

IEOR 4701: Stochastic Models in FE
Professor Whitt, Summer, 2007
Problem for Discussion, Monday, July 23

Trip to the Post Office

Five students from IEOR 4701 – Nicolas Abadie-Vennin, Pierre-Dimitri Gore-Coty, Rohit Saraf, Saraswati Rachupalli, and Thayne Batty – simultaneously enter an empty post office, where there are three clerks ready to serve them. Nicolas, Pierre-Dimitri and Rohit begin to receive service immediately, while Saraswati and Thayne wait in a single line, ready to be served by the first free clerk, with Saraswati at the head of the line (to be served first when a server becomes free). Suppose that the service times of the three clerks (for all customers) are independent exponential random variables, each with mean 2 minutes.

- (a) What is the expected time (from the moment the students enter the post office) until Rohit completes service?
- (b) What is the probability that Rohit is still in service after 6 minutes?
- (c) What is the *conditional* probability that Rohit is still in service after 10 minutes, given that Rohit has not yet been served after 4 minutes?
- (d) What is the *conditional* probability that Rohit is still in service after 10 minutes, given that Nicolas has not yet been served after 4 minutes?
- (e) What is the probability that Rohit is the first to complete service?
- (f) What is the expected time (from the moment the students enter the post office) until the first student completes service?
- (g) What is the variance of the time (from the moment the students enter the post office) until the first student completes service?
- (h) What is the expected time (from the moment the students enter the post office) until Saraswati completes service?
- (i) What is the expected time (again since entering the post office) until *all* five students finish service?
- (j) What is the variance of the time until *all* five students finish service?
- (k) What is the probability that Saraswati is the *third* student to finish service?
- (l) Suppose that you wanted to calculate the probability that the time required for all five students to complete service will exceed 10 minutes. What computational tool makes that calculation easy to perform? Briefly explain why.