Copulas.

In de la Peña, Ibragimov and Sharakhmetov (2006) we obtain general representations for the copulas and joint distributions of arbitrary dependent r.v.'s absolutely continuous with respect to the product of given one-dimensional marginal distributions. Using the above representations, we obtain complete characterizations of important classes of dependent r.v.'s that give, in particular, methods for constructing new copulas and modeling different dependence structures. The results obtained in the paper provide a device for reducing the analysis the of convergence in distribution of sums of double arrays of dependent random variables to the case of convergence to zero of various information based measures of dependence plus convergence in distribution of a completely decoupled version of the original double array. The tools used involve new complete decoupling moment and probability inequalities for dependent r.v.'s in terms of their dependence characteristics, extending complete decoupling inequalities obtained in de la Peña (1990).

Change point detection in Geo-Physics.

Our work in this direction derived from becoming aware of a puzzle that appears in many climatological time series when two time series are independent (and hence uncorrelated). Climatologists found that on the empirical sample correlations there appeared to be a fluctuation pattern indicating a change in the underlying correlation structure. We pointed that this appears because the sample covariances were calculated on the basis of moving windows of ten years (decadal oscillations) and that data was added and removed gradually.

In Robinson, de la Peña and Kushnir (2008) we provide a method for detecting shifts in correlation and variability with application to ENSO(El Niño/Southern Oscillation)-monsoon rainfall relationships. In general, we addresses the retrospective detection of step changes at unknown time points in the correlation structure of two or more climate time series. Both the variance of the individual series and the covariance between series are addressed. For a sequence of vector-valued observations with an approximate multivariate normal distribution, the proposed method is a parametric likelihood ration test of the hypotheses of constant covariance against the hypothesis of at least one shift in covariance (Chen and Gupta (2000)). This test is applied to the series of NINO3 and Northeaset Brazil rainfall observations from the years 1856-2001. A shift was detected in 1982 which is significant at the 1 percent level. Some or all of this shift in the covariance matrix can be attributed to a change in the variance of the Northeast Brazil rainfall. We use probabilistic arguments to introduce a methodology that increases the power of the test under certain change point alternatives, specifically when a shift is followed by a reversal. Our work is backed by simulations to asses the power of the test under various alternatives.

Applications of Probability to Banking.

Our joint paper with R. Rivera and J. Ruiz-Mata (2006/2007) introduces a new approach to assesing the quality of risk measures: quality control of risk measures (QCRM). The approach is applied to the problem of backtesting value-at-risk (VaR) models. VAR models are used to predict the maximum likely losses in a bank's portfolio at a specified confidence level and time horizon. The widely accepted VaR backtesting procedure outlined by the Basel Committee for Banking Supervision controls the probability of rejecting the model when the model is correct. A drawback of the Basel apporach is (as stated in Basel's document) " *is*

its limited power to control the probability of accepting an incorrect VaR model". By exploiting the Binomial structure of the testing problem, QCRM provides a more balanced testing procedure, which results in a drastic uniform reduction of the probability of acepting a wrong model.

In de la Peña and Rivera (2007/2008) we extend the typical Back Testing approach to the case of dymamic backtesting of value-at-Risk models under regime change. The dislocation of the financial markets during the third quarter of 2007 and 2008 has caused an unusual number of exceptions in baks' VaR models. In the presence of regime change, the standard VaR backtesting method is inappropriate to assess the accuracy of the model. This paper introduces a dynamic (multi-period) method for backtesting models. The method is used to sequentially validate the VaR model for consecutive time periods (moving window samples). Banks' risk managers tend to think about a large number of exceptions as a temporary phenomenon and assume that the model is fundamentally correct. Consequently, they do not incorporate the new data into the decison-making process. In contrast, our method fully uses the data to estimate the probability of an exception and to test whether or not the observed number of exceptions is consistent with the VaR model's assumptions. Under given conditions, we find that the acceptance or rejection of the model is a function of the number of exceptions that occurred in the current and past time periods. The dynamic backtesting approach is used to detect the emergence of a regime shift that can alert risk managers to revise the model. It also provides a way to estimate the VaR model risk under stress conditions. We show how the current Basel Backtesting procedure can be modified to incorporate our approach. The dynamic approach can be extended to validate other risk management pricing models.

Sharp Inequalities and LIL's for Self-Normalized Processes.

The development of sharp inequalities for self-normalized processes is important for their connection to the study of central limit theorems and as pivotal quantities in the construction of confidence intervals as in the case of the t-statistic (see Giné, Götze and Mason (1997) for recent work on this). A salient property of this type of estimator is that it frequently obviates the need for moment assumptions on the variables involved, thus providing natural non-parametric estimators. For example, Shao (1997) provides large deviation results for self-normalized sums of i.i.d. random variables without moment conditions. Self-normalized estimators are important especially in cases for which estimators based purely on the available data are desired, and in the study of the properties of pivotal quantities for the construction of confidence intervals

Other uses of self-normalization can be found in Caballero, Fernandez and Nualart (1998) where they apply a moment bound (see also de la Peña (1999)) to provide estimates on the density of a difussion process which is the solution to a stochastic differential equation. They also use moment bounds to establish the continuity (through Kolmogorov's criteria) and uniqueness of the solutions of a nonlinear stochastic partial differential equations.

Our work in the area (presented in de la Peña, Lai and Shao (2009)) consists in the development of optimal L_p, exponential and LIL-type bounds for self-normalized process both in univariate and multivariate settings.