\varphi_j}{\varphi_j}$: L_1 assigns positive probability to i and k, L_2 to j and k.

(iii) $0 < \delta < \frac{\varphi_k}{1-\varphi_k}\frac{1-\varphi_j}{\varphi_j}$: both lobbyists assign positive probability to j and k.

(a)$\varphi_j = \varphi_k \equiv \varphi > \frac{1}{2}$

Choosing the unbiased decision-maker is indeed a best response for L_1 to L_2's strategy if

$$
\begin{aligned}
&\frac{1}{1+\delta} + \varphi + \frac{\delta\varphi}{\delta\varphi + (1-\varphi)} \\
&\quad (\sigma^1(i)\sigma^2(j) + \sigma^1(i)(1-\sigma^2(j)))(\frac{1}{1+\delta} + \varphi + \frac{\delta\varphi}{\delta\varphi + (1-\varphi)}) \\
&\quad + (\sigma^1(j)\sigma^2(j) + (1-\sigma^1(i)-\sigma^1(j))(1-\sigma^2(j)))(\frac{1}{2} + 2\varphi) \\
&\quad + (\sigma^1(j)(1-\sigma^2(j)) + (1-\sigma^1(i)-\sigma^1(j))\sigma^2(j))(\frac{1}{2} + \frac{\varphi}{\varphi+\delta(1-\varphi)} + \frac{\delta\varphi}{\delta\varphi+1-\varphi})
\end{aligned}
$$

Simplifying yields

$$
\begin{aligned}
&(1-\sigma^1(i))(\frac{1}{1+\delta} + \varphi + \frac{\delta\varphi}{\delta\varphi + (1-\varphi)}) \\
&\quad (\sigma^1(j)\sigma^2(j) + (1-\sigma^1(i)-\sigma^1(j))(1-\sigma^2(j)))(\frac{1}{2} + 2\varphi) \\
&\quad + (\sigma^1(j)(1-\sigma^2(j)) + (1-\sigma^1(i)-\sigma^1(j))\sigma^2(j))(\frac{1}{2} + \frac{\varphi}{\varphi+\delta(1-\varphi)} + \frac{\delta\varphi}{\delta\varphi+1-\varphi}) \\
&\Leftrightarrow \\
&\frac{1}{1+\delta} + \varphi + \frac{\delta\varphi}{\delta\varphi + (1-\varphi)} \\
&\quad (x\sigma^2(j) + (1-x)(1-\sigma^2(j)))(\frac{1}{2} + 2\varphi) \\
&\quad + (x(1-\sigma^2(j)) + (1-x)\sigma^2(j))(\frac{1}{2} + \frac{\varphi}{\varphi+\delta(1-\varphi)} + \frac{\delta\varphi}{\delta\varphi+1-\varphi}),
\end{aligned}
$$

where $x = \frac{\sigma^1(j)}{1-\sigma^1(i)}$ and $1-x = \frac{1-\sigma^1(i)-\sigma^1(j)}{1-\sigma^1(i)}$. As $2\varphi > \frac{\varphi}{\varphi+\delta(1-\varphi)} + \frac{\delta\varphi}{\delta\varphi+1-\varphi}$, $L_1's$ problem is

$$
\max_x \quad \big(x(1-\sigma^2(j)) + (1-x)\sigma^2(j)\big)
$$

If $\sigma^2(j) = \frac{1}{2}$, any value of x is a solution – i.e., L_1 is indifferent between choosing decision-makers 2 and 3. But in this case, choosing $\sigma^1(i) = 1$ is the unique best response as

$$\frac{1}{1+\delta}+\varphi+\frac{\delta\varphi}{\delta\varphi+(1-\varphi)}>\frac{1}{2}(\frac{1}{2}+2\varphi)+\frac{1}{2}(\frac{1}{2}+\frac{\varphi}{\varphi+\delta(1-\varphi)}+\frac{\delta\varphi}{\delta\varphi+1-\varphi})$$

If $\sigma^2(j)<\frac{1}{2}$, L_1 chooses $\sigma^1(j)=0$ and if $\sigma^2(j)>\frac{1}{2}$, $\sigma^1(k)=0$. If $\sigma^2(j)<\frac{1}{2}$, L_1 prefers decision-maker i to decision-maker k if $\sigma^2(j)>\frac{1}{2}-\frac{\delta(2\varphi-1)}{2(1-\delta^2)\varphi(1-\varphi)}$. And if $\sigma^2(j)>\frac{1}{2}$, L_1 prefers decision-maker i to decision-maker j if $\sigma^2(j)<\frac{1}{2}+\frac{\delta(2\varphi-1)}{2(1-\delta^2)\varphi(1-\varphi)}$, which establishes the given boundaries. Given L_1 chooses $\sigma^1(i)=1$, L_2 is indifferent between the biased decision-makers and prefers the biased decision-makers to the neutral ones as

$$1-\frac{1}{3}(\frac{1}{1+\delta}+\varphi+\frac{\delta\varphi}{\delta\varphi+(1-\varphi)})>1-\frac{1}{3}(\frac{1}{2}+2\varphi)$$

for the specified levels of δ.

The question that remains is whether the set of Nash equilibria is unique. To check this consider the possible strategies of L_2. Suppose first that L_2 assigns positive probability to all decision-makers. L_1 is never indifferent between all decision-makers in this case as when he is indifferent between j and k, he strictly prefers the unbiased decision-maker i to the biased ones. For this same reason it can never be a best response of L_1 to mix between the biased decision-makers alone. It might be a best response for L_1 to mix between either i and j or i and k. But if L_1 mixes between i and j (k), L_2 chooses k (j) and does not mix anymore. So this can also not be a Nash equilibrium. If L_1 chooses a pure strategy, then L_2 also has an incentive to deviate. L_2 will then be better off choosing the decision-makers, L_1 assigns zero probability to. Therefore, there cannot be a Nash equilibrium that involves L_2 mixing between all his strategies. Next, suppose L_2 mixes between the unbiased and one of the biased decision-makers. Then, L_1 will never mix between all strategies as he will never be indifferent between the biased decision-makers. Therefore, he will also not randomize over the biased decision-makers only. It can be a best response to assign positive probability to the unbiased decision-maker and the same biased decision-maker L_2 chooses. But then, L_2 will deviate to the decision-maker, L_1 does not lobby with positive probability. Last, L_1 might choose a pure strategy. But the best response to a pure strategy is never mixing between one biased decision-maker and the unbiased one. Thus, it can also not be part of a Nash equilibrium that L_2 mixes between the unbiased and one biased decision-maker. We already checked what happened when L_2 mixes between the biased decision-makers. And last, playing a pure strategy can never be part of a Nash equilibrium as L_1 will always have an incentive to choose the same decision-maker, which then leads L_2 to choose a different one. This shows that there is indeed only the specified set of Nash equilibria.

(b)$\varphi_k > \varphi_j \equiv \varphi > \frac{1}{2}$

(i) $\frac{1-\varphi_k}{\varphi_k} < \delta < \frac{1-\varphi_j}{\varphi_j}$

If the discount factor lies in this range, the unique Nash equilibrium is given by

$$\sigma^1(i) = 1 - \sigma^1(j), \quad \sigma^1(j) = \frac{\delta + \varphi_j - 2\delta\varphi_j - \delta^2\varphi_j - \varphi_j^2 + \delta^2\varphi_j^2}{2\varphi_j - 2\delta^2\varphi_j - 2\varphi_j^2 + 2\delta^2\varphi_j^2}$$
$$\sigma^2(i) = 1 - \sigma^2(j), \quad \sigma^2(j) = \frac{-\delta + \varphi_j + 2\delta\varphi_j - \delta^2\varphi_j - \varphi_j^2 + \delta^2\varphi_j^2}{2\varphi_j - 2\delta^2\varphi_j - 2\varphi_j^2 + 2\delta^2\varphi_j^2}$$

It is straightforward to verify that the proposed strategies define a Nash equilibrium. It remains to show that the Nash equilibrium is unique. Suppose L_2 assigns positive probability to all decision-makers. For L_1 it is never a best response to mix between all nodes, as he is never indifferent between them. However, mixing between i and j can be a best response, but then L_2 does not have an incentive to mix between all three nodes. Mixing between i and k as well as between j and k cannot be a best response for the given range of δ. And choosing a pure strategy leads L_2 to prefer some pure strategy to mixing. Therefore, L_2 can assign positive probability to at most two decision-makers. Suppose next, L_2 assigns positive probability to i and k. In this case, L_1 is never indifferent between i and j and therefore, mixing between all nodes or mixing between i and j can never be a best response. Also, mixing between i and k is not a best response for the specified δ. Mixing between j and k is not a best response as choosing i is always better than choosing j. As before, if $L1$ chooses a pure strategy, L_2 will not mix. Now suppose, L_2 chooses to mix between j and k. As before, mixing between all nodes will not be a best response for L_1. It is a best response for L_1 to mix between i and j. But then mixing between j and k is not a best response for L_2. Mixing between i and k or between j and k is not best response for the given range of δ. So, what remains is that L_2 chooses a pure strategy. If L_2 chooses i (j), L_1 chooses i (j) as well. But given L_1 chooses i (j), choosing i (j) is strictly dominated for L_2. For $\frac{1-\varphi_k}{\varphi_k} < \delta < \frac{1-\varphi_j}{\varphi_j}$, if L_2 chooses k, L_1

chooses i. But L_2's best response is then choosing j. This establishes uniqueness. For all additional cases uniqueness can be establishes the same way and is therefore omitted.

(ii) $\frac{1-\varphi_j-\varphi_k+\varphi_j\varphi_k}{\varphi_j\varphi_k} < \delta < \frac{1-\varphi_k}{\varphi_k}$

In this case the unique Nash equilibrium is given by

$$\sigma^1(i) = 1 - \sigma^1(j), \quad \sigma^1(j) = \frac{(\delta(1-\varphi_j)+\varphi_j)(\varphi_k-\varphi_j)(-1+\varphi_k+\varphi_j(1-(1-\delta)\varphi_k))}{(1-\delta)\varphi_j(3\varphi_j-1-2\varphi_j^2)(1-(1-\delta)\varphi_k)}$$
$$\sigma^2(j) = 1 - \sigma^2(k), \quad \sigma^2(k) = \frac{\delta+\varphi_j-2\delta\varphi_j-\delta^2\varphi_j-\varphi_j^2+\delta^2\varphi_j^2}{2\varphi_j-2\delta^2\varphi_j-2\varphi_j^2+2\delta^2\varphi_j^2}$$

(iii) $0 < \delta < \frac{1-\varphi_j-\varphi_k+\varphi_j\varphi_k}{\varphi_j\varphi_k}$

The unique Nash equilibrium is given by

$$\sigma^1(i) = 1 - \sigma^1(k), \quad \sigma^1(k) = \frac{(\varphi_k-\varphi_j)(\delta(1-\varphi_k)+\varphi_k)(1-\varphi_k-\varphi_j(1-(1-\delta)\varphi_k))}{(1-\delta)(1-(1-\delta)\varphi_j)\varphi_k(3\varphi_k-1-2\varphi_k^2)}$$
$$\sigma^2(j) = 1 - \sigma^2(k), \quad \sigma^2(k) = \frac{-\delta+\varphi_k+2\delta\varphi_k-\delta^2\varphi_k-\varphi_k^2+\delta^2\varphi_k^2}{2\varphi_k-2\delta^2\varphi_k-2\varphi_k^2+2\delta^2\varphi_k^2}$$

It is straightforward to verify that this is indeed a Nash equilibrium. Uniqueness can be shown along the same lines as previously and is therefore omitted.

(c)$\varphi_j = 1 - \varphi_k > \frac{1}{2}$ The set of Nash equilibria is given by

$$\sigma^1(j) \in [0, \frac{\delta+\varphi_j-2\delta\varphi_j-\delta^2\varphi_j-\varphi_j^2+\delta^2\varphi_j^2}{2\varphi_j-2\delta^2\varphi_j-2\varphi_j^2+2\delta^2\varphi_j^2}], \quad \sigma^1(k) = 1 - \sigma^1(k)$$
$$\sigma^2(j) \in [\frac{\delta+\varphi_k-2\delta\varphi_k-\delta^2\varphi_k-\varphi_k^2+\delta^2\varphi_k^2}{2\varphi_k-2\delta^2\varphi_k-2\varphi_k^2+2\delta^2\varphi_k^2}, 1], \quad \sigma^2(k) = 1 - \sigma^2(j)$$

It is again easy to verify that these are Nash equilibria as well as that these are the only Nash equilibria and is therefore omitted.

(d)$\varphi_j > 1 - \varphi_k > \frac{1}{2}$

(i) $\frac{1-\varphi_j}{\varphi_j} < \delta < \frac{\varphi_k}{1-\varphi_k}$

The unique Nash equilibrium is given by

$$\sigma^1(i) = \frac{\delta+\varphi_k-2\delta\varphi_k-\delta^2\varphi_k-\varphi_k^2+\delta^2\varphi_k^2}{2\varphi_k-2\delta^2\varphi_k-2\varphi_k^2+2\delta^2\varphi_k^2}, \quad \sigma^1(k) = 1 - \sigma^1(i)$$
$$\sigma^2(i) = \frac{-\delta+\varphi_k+2\delta\varphi_k-\delta^2\varphi_k-\varphi_k^2+\delta^2\varphi_k^2}{2\varphi_k-2\delta^2\varphi_k-2\varphi_k^2+2\delta^2\varphi_k^2}, \quad \sigma^2(k) = 1 - \sigma^2(i)$$

(ii) $\frac{\varphi_k}{1-\varphi_k}\frac{1-\varphi_j}{\varphi_j} < \delta < \frac{1-\varphi_j}{\varphi_j}$

The unique Nash equilibrium is given by

$$\sigma^1(i) = \frac{(-1+\varphi_j+\varphi_k)(1+(-1+\delta)\varphi_k)(\delta\varphi_j(-1+\varphi_k)+\varphi_k-\varphi_j\varphi_k)}{(-1+\delta)(1+(-1+\delta)\varphi_j)\varphi_k(1-3\varphi_k+2\varphi_k^2)}, \quad \sigma^1(k) = 1-\sigma^1(i)$$
$$\sigma^2(j) = \frac{\delta+\varphi_k-2\delta\varphi_k-\delta^2\varphi_k-\varphi_k^2+\delta^2\varphi_k^2}{2\varphi_k-2\delta^2\varphi_k-2\varphi_k^2+2\delta^2\varphi_k^2}, \quad \sigma^2(k) = 1-\sigma^2(j)$$

(iii) $0 < \delta < \frac{\varphi_k}{1-\varphi_k}\frac{1-\varphi_j}{\varphi_j}$

The unique Nash equilibrium is given by

$$\sigma^1(j) = \frac{((\delta(-1+\varphi_j)-\varphi_j)(1+(-1+\delta)\varphi_k)(\delta\varphi_j(-1+\varphi_k)+\varphi_k-\varphi_j\varphi_k))}{C}, \sigma^1(k) = 1-\sigma^1(j)$$
$$\sigma^2(j) = \frac{((1+(-1+\delta)\varphi_j)(\delta(-1+\varphi_k)-\varphi_k)(-\delta\varphi_k+\varphi_j(1+(-1+\delta)\varphi_k)))}{C}, \quad \sigma^2(k) = 1-\sigma^2(j)$$

with

$$C = ((-1+\delta)(2(-1+\varphi_j)\varphi_j(-1+\varphi_k)\varphi_k + 2\delta^2(-1+\varphi_j)\varphi_j(-1+\varphi_k)\varphi_k$$
$$+\delta(\varphi_k-2\varphi_k^2+\varphi_j^2(-2+4\varphi_k-4\varphi_k^2)+\varphi_j(1-2\varphi_k+4\varphi_k^2))))$$

Case 2: All decision-makers i, j, k are biased Let $\delta \to 0$ and let all decision-makers be biased – i.e., $\varphi_i \neq \frac{1}{2}, \forall i \in N$. If

(a)$\varphi_i > \frac{1}{2}, \forall i$: in the unique Nash equilibrium, both lobbyists assign positive probability to all decision-makers.

(b)$\varphi_i \geq \varphi_j > \frac{1}{2}, 1-\varphi_i > \varphi_k > 0$: in the unique Nash equilibrium, both lobbyists assign positive probability to all decision-makers.

(c)$\varphi_i > \varphi_j > \frac{1}{2}, 1-\varphi_j < \varphi_k$: in the unique Nash equilibrium, L_1 assigns positive probability to i and k and L_2 mixes between i and j.

(d)$\varphi_i = \varphi_j > \frac{1}{2}, 1-\varphi_j < \varphi_k$: in the set of Nash equilibria, L_1 chooses k, L_2 randomizes between i and j.

(e)$1 > \varphi_i > 1-\varphi_k \geq \varphi_j > \frac{1}{2}$: in the unique Nash equilibrium, L_1 chooses i and k and L_2 chooses between i and j.

(f) $1 > \varphi_i = 1-\varphi_k > \varphi_j > \frac{1}{2}$: in the unique Nash equilibrium, both lobbyists mix between i and k.

We first take the limit of the payoff matrix:

$$\lim_{\delta \to 0} A = \begin{pmatrix} \varphi_1 + \varphi_2 + \varphi_3 & 1 + \varphi_3 & 1 + \varphi_2 \\ 1 + \varphi_3 & \varphi_1 + \varphi_2 + \varphi_3 & 1 + \varphi_1 \\ 1 + \varphi_2 & 1 + \varphi_1 & \varphi_1 + \varphi_2 + \varphi_3 \end{pmatrix}$$

Based on this, we find the Nash equilibria.

(a)$\varphi_i > \frac{1}{2}, \forall i$:

The unique Nash equilibrium $\forall x \in \{1,2\}, \forall i \in \{1,2,3\}$, is given by

$$\begin{aligned} \sigma^x(i) &= \frac{(-1 + 2\varphi_i)(-1 + \varphi_j + \varphi_k)}{3 + 4\varphi_j(-1 + \varphi_k) - 4\varphi_k + 4\varphi_i(-1 + \varphi_j + \varphi_k)} \\ \sigma^x(j) &= \frac{(-1 + 2\varphi_j)(-1 + \varphi_i + \varphi_k)}{3 + 4\varphi_j(-1 + \varphi_k) - 4\varphi_k + 4\varphi_i(-1 + \varphi_j + \varphi_k)} \\ \sigma^x(k) &= 1 - \sigma^x(i) - \sigma^x(j) \end{aligned}$$

There cannot be another Nash equilibrium where one lobbyist assigns positive probability to all three decision-makers and the other one does not. A lobbyist can only be indifferent between all his pure strategies if the other one assigns positive probability to all three decision-makers. If L_1 assigns positive probability to two decision-makers, then L_2 will choose the decision-maker that L_1 does certainly not lobby. But then L_1 has an incentive to deviate and to assign positive probability to the decision-maker chosen by L_2. Last, a pure strategy cannot be a part of an equilibrium, as L_1 prefers to be at the same node as L_2, but L_2 at a different one.

(b)$\varphi_i \geq \varphi_j > \frac{1}{2}, 1 - \varphi_i > \varphi_k > 0$

The unique Nash equilibrium is the same as given in the previous subcase. Again, there cannot be another Nash equilibrium, which can be shown along the same lines as for the previous case.

(c)$\varphi_i > \varphi_j > \frac{1}{2}, 1 - \varphi_j < \varphi_k$

Here, the Nash equilibrium is

$$\begin{aligned} \sigma^1(i) &= \frac{\varphi_i - \varphi_j}{-1 + 2\varphi_i}, \quad \sigma^1(k) = 1 - \sigma^1(i) \\ \sigma^2(i) &= \frac{\varphi_i - \varphi_k}{-1 + 2\varphi_i}, \quad \sigma^2(j) = 1 - \sigma^2(i) \end{aligned}$$

(d)$\varphi_i = \varphi_j > \frac{1}{2}, 1 - \varphi_j < \varphi_k$:

The Nash equilibria are given by

$$\sigma^1(k) = 1$$
$$\sigma^2(i) \in (\frac{-1+\varphi_j+\varphi_k}{-1+2\varphi_j}, \frac{\varphi_j-\varphi_k}{-1+2\varphi_j}), \quad \sigma^2(j) = 1-\sigma^2(i)$$

(e)$1 > \varphi_i > 1-\varphi_k \geq \varphi_j > \frac{1}{2}$:

The unique Nash equilibrium is

$$\sigma^1(i) = \frac{\varphi_i-\varphi_j}{-1+2\varphi_i}, \quad \sigma^1(k) = 1-\sigma^1(i)$$
$$\sigma^2(i) = \frac{\varphi_i-\varphi_k}{-1+2\varphi_i} \quad \sigma^2(j) = 1-\sigma^2(i)$$

(f) $1 > \varphi_i = 1-\varphi_k > \varphi_j > \frac{1}{2}$:

In the Nash equilibria both lobbyists mix between i and k

$$\sigma^1(i) \in (0, \frac{\varphi_i-\varphi_j}{-1+2\varphi_i}), \quad \sigma^1(k) = 1-\sigma^1(i)$$
$$\sigma^2(i) \in (\frac{\varphi_i-\varphi_j}{-1+2\varphi_i}, 1) \quad \sigma^2(k) = 1-\sigma^2(i)$$

Proof of Proposition 6: Central decision-makers are lobbied

The payoff matrix is given by

$$A = \frac{1}{3}\begin{pmatrix} \varphi_1+\varphi_2+\varphi_3 & \frac{\varphi_1}{\varphi_1+\delta(1-\varphi_1)} + \frac{\delta\varphi_2}{\delta\varphi_2+(1-\varphi_2)} + \frac{\delta\varphi_3}{\delta\varphi_3+(1-\varphi_3)} & \frac{\varphi_1}{\varphi_1+\delta^2(1-\varphi_1)} + \varphi_2 + \frac{\delta^2\varphi_3}{\delta^2\varphi_3+(1-\varphi_3)} \\ \frac{\delta\varphi_1}{\delta\varphi_1+(1-\varphi_1)} + \frac{\varphi_2}{\varphi_2+\delta(1-\varphi_2)} + \frac{\varphi_3}{\varphi_3+\delta(1-\varphi_3)} & \varphi_1+\varphi_2+\varphi_3 & \frac{\varphi_1}{\varphi_1+\delta(1-\varphi_1)} + \frac{\varphi_2}{\varphi_2+\delta(1-\varphi_2)} + \frac{\delta\varphi_3}{\delta\varphi_3+(1-\varphi_3)} \\ \frac{\delta^2\varphi_1}{\delta^2\varphi_1+(1-\varphi_1)} + \varphi_2 + \frac{\varphi_3}{\varphi_3+\delta^2(1-\varphi_3)} & \frac{\delta\varphi_1}{\delta\varphi_1+(1-\varphi_1)} + \frac{\delta\varphi_2}{\delta\varphi_2+(1-\varphi_2)} + \frac{\varphi_3}{\varphi_3+\delta(1-\varphi_3)} & \varphi_1+\varphi_2+\varphi_3 \end{pmatrix}$$

(a)$\varphi_1 = \varphi_2 = \frac{1}{2}$, $\varphi_3 \leq \frac{1}{2}$

Then choosing decision-maker 2 strictly dominates choosing decision-maker 1 for L_1 as

$$1 + \frac{\varphi_3}{\varphi_3+\delta(1-\varphi_3)} > 1+\varphi_3$$
$$\frac{2}{1+\delta} + \frac{\delta\varphi_3}{\delta\varphi_3+(1-\varphi_3)} > 1 + \frac{\delta\varphi_3}{\delta\varphi_3+(1-\varphi_3)}$$
$$\frac{2}{1+\delta} + \frac{\delta\varphi_3}{\delta\varphi_3+(1-\varphi_3)} > \frac{1}{1+\delta^2} + \frac{1}{2} + \frac{\delta^2\varphi_3}{\delta^2\varphi_3+(1-\varphi_3)}$$

The same holds for L_2 as

$$
\begin{aligned}
1 + \varphi_3 &> 1 + \frac{\delta\varphi_3}{\delta\varphi_3 + (1-\varphi_3)} \\
1 + \frac{\varphi_3}{\varphi_3 + \delta(1-\varphi_3)} &> 1 + \varphi_3 \\
\frac{\delta^2}{1+\delta^2} + \frac{1}{2} + \frac{\varphi_3}{\varphi_3 + \delta^2(1-\varphi_3)} &> \frac{2\delta}{1+\delta} + \frac{\varphi_3}{\varphi_3 + \delta(1-\varphi_3)}
\end{aligned}
$$

Then, for L_1 choosing decision-maker two strictly dominates decision-maker three. But then, also for L_2 choosing decision-maker three is strictly dominated by choosing decision-maker two. Therefore, the unique Nash equilibrium is for both lobbyists to choose decision-maker two.

(b)$\varphi_1 = 1 - \varphi_3$

It is straightforward to verify that choosing the central decision-makers is a strictly dominant strategy, independently of his bias.