IEOR 4701: Stochastic Models in Financial Engineering

Summer 2007, Professor Whitt

Conditional Probability

Chapter 1 of Ross. Here are key points:

1. Probability theory is a branch of *mathematics*, so it is important to pay attention to the *definitions* and *axioms*.

In particular, Chapter 1 emphasizes that a key idea overall is to remember and apply the definition of *conditional probability*:

$$P(A|B) \equiv \frac{P(AB)}{P(B)} \ ,$$

where $AB \equiv A \cap B$ denotes the *intersection* of the sets (events) A and B.

2. We are focusing on problem solving. For that purpose, a good general strategy is *divide and conquer*: break the problem into smaller pieces that are easier to analyze. Skipping steps can cause errors.

3. It is helpful to draw *pictures*.

The following are exercises at the end of Chapter 1 in Ross.

1.28 If the occurrence of event B makes event A more likely, does the occurrence of event A make event B more likely?

1.29 Suppose that $P(E) = 0.6$. What can you say about $P(E|F)$ when
(a) E and F are mutually exclusive?
(b) $E \subseteq F$?
(c) $F \subseteq E$?

1.18 (a) A family has two children. What is the probability that both are girls, given that at least one is a girl? (Assume that each child is equally likely to be a boy or a girl.)

(b) Does the answer change if we rephrase the question: What is the probability that both are girls, given that the older child is a girl?

1.33 In a class there are four freshman boys, six freshman girls, and six sophomore boys. How many sophomore girls must be in the class if sex and class are to be independent when a student is selected at random?

1.42 There are three coins in a box. One is a two-headed coin, another is a fair coin, and the third is a biased coin that comes up heads 75 percent of the time. Suppose that one of these three coins is selected at random and flipped. What is the conditional probability that the randomly selected coin was the two-headed coin, given that the outcome of the flipping showed heads?

1.26 and 1.27 Four hands of 13 cards each are to be dealt from a deck of 52 playing cards. What is the probability that there is one ace in each of the four hands?