IEOR 6711: Stochastic Models I

Fall 2013, Professor Whitt

Homework Assignment 6, Tuesday, October 8

Due on Tuesday, October 15.

Problems on renewal theory from Ch. 3 of *Stochastic Processes* by Sheldon Ross.

Problem 3.1 (Hint: This problem is in the same spirit as Problem 2 in Hmwk 1, but somewhat different. For each sample path, $N(t)$ is a right-continuous nondecreasing function, but S_n is not directly the inverse of $N(t)$. Nevertheless, the same kind of reasoning applies.)

Problem 3.2

Problem 3.3 (Hint: For the first part, condition on the last renewal before t. The inequality expresses *stochastic order*; see the first two pages of Chapter 9.)

Problem 3.4 (Hint: Condition on the time of the first renewal, and then uncondition.)

Problem 3.5 (Hint: Use Laplace transforms.)

Added Problem. Consider a renewal process in which the time between renewals has the distribution of $X + Y$, where X and Y are independent exponential random variables with means $EX = 2$ and $EY = 3$.

(a) Calculate the Laplace transform of $m(t)$.

(b) Apply Laplace transforms and numerical inversion (using your inversion code being written) to compute the renewal function $m(t)$ for $t = 10$ and $t = 20$. The discretization error bound needs to be treated somewhat differently here, because $m(t)$ is not bounded above by a constant. (See part (c) below.)

(c) It can be shown that $m(t)$ is asymptotically of the form $c + dt$ for constant c and d, where d is the reciprocal of the mean interrenewal time; see Theorem 3.3.4 on p. 107 and Corollary 3.4.7 on p. 121. Thus it is not difficult to control the discretization error; act as if $m(t) \leq c + dt$ for all t, for the correct constants c and d. Using this approach, calculate the appropriate transform inversion parameter A in order to make the discretization error be 10^{-8}.

Problem 3.6

Problem 3.7 (answer in back)

Problem 3.9

Problem 3.10

Problem 3.11