IEOR 3106: Second Midterm Exam, Chapters 5-6, November 7, 2013

This exam is closed book. YOU NEED TO SHOW YOUR WORK.

Honor Code: Students are expected to behave honorably, following the accepted code of academic honesty. You may keep the exam itself. Solutions will eventually be posted on line.

1. Copier Breakdown and Repair (35 **points)**

Three copier machines operate continuously and independently through time. They are maintained by a single repairman. Each copier functions for an exponentially distributed amount of time with mean 10 days before it breaks down and the repairman is notified. The repair times for each copier are exponential with mean 1 day, but the repairman can only work on one machine at a time. The machines are repaired in the order in which they fail.

(a) Suppose that all three copiers are initially working. What is the expected time until one of the copiers breaks down?

(b) After the first copier breaks down, what is the probability that a second copier fails before the first one is repaired?

(c) Let $X(t)$ be the number of copiers not working at time t. Multiple choice; pick the best answer (and explain):

(i) The stochastic process $\{X(t) : t \geq 0\}$ is a Markov process.
(ii) The stochastic process $\{X(t) : t \geq 0\}$ is a birth-and-death process.
(iii) Both of the above.
(iv) None of the above.

(d) What is the long-run proportion of time that no copier is working?

(e) What is the long-run proportion of time that the repairman is busy doing repair work on these copiers?

2. Fishing (35 **points)**

Suppose that a fisherman catches fish at random times, according to a Poisson process with rate 4 fish per hour. Suppose that each fish is either a grouper or a snapper, with the probability of being a grouper being 1/4 (independent of the history up to that time). Let W_g and W_s be the random weights of each grouper and snapper, respectively, (also independent of the history), with means and standard deviations:

$$E[W_g] = 100, \quad SD[W_g] = 20, \quad E[W_s] = 20, \quad \text{and} \quad SD[W_s] = 10,$$

measured in pounds.

(a) What are the mean and variance of the time until the fisherman catches his fourth fish?

(b) What is the probability that the fisherman catches exactly 6 fish in a given 2-hour period?

(c) What is the conditional probability that the fisherman catches exactly 6 fish in a given 2-hour period, given that he catches no fish in the previous two hours?

(d) What is the probability that the fisherman catches exactly 4 grouper in a given 2-hour period (along with an unspecified number of snapper)?

(e) What is the probability that the fisherman catches exactly 4 grouper and 5 snapper in a given 2-hour period?

(f) What are the mean and variance of the total weight of all fish caught by the fisherman in a given two-hour period?

(g) What is the approximate probability that the total weight of all fish caught by the fisherman in a given two-hour period exceeds 400 pounds? Multiple choice; pick the best answer (and explain, justify your answer):

(i) 1.00
(ii) 0.50
(iii) 0.30
(iv) 0.03
(v) 0.00

3. Cars in a Highway Segment (30 points)

Suppose that cars enter a (one-way) highway segment at an increasing rate over some interval of time. Specifically, suppose that cars enter the highway segment according to a nonhomogeneous Poisson process with rate $\lambda(t) = 18t$ per minute at time t, starting at time 0. Assume that different cars do not interact. Suppose that the time each car remains in the highway segment is a random variable uniformly distributed on the interval $[2, 4]$ minutes. Suppose that these random times for different cars are mutually independent. Let $A(t)$ be the number of cars to enter the highway segment during $[0, t]$ and let $X(t)$ be the number of cars in the highway segment at time t.

(a) Give an (exact) expression for $E[A(10)]$.

(b) Give an approximate expression for $P(A(10) > 950)$.

(c) Give an (exact) expression for $P(A(2) = 40|A(1) = 20)$.

(d) Give an (exact) expression for the covariance $Cov[A(10)-A(0), A(30)-A(20)]$. (Recall that $Cov(X, Y) = E[XY] - E[X]E[Y]$.)

(e) Give an (exact) expression for $E[X(10)]$.

(f) Multiple choice; pick the best answer(and explain):

(i) The stochastic process $\{X(t) : t \geq 0\}$ is a nonhomogeneous Poisson process.
(ii) The stochastic process $\{X(t) : t \geq 0\}$ is a Markov process.
(iii) Both of the above.
(iv) None of the above.