IEOR 4701: Stochastic Models in Financial Engineering

Summer 2007, Professor Whitt

Lecture notes for Wednesday, August 8

1. Conditional Expectation

It is important to understand that $E[X|Y]$ is a random variable, more specifically, a function of Y. In the discrete case, $E[X|Y]$ assumes the value $E[X|Y = y]$ on the event $\{Y = y\}$. Hence, $E[E[X|Y]] = E[X]$; see p. 106 of Ross.

It is important to recognize that

$$E[E[X|Y_1, Y_2]|Y_1] = E[E[X|Y_1]|Y_1, Y_2] = E[X|Y_1]$$

There is some discussion in the textbook, but you might well want to read more. As usual, you can Google for more. I liked this one:

http://www.ds.unifi.it/VL/VL_EN/expect/expect5.html

2. Martingales

I handed out Professor Sigman's notes on martingales. You should read Sections 1.1, 1.3 and 1.4, omitting the proof of Proposition 1.5 in Section 1.3. In Section 1.2, the example at the beginning and the statement of Proposition 1.2 add insight.

3. Brownian motion

We discussed the conditional distribution of BM at time s given the value at time t, for both cases: $s < t$ and $s > t$. We emphasized that this conditional distribution is also normal. All joint distributions are normal, so the process is a Gaussian process. The homework thus emphasizes properties of multivariate normal distributions.

We discussed the reflection principle and its use to calculate the distribution of the maximum of BM over an interval.

We showed how martingales can be used to calculate $P(T_a < T_{-b})$ and $E[\min\{T_a, T_{-b}\}]$, where T_a is the hitting time of a. For the first, we use the optional stopping theorem (OST) with the martingale (MG) $\{B(t) : t \geq 0\}$, standard BM. For the second, we use the OST with the MG $\{B(t)^2 - t : t \geq 0\}$, plus the first result. Analogous results for random walks are in the Sigman notes.