APPENDIX to
Choosing Arrival Process Models for Service Systems: Tests of a Nonhomogeneous Poisson Process

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Abstract

Service systems such as call centers and hospital emergency rooms typically have strongly time-varying arrival rates. Thus, a nonhomogeneous Poisson process (NHPP) is a natural model for the arrival process in a queueing model for performance analysis. Nevertheless, it is important to perform statistical tests with service system data to confirm that an NHPP is actually appropriate, as emphasized by Brown et al. (2005). They suggested a specific statistical test based on the Kolmogorov-Smirnov (KS) statistic after exploiting the conditional-uniform (CU) property to transform the NHPP into a sequence of i.i.d. random variables uniformly distributed on $[0,1]$ and then performing a logarithmic transformation of the data. We investigate why it is important to perform the final data transformation and consider what form it should take. We conduct extensive simulation experiments to study the power of these alternative statistical tests. We conclude that the general approach of Brown et al. (2005) is excellent, but that an alternative data transformation proposed by Lewis (1965), drawing upon Durbin (1961), produces a test of an NHPP test with consistently greater power. We also conclude that the KS test after the CU transformation, without any additional data transformation, tends to be best to test against alternative hypotheses that primarily differ from an NHPP only through stochastic and time dependence. This appendix provides additional details for the main paper.

Keywords: nonhomogeneous Poisson process; power of statistical tests; data transformations for statistical tests; service systems; arrival processes; time-varying arrival rate.

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1 Overview

In this appendix, we present supporting material complementing the main paper, Kim and Whitt (2013). In §2 we present alternative tests of the Poisson property and introduce our experimental design. We present detailed results for our main experimental setting with fixed length of time in §3. We extend our main setting by examining longer intervals in §4.1 and also by examining the effect of introducing subintervals in §4.2. In §5, we do similar analyses with fixed sample size, and show that the results are similar to those for a fixed length of time. In §6, we present experiment results with varying significance levels for the standard Kolmogorov-Smirnov (KS) test with estimated mean. Finally, in §7, we provide additional figures to supplement Section 7 of the main paper where we discuss the asymptotics of the KS tests.

2 Study Setting

Suppose we are given arrival data and we want to test whether the data can be regarded as a sample from a Poisson process (PP) We note that the data can be given in two forms: (1) arrivals over a fixed length of time t such that a random number $N(t)$ events occur at times T_j, $1 \leq j \leq N(t)$, during the interval $[0, t]$, from which the interarrival times can be derived such that $X_j = T_j - T_{j-1}$ with $T_0 = 0$, or (2) a fixed sample size of n interarrival times X_j, $1 \leq j \leq n$, such that X_j is the j^{th} interarrival time.

2.1 The Alternative Kolmogorov-Smirnov Statistical Tests

The Kolmogorov-Smirnov (KS) test is is commonly used to determine if n observations can be regarded as a sample from a sequence of independent and identically distributed (i.i.d.) random variables $\{X_n : n \geq 1\}$, each distributed as a random variable X with a specified continuous cdf $F(x) \equiv P(X \leq x)$, $x \in \mathbb{R}$. The test is based on the maximum difference between the *empirical cdf* (ecdf)

$$F_n(x) \equiv n^{-1} \sum_{k=1}^{n} 1_{\{X_k \leq x\}}, \quad x \in \mathbb{R}, \tag{1}$$

and the underlying cdf F, where 1_A is an indicator function, equal to 1 if the event A occurs, and equal to 0 otherwise, i.e.,

$$D_n \equiv \sup_x \{|F_n(x) - F(x)|\}, \tag{2}$$

which has a distribution that is independent of the cdf F. For any observed maximum y from a sample of size n, we compute the P-value $P(D_n > y)$ using the Matlab program *ksstat* and compare it to the significance level α, i.e., for specified probability of rejecting the null hypothesis when it is in fact correct (type I error), which we take to be $\alpha = 0.05$. Sometimes it is preferable to use corresponding one-sided KS tests, but we will concentrate on the two-sided test.

As a step in constructing alternative KS tests, we sometimes transform the initial sequence $\{X_n : n \geq 1\}$ into a new sequence of i.i.d. random variables with a new cdf. In particular, we sometimes replace X_n by $F(X_n)$, which has a uniform cdf on $[0, 1]$. It is important to note that the KS statistic is unchanged by this transformation. It is also unchanged if we consider associated transformations such as $-\log\{1 - F(X_n)\}$, which has a mean-1 exponential cdf.

As a basis for comparison in our study of the CU, Log and Lewis KS tests, we also consider variants of the standard KS test as described above. The standard KS test applies to a fixed number n of observations with a fully specified cdf, including the mean of the exponential interarrival times. However, in our application to a Poisson process (PP) or a nonhomogeneous Poisson process (NHPP), both these requirements are violated. First, the number of arrivals in each interval is actually random; second, in an application we would not know the rate of the Poisson process over each subinterval, and thus we do not know the mean of the exponential interarrival times in the PP of the null hypothesis. Nevertheless, we include the standard KS test for comparison. In doing so, we let the required fixed number of interarrival times, n, be the random number observed in that sample, and we use the known mean 1. We find that the Lewis test, without using information about the rate of the PP, is usually superior, but the difference is not great.

To understand the implications of the invalid standard KS test, we also study variants of this base standard KS test. First, we consider alternative standard KS tests with the same random sample size but (i) using the estimated mean and (ii) the associated Lilliefors (1969) test for the exponential distribution with unknown mean, using the Matlab program *lillietest*. Second, we consider alternative experiments based on a fixed number n of interarrival times. Then the standard KS test of the PP is valid, provided that the mean is known. In this setting with fixed n, we also consider the consequence of using the estimated mean and the Lilliefors (1969) test.

We now specify the four main KS tests that we consider.

Standard Test. We use the standard KS test described above to test whether the random number of observations in the interval $[0, t]$ is consistent with a rate-1 PP. We act as if the observed random number is a fixed number and we use the known mean 1 of an exponential interarrival time in the rate-1 PP.

Conditional-Uniform (CU) Test. In this test, we exploit the basic conditioning property of a PP. Given an arrival process over an interval $[0, t]$, we observe the number n of arrival in this interval and their arrival times T_j, $1 \le j \le n$. Under the null PP hypothesis, these random variables are distributed as the order statistics of i.i.d. random variables uniformly distributed over $[0, t]$. Thus, the random variables T_j/t, $1 \le j \le n$, are distributed as the order statistics of i.i.d. random variables uniformly distributed over $[0, 1]$. Thus the ecdf can be computed via

$$F_n(x) \equiv n^{-1} \sum_{k=1}^{n} 1_{\{T_k/t \le x\}}, \quad 0 \le x \le 1,$$

and the KS statistic can be computed as in (9) with uniform cdf $F(x) = x$, $0 \le x \le 1$.

Log Test. Brown et al. (2005) observed (as we show in the next section) that, given the n observed arrival times $\{T_j : 1 \le j \le n\}$ during the interval $[0, t]$,

$$X_j^{Log} \equiv -(n + 1 - j)\ \log_e \left(\frac{t - T_j}{t - T_{j-1}} \right), 1 \le j \le n,$$

are n i.i.d. mean-1 exponential random variables. The KS test in (9) can then be applied using the exponential cdf $F(x) \equiv 1 - e^{-x}$. A variant of the Log test applies to a fixed sample of size n. With T_j again denoting the time of the $j^{\rm th}$ arrival,

$$X_j^{Log,n} \equiv -j \ \log_e \left(\frac{T_j}{T_{j+1}} \right), 1 \le j \le n-1,$$

are again i.i.d. rate-1 exponential random variables.

Lewis Test. Lewis (1965) proposed using a different modification of the CU test, exploiting a transformation due to Durbin (1961). Durbin (1961) started with a sample U_j, $1 \le j \le n$, hypothesized to be uniformly distributed on $[0,1]$. Then let $U_{(j)}$ be the $j^{\rm th}$ smallest of these, $1 \le j \le n$, so that $U_{(1)} < \cdots < U_{(n)}$. Lewis (1965) applies this with $U_{(j)} = T_j/t$ from the CU test. Next we look at the successive *intervals* between these ordered observations:

$$C_1 = U_{(1)}, \, C_j = U_{(j)} - U_{(j-1)} \quad 2 \le j \le n, \quad \text{and} \quad C_{n+1} = 1 - U_{(n)}.$$

Then let $C_{(j)}$ be the $j^{\rm th}$ smallest of these intervals, $1 \le j \le n$, so that $0 < C_{(1)} < \cdots < C_{(n+1)} < 1$. Now let Z_j be scaled versions of the intervals between these new variables, i.e.,

$$Z_j = (n+2-j)(C_{(j)} - C_{(j-1)}), \quad 1 \le j \le n+1, \quad (\text{with} \quad C_{(0)} \equiv 0). \tag{3}$$

Remarkably, Durbin (1961) showed in a simple direct argument (by giving explicit expressions for the joint density functions, exploiting the transformation of random vectors by a function) that, under the PP null hypothesis, the random vector $(Z_1, \ldots, Z_n)$ is distributed the same as the random vector $(C_1, \ldots, C_n)$. Hence, again under the PP null hypothesis, the vector of associated partial sums $(S_1, \ldots, S_n)$, where

$$S_k \equiv Z_1 + \cdots + Z_k, \quad 1 \le k \le n, \tag{4}$$

has the same distribution as the original random vector $(U_{(1)}, \ldots, U_{(n)})$ of ordered uniform random variables. Hence, we can apply the KS test with the ecdf

$$F_n(x) \equiv n^{-1} \sum_{k=1}^{n} 1_{\{S_k \le x\}}, \quad 0 \le x \le 1,$$

for S_k in (4) and (3), comparing it to the uniform cdf $F(x) \equiv x$, $0 \le x \le 1$. Durbin (1961) showed that by doing this transformation starting from a sequence of i.i.d. uniform random variables, we should gain an increase in power. Lewis (1965) showed that this transformation increases power after the CU transformation, which is a different setting than considered by Durbin (1961).

2.2 Proof of the Log Test

Here we show why the Log transformation in Section 2.1 produces i.i.d. rate-1 exponential random variables. We use a property of uniform order statistics, giving the conditional distribution of the remaining variables, given the minimum. Given n i.i.d. random variables uniformly distributed over an interval $[0, L]$, conditional on the minimum, say M_n, the remaining $n-1$ random variables are i.i.d. random variables uniformly distributed on the interval $[M_n, L]$. We can repeat that construction to generate independent random variables. Given this recursive procedure, it suffices to convert the distribution of the minimum of i.i.d. uniform random variables into a standard exponential. For simplicity, suppose that $L = 1$. Let us first observe that

$$P(M_n > 1-t) = P(U > 1-t)^n = t^n, \quad 0 \le t \le 1. \tag{5}$$

Lemma 2.1 *If M_n is the minimum of n i.i.d. uniform random variables on the interval $[0, 1]$, then*

$$P(-n \log_e (1-M_n) > t) = e^{-t}, \quad t \ge 0.$$

Proof Using (5) in the last step,

$$\begin{aligned} P(-n \log_e (1-M_n) > t) &= P(\log_e (1-M_n) < -t/n) \\ &= P((1-M_n) < e^{-t/n}) \\ &= P(M_n > 1 - e^{-t/n}) \\ &= (e^{-t/n})^n = e^{-t}. \quad \blacksquare \end{aligned}$$

2.3 Multiple Test Procedures

Since different tests may have advantages, it is natural to investigate if we can gain power by considering more than one test, but care is needing in doing so. Fortunately, multiple statistical inference has been extensively studied; e.g., Miller (1981).

If we perform one significance test at level α, the probability of rejecting the true null hypothesis (type I error) is the individual error rate, α. If we instead conduct multiple tests of significance with testing hypotheses $H_1, \ldots, H_n$, each with significance level α, where the n tests are *independent*, then the probability of not rejecting the global test of the intersection hypothesis $H_0 = \{H_1, \ldots, H_n\}$ is $(1-\alpha)^k$. That is, the probability of rejecting at least one of the k independent null hypotheses when all are true is the familywise error rate $1-(1-\alpha)^k$. Hence, performing multiple tests alters the overall significance level.

The simplest multiple test procedure is the well-known Bonferroni method (Miller 1981), which is performed by rejecting H_0 if any p-value is less than α/n. It is very simple to use and requires no distributional assumptions, but studies have shown that it should only be used when the number of tests is small and when the test statistics are not highly-correlated. There are a number of improvements of the Bonferroni method, including the Holm procedure (Holm 1979). In this procedure, letting $P_{(1)}, \ldots, P_{(n)}$ be the ordered p-values for testing hypotheses $H_0 = \{H_1, \ldots, H_n\}$, H_0 is rejected if $P_{(k)} \le \alpha/(n+1-k)$ for any $k = 1, \ldots, n$. We note that this method also does not have any restriction on the joint distribution of the test statistics. The last method we consider is the Simes procedure (Simes 1986). In this method, H_0 is rejected if $P_{(k)} \le k\alpha/n$

for any $k = 1, \ldots, n$. Designed to handle independent hypotheses as well as some forms of positively dependent hypotheses well, this method is shown to be valid on average (Rodland 2006). See Levin (1996) for more discussion on this as well as comparison of the three tests.

2.4 Study Cases

In this section we specify the stochastic processes to which we apply the different KS tests of a PP. We consider nine cases, each with one-to-five subcases, yielding a total of 26 cases in all. We specify these cases in terms of the sequence $\{X_n : n \geq 1\}$ of interarrival times, each distributed as a random variable X. In all cases, the sequence is assumed to be stationary with $E[X] = 1$. The first five cases are renewal arrival processes, with i.i.d. interarrival times. The first i.i.d. case is our PP null hypothesis with exponential interarrival times. The other i.i.d. cases have non-exponential interarrival times. Cases 2 and 3 contain Erlang and hyperexponential interarrival times, which are, respectively, stochastically less variable and stochastically more variable than the exponential distribution in convex stochastic order, as in §9.5 of Ross (1996). Thus, they have *squared coefficient of variation* (scv, variance divided by the square of the mean, denoted by c^2), respectively, $c^2 < 1$ and $c^2 > 1$. Cases 4 and 5 contain non-exponential cdf's with $c_X^2 = 1$ as well as $E[X] = 1$, just like the exponential cdf.

Case 1, Exponential. The PP null hypothesis with exponential interarrival times (Base Case).

Case 2, Erlang, $\mathbf{E_k}$. Erlang-k (E_k) interarrival times, a sum of k i.i.d. exponentials for $k = 2, 4, 6$ with $c_X^2 \equiv c_k^2 = 1/k$.

Case 3, Hyperexponetial, $\mathbf{H_2}$. Hyperexponential-2 (H_2) interarrival times, a mixture of 2 exponential cdf's with $c_X^2 = 1.25$, 1.5, 2, 4 and 10. The cdf is $P(X \leq x) \equiv 1 - p_1 e^{-\lambda_1 x} - p_2 e^{-\lambda_2 x}$. We further assume balanced means ($p_1 \lambda_1^{-1} = p_2 \lambda_2^{-1}$) as in (3.7) of Whitt (1982) so that given the value of c_X^2, $p_i = [1 \pm \sqrt{(c_X^2 - 1)/(c_X^2 + 1)}]/2$ and $\lambda_i = 2p_i$.

Case 4, mixture with $\mathbf{c_X^2 = 1}$. A mixture of a more variable cdf and a less variable cdf so that the $c_X^2 = 1$; $P(X = Y) = p = 1 - P(X = Z)$, where Y is H_2 with $c_Y^2 = 4$, Z is E_2 with $c_Z^2 = 1/2$ and $p = 1/7$.

Case 5, lognormal, $\mathbf{LN(1,1)}$ with $c_X^2 = 1$. Lognormal distribution with mean and variance both equal to 1, so that $c_X^2 = 1$.

Cases 6 and 7 are stationary point processes that deviate from a Poisson process only through dependence among successive interarrival times, each exponentially distributed with mean 1:

Case 6, RRI, dependent exponential interarrival times. Randomly Repeated Interarrival (RRI) times with exponential interarrival times, constructed by letting each successive interarrival time be a mixture of the previous interarrival time with probability p or a new independent interarrival time from an exponential distribution with mean 1, with probability $1 - p$ (a special case of a first-order discrete autoregressive process, DAR(1), studied by Jacobs and Lewis (1978, 1983)). Its serial correlation is $Corr(X_j, X_{j+k}) = p^k$. We consider three values of p: 0.1, 0.5 and 0.9.

Case 7, EARMA, dependent exponential interarrival times. A stationary sequence of dependent exponential interarrival times with the correlation structure of an autoregressive-moving average process, called EARMA(1,1) in Jacobs and Lewis (1977). Starting from three independent sequences of i.i.d. random variables $\{X_n : n \geq 0\}$, $\{U_n : n \geq 1\}$, and $\{V_n : n \geq 1\}$, where X_n is exponentially distributed with mean m, while

$$P(U_n = 0) = 1 - P(U_n = 1) = \beta \quad \text{and} \quad P(V_n = 0) = 1 - P(V_n = 1) = \rho, \tag{6}$$

the EARMA sequence $\{S_n : n \geq 1\}$ is defined recursively by

$$\begin{aligned} S_n &= \beta X_n + U_n Y_{n-1}, \\ Y_n &= \rho Y_{n-1} + V_n X_n, \quad n \geq 1. \end{aligned} \tag{7}$$

Its serial correlation is $Corr(S_j, S_{j+k}) = \gamma\rho^{k-1}$ where $\gamma = \beta(1-\beta)(1-\rho) + (1-\beta)^2\rho$. We consider five cases of (β, ρ): $(0.75, 0.50)$, $(0.5, 0.5)$, $(0.5, 0.75)$, $(0.00, 0.75)$, $(0.25, 0.90)$ so that the cumulative correlations $\sum_{k=1}^{\infty} Corr(S_j, S_{j+k})$ increase: 0.25, 0.50, 1.00, 3.00, and 5.25. For more details, see Pang and Whitt (2012). We specify these cases by these cumulative correlations.

The final two cases are stationary point processes that have *both* non-exponential interarrival times and dependence among successive interarrival times:

Case 8, $\mathbf{mH_2}$, superposition of $\mathbf{m}$ i.i.d. $\mathbf{H_2}$ renewal processes. Superposition of m i.i.d. equilibrium renewal processes, where the times between renewals (interarrival times) in each renewal process has a hyperexponential (H_2) distribution with $c_a^2 = 4$ ($m\,H_2$). As the number m of component renewal processes increases, the superposition process converges to a PP, and thus looks locally more like a PP, with the interarrival distribution approaching exponential and the lag-k correlations approaching 0, but small correlations extending further across time, so that the superposition process retains an asymptotic variability parameter, $c_A^2 = 4$. We consider four values of m: 2, 5, 10 and 20.

Case 9, RRI $(\mathbf{H_2})$, dependent $\mathbf{H_2}$ interarrival times with $\mathbf{c^2 = 4}$. Randomly Repeated Interarrival (RRI) times with H_2 interarrival times, each having mean 1, $c^2 = 4$ and balanced means (as specified in Case 3). The repetition is done just as in Case 6. We again consider three values of p: 0.1, 0.5 and 0.9.

Cases 6 and 7 above have short-range dependence, whereas Case 8 for large m tends to have nearly exponential interarrival times, but longer-range dependence. For small m, the mH_2 superposition process should behave much like the H_2 renewal process in Case 3 with the component $c^2 = 4$; for large m, the mH_2 superposition process should behave more like Cases 6 and 7 with dependence and exponential interarrival times.

2.5 Performance Implications of the Alternative Hypotheses

In our experiments we will see how well the different KS tests can detect the deviation of each of the alternative hypotheses from the PP null hypothesis. To show how different these alternative processes are from a PP from a performance perspective, we now show steady-state performance measures for a $G/G/s(+M)$ queueing models in which each alternative process serves as the arrival process.

We start with the models in Table 1 that have i.i.d. exponential service times, s servers and customer abandonment with i.i.d. exponential patience times. We let the arrival, service and abandonment rates be $\lambda = 25$, $\mu = 1$ and $\theta = 1$, respectively. These simulation estimates are based on 100 replications of 10^5 customers, with the first 10^3 removed to allow the system to approach steady state. Table 1 shows five performance measures for three levels of staffing $s = 28$, 30 and 35. The five performance measures are: the expected waiting time of (i) all customers, (ii) served customers and (iii) abandoning customers, and the percentage of customers that (i) must wait before entering service and (ii) who abandon. For example, we see that the abandonment probability of 0.0515 for an EARMA (5) arrival process with staffing $s = 35$ and 0.0493 for an H_2 arrival process with $c^2 = 4$ and staffing $s = 30$ are higher than the abandonment probability of 0.0349 for a PP arrival process with staffing $s = 28$. Hence, those alternative arrival processes require 7 and 2 extra servers than for a PP to achieve comparable performance, respectively.

Corresponding results for models with the same arrival and service rate ($\lambda = 25$, $\mu = 1$) but without customer abandonment are in Table 2; with smaller arrival rate ($\lambda = 10$) but with the same service and abandonment rates are in Table 3; and with general interarrival and service time distributions with specified cdfs and with customer abandonment are given in Table 4. In all cases, the results show the importance of detecting deviations from a PP from a performance perspective is non-negligible.

2.6 How the Simulation Experiment Was Conducted

For each arrival process, we simulate 10^4 replications of 10^4 interarrival times. We generate much more data than needed in order to get rid of any initial effects. We are supposing that we observe stationary point processes, which is achieved by having the system operate for some time before collecting data. The initial effect was observed to matter for the cases with dependent interarrival times and relatively small sample sizes.

We use this simulation output to generate arrival data for both time intervals of a fixed length t and sample sizes of a fixed size n. Our main results are for time intervals of fixed length, but we also consider the other scenario. For the first scenario with specified intervals $[0, t]$ with $t = 200$, in each replication we transform the 10^4 interarrival times to 10^4 arrival times starting at $t = 0$ by taking cumulative sums and then consider the arrival process in the interval $[10^3, 10^3 + 200]$. We treat this as observations from a stationary point process over the interval $[0, 200]$.

To observe the effect of longer intervals, we subsequently consider the arrival process in the interval $[10^3, 10^3 + 2000]$; we treat that interval as $[0, 2000]$. To examine the impact of introducing subintervals,

Table 1 Simulation estimates of steady-state performance measures in the $G/M/s+M$ model ($\lambda = 25$, $\mu = 1$, $\theta = 1$) with different staffing levels based on 100 replications of 10^5 customers (first 10^3 customers removed to avoid initial transient behavior). The 95% confidence intervals are shown.

s	$Arrival$	$E[W\|All]$	$E[W\|Serve]$	$E[W\|Abandon]$	$\%Wait$	$\%Abandon$
28	M	0.0324 ± 0.0003	0.0348 ± 0.0003	0.1004 ± 0.0006	29.96 ± 0.15	3.49 ± 0.03
($\beta = 0.5$ for M)	$H_2/c^2 = 2$	0.0461 ± 0.0003	0.0496 ± 0.0004	0.1165 ± 0.0006	35.93 ± 0.16	4.96 ± 0.03
	$H_2/c^2 = 4$	0.0680 ± 0.0005	0.0731 ± 0.0005	0.1384 ± 0.0005	43.23 ± 0.18	7.32 ± 0.05
	E_2	0.0241 ± 0.0002	0.0258 ± 0.0002	0.0881 ± 0.0006	25.72 ± 0.16	2.58 ± 0.02
	E_4	0.0197 ± 0.0002	0.0210 ± 0.0002	0.0807 ± 0.0005	23.03 ± 0.16	2.10 ± 0.02
	$RRI(p = 0.5)$	0.0562 ± 0.0004	0.0608 ± 0.0005	0.1316 ± 0.0007	38.79 ± 0.19	6.07 ± 0.05
	$RRI(p = 0.9)$	0.1401 ± 0.0009	0.1552 ± 0.0009	0.2371 ± 0.0009	53.58 ± 0.22	15.52 ± 0.09
	$EARMA(1)$	0.0547 ± 0.0005	0.0590 ± 0.0005	0.1271 ± 0.0006	38.81 ± 0.20	5.91 ± 0.05
	$EARMA(5)$	0.1108 ± 0.0008	0.1218 ± 0.0008	0.2007 ± 0.0008	49.29 ± 0.23	12.19 ± 0.08
30	M	0.0168 ± 0.0002	0.0181 ± 0.0002	0.0878 ± 0.0006	18.23 ± 0.13	1.81 ± 0.02
($\beta = 1$ for M)	$H_2/c^2 = 2$	0.0272 ± 0.0003	0.0295 ± 0.0003	0.1031 ± 0.0006	24.75 ± 0.15	2.94 ± 0.03
	$H_2/c^2 = 4$	0.0454 ± 0.0004	0.0492 ± 0.0004	0.1233 ± 0.0006	33.33 ± 0.18	4.93 ± 0.04
	E_2	0.0108 ± 0.0001	0.0115 ± 0.0001	0.0762 ± 0.0007	13.69 ± 0.12	1.15 ± 0.01
	E_4	0.0079 ± 0.0001	0.0084 ± 0.0001	0.0690 ± 0.0006	11.01 ± 0.11	0.84 ± 0.01
	$RRI(p = 0.5)$	0.0360 ± 0.0003	0.0392 ± 0.0004	0.1181 ± 0.0007	28.47 ± 0.17	3.91 ± 0.04
	$RRI(p = 0.9)$	0.1151 ± 0.0008	0.1290 ± 0.0009	0.2233 ± 0.0009	47.74 ± 0.22	12.90 ± 0.09
	$EARMA(1)$	0.0345 ± 0.0003	0.0374 ± 0.0004	0.1132 ± 0.0007	28.29 ± 0.19	3.75 ± 0.04
	$EARMA(5)$	0.0869 ± 0.0007	0.0964 ± 0.0007	0.1860 ± 0.0008	42.51 ± 0.22	9.66 ± 0.07
35	M	0.0022 ± 0.0001	0.0024 ± 0.0001	0.0647 ± 0.0012	3.41 ± 0.06	0.24 ± 0.01
($\beta = 2$ for M)	$H_2/c^2 = 2$	0.0057 ± 0.0001	0.0061 ± 0.0001	0.0776 ± 0.0009	7.11 ± 0.09	0.61 ± 0.01
	$H_2/c^2 = 4$	0.0140 ± 0.0002	0.0152 ± 0.0002	0.0939 ± 0.0008	14.11 ± 0.13	1.53 ± 0.02
	E_2	0.0008 ± 0.0000	0.0009 ± 0.0000	0.0562 ± 0.0014	1.50 ± 0.04	0.09 ± 0.00
	E_4	0.0004 ± 0.0000	0.0004 ± 0.0000	0.0477 ± 0.0018	0.80 ± 0.02	0.04 ± 0.00
	$RRI(p = 0.5)$	0.0101 ± 0.0002	0.0110 ± 0.0002	0.0924 ± 0.0008	10.61 ± 0.12	1.10 ± 0.02
	$RRI(p = 0.9)$	0.0696 ± 0.0006	0.0796 ± 0.0007	0.1949 ± 0.0010	34.50 ± 0.21	7.96 ± 0.07
	$EARMA(1)$	0.0089 ± 0.0001	0.0097 ± 0.0002	0.0869 ± 0.0008	9.96 ± 0.12	0.96 ± 0.02
	$EARMA(5)$	0.0458 ± 0.0005	0.0515 ± 0.0005	0.1558 ± 0.0008	27.70 ± 0.21	5.15 ± 0.05

we use the same arrival process in the interval $[10^3, 10^3 + 200]$, again treated as $[0, 200]$, and divide it into ten disjoint contiguous subintervals, each of length 20. In forming subintervals, we necessarily break up the subintervals crossing the boundary points. For these sample sizes, that boundary effect matters; about 5% of the subintervals are altered when the subintervals are of length 20.

In the second scenario with fixed sample size $n = 200$, in each replication of the 10^4 simulated interarrival times we use interarrival times from the 10^3th interarrival time to the $10^3 + 200$th interarrival time. We then consider the interarrival times from the 10^3th interarrival time to the $10^3 + 2000$th interarrival time to observe the effect of larger sample size. In addition, the interarrival times from the 10^3th interarrival time to the $10^3 + 200$th interarrival time are used to examine the impact of introducing ten equally sized sub-samples, each with sample size 20, but now no intervals are split up.

For each sample we checked our simulation results by estimating the mean and scv of each interarrival-time cdf both before and after transformations; we provide tables of the results and plots of the average of the ecdf's appear in corresponding sections.

Table 2 Simulation estimates of steady-state performance measures in the $G/M/s$ model ($\lambda = 25$, $\mu = 1$) with different staffing levels based on 100 replications of 10^5 customers (first 10^3 customers removed to get rid of the initial effect). Associated 95% confidence intervals are shown.

	$Arrival$	$E[W]$	$\%Wait$
28	M	0.1523 ± 0.0034	45.70 ± 0.32
	$H_2/c^2 = 2$	0.2649 ± 0.0060	54.81 ± 0.32
	$H_2/c^2 = 4$	0.5074 ± 0.0155	64.80 ± 0.38
	E_2	0.0977 ± 0.0018	38.81 ± 0.30
	E_4	0.0734 ± 0.0016	34.45 ± 0.31
	$RRI(p = 0.5)$	0.3781 ± 0.0101	59.73 ± 0.39
	$RRI(p = 0.9)$	2.4417 ± 0.0933	81.15 ± 0.43
	$EARMA(1)$	0.3714 ± 0.0090	59.40 ± 0.39
	$EARMA(5)$	1.5006 ± 0.0572	75.52 ± 0.44
30	M	0.0501 ± 0.0010	25.00 ± 0.22
	$H_2/c^2 = 2$	0.0977 ± 0.0019	34.52 ± 0.25
	$H_2/c^2 = 4$	0.2087 ± 0.0049	46.63 ± 0.31
	E_2	0.0279 ± 0.0005	18.36 ± 0.18
	E_4	0.0189 ± 0.0004	14.59 ± 0.17
	$RRI(p = 0.5)$	0.1509 ± 0.0035	40.52 ± 0.32
	$RRI(p = 0.9)$	1.2162 ± 0.0347	69.90 ± 0.41
	$EARMA(1)$	0.1450 ± 0.0030	40.04 ± 0.32
	$EARMA(5)$	0.7182 ± 0.0196	62.15 ± 0.41
35	M	0.0040 ± 0.0001	3.99 ± 0.07
	$H_2/c^2 = 2$	0.0115 ± 0.0003	8.53 ± 0.11
	$H_2/c^2 = 4$	0.0348 ± 0.0009	17.39 ± 0.19
	E_2	00013 ± 0.0001	1.72 ± 0.04
	E_4	0.0006 ± 0.0000	0.90 ± 0.03
	$RRI(p = 0.5)$	0.0236 ± 0.0006	13.15 ± 0.17
	$RRI(p = 0.9)$	0.3833 ± 0.0092	46.91 ± 0.35
	$EARMA(1)$	0.0206 ± 0.0005	12.26 ± 0.17
	$EARMA(5)$	0.1995 ± 0.0046	36.75 ± 0.32

Table 3 Simulation estimates of steady-state performance measures in the $G/M/s + M$ model ($\lambda = 25$, $\mu = 1$, $\theta = 1$) with different staffing levels based on 100 replications of 10^5 customers (first 10^3 customers removed to get rid of the initial effect). Associated 95% confidence intervals are shown.

s	$Arrival$	$E[W\|All]$	$E[W\|Serve]$	$E[W\|Abandon]$	$\%Wait$	$\%Abandon$
12	M	0.0474 ± 0.0003	0.0531 ± 0.0003	0.1548 ± 0.0006	30.34 ± 0.11	5.32 ± 0.03
	$H_2/c^2 = 2$	0.0667 ± 0.0003	0.0747 ± 0.0004	0.1735 ± 0.0006	37.15 ± 0.12	7.46 ± 0.04
	$H_2/c^2 = 4$	0.0977 ± 0.0005	0.1085 ± 0.0005	0.1971 ± 0.0006	46.09 ± 0.13	10.84 ± 0.05
14	M	0.0167 ± 0.0001	0.0187 ± 0.0002	0.1260 ± 0.0007	13.57 ± 0.08	1.87 ± 0.01
	$H_2/c^2 = 2$	0.0286 ± 0.0002	0.0322 ± 0.0002	0.1419 ± 0.0007	20.19 ± 0.10	3.22 ± 0.02
	$H_2/c^2 = 4$	0.0499 ± 0.0003	0.0562 ± 0.0004	0.1623 ± 0.0007	29.86 ± 0.12	5.62 ± 0.03
17	M	0.0025 ± 0.0000	0.0028 ± 0.0000	0.0961 ± 0.0015	2.71 ± 0.03	0.27 ± 0.01
	$H_2/c^2 = 2$	0.0063 ± 0.0001	0.0070 ± 0.0001	0.1088 ± 0.0011	5.93 ± 0.06	0.70 ± 0.01
	$H_2/c^2 = 4$	0.0152 ± 0.0002	0.0171 ± 0.0002	0.1244 ± 0.0008	12.30 ± 0.09	1.72 ± 0.02

Table 4 Simulation estimates of steady-state performance measures in the $G/G/s+M$ model ($\lambda = 25$, $\mu = 1$, $\theta = 1$) with different staffing levels based on 100 replications of 10^5 customers (first 10^3 customers removed to get rid of the initial effect). Associated 95% confidence intervals are shown.

$Model$	s	$E[W\|All]$	$E[W\|Serve]$	$E[W\|Abandon]$	$\%Wait$	$\%Abandon$
$M/M/s+M$	28	0.0324 ± 0.0003	0.0348 ± 0.0003	0.1004 ± 0.0006	29.96 ± 0.15	3.49 ± 0.03
	30	0.0168 ± 0.0002	0.0181 ± 0.0002	0.0878 ± 0.0006	18.23 ± 0.13	1.81 ± 0.02
	35	0.0022 ± 0.0001	0.0024 ± 0.0001	0.0647 ± 0.0012	3.41 ± 0.06	0.24 ± 0.01
$H_2/M/s+M$	28	0.0461 ± 0.0003	0.0496 ± 0.0004	0.1165 ± 0.0006	35.93 ± 0.16	4.96 ± 0.03
	30	0.0272 ± 0.0003	0.0295 ± 0.0003	0.1031 ± 0.0006	24.75 ± 0.15	2.94 ± 0.03
	35	0.0057 ± 0.0001	0.0061 ± 0.0001	0.0776 ± 0.0009	7.11 ± 0.09	0.61 ± 0.01
$M/H_2/s+M$	28	0.0342 ± 0.0003	0.0370 ± 0.0004	0.1096 ± 0.0007	29.84 ± 0.19	3.70 ± 0.03
	30	0.0178 ± 0.0002	0.0193 ± 0.0002	0.0958 ± 0.0008	18.26 ± 0.15	1.93 ± 0.02
	35	0.0024 ± 0.0001	0.0025 ± 0.0001	0.0707 ± 0.0013	3.42 ± 0.06	0.26 ± 0.01
$H_2/H_2/s+M$	28	0.0462 ± 0.0004	0.0500 ± 0.0004	0.1234 ± 0.0006	35.12 ± 0.20	5.00 ± 0.04
	30	0.0270 ± 0.0003	0.0294 ± 0.0003	0.1089 ± 0.0007	24.01 ± 0.17	2.94 ± 0.03
	35	0.0054 ± 0.0001	0.0058 ± 0.0001	0.0810 ± 0.0011	6.64 ± 0.08	0.58 ± 0.01

3 Experiments with Fixed Length of Time, $[0, 200]$

3.1 First Look at the (Transformed) Interarrival Times

Before applying any statistical tool to test for departures from a Poisson process, one can first check the summary statistics of a given arrival process. Table 5 provides such information for the arrival processes described in Section 2.4. Note that all results can be compared to that of Exp in the first row. For the renewal processes (E_k, H_2, Z and LN), we can tell departures from the Poisson property when the interarrival times have c^2 value different from 1, such as the cases with E_k and H_2. However, when the c^2 value is not different from 1, such as the cases with Z and LN, checking the average and c^2 is not sufficient. The same is true for the processes which deviate from a Poisson process via local dependence (RRI and $EARMA$). On the other hand, for mH_2, it is possible to detect departures when m is small (c^2 is greater than 1), but it becomes increasingly harder as m increases. Lastly, the c^2 values of $RRI(H_2)$ cases reveal their departures from a Poisson process. One noticeable thing in Table 5 is the value of $E[X]$ for $RRI(H_2)$ with $p = 0.9$. This high average is caused by sample paths that have only small number of long interarrival times. Note that the minimum number of arrivals among the 1000 sample paths is 4.

Table 5 Summary statistics of the arrival processes on $[0, 200]$ with associated 95% confidence intervals. All results are based on 10000 replications. $\{X_n : n \geq 1\}$ are the interarrival times where n is the number of arrivals in $[0, 200]$.

$Case$	$Subcase$	$E[X]$	$c^2[X]$	$Min[n]$	$Max[n]$	$E[n]$	t_n	$Min[X]$	$Max[X]$
Exp	–	1.00 ± 0.0014	1.00 ± 0.0028	146	257	200.0 ± 0.3	199 ± 0.02	0.0051 ± 0.00010	5.9 ± 0.02
E_k	$k = 2$	1.00 ± 0.0010	0.50 ± 0.0013	165	245	200.0 ± 0.2	199 ± 0.01	0.0448 ± 0.00049	4.1 ± 0.02
	$k = 4$	1.00 ± 0.0007	0.25 ± 0.0005	175	224	200.0 ± 0.1	199 ± 0.01	0.1402 ± 0.00107	2.9 ± 0.01
	$k = 6$	1.00 ± 0.0006	0.17 ± 0.0004	179	222	199.9 ± 0.1	199 ± 0.01	0.2071 ± 0.00150	2.5 ± 0.01
H_2	$c^2 = 1.25$	1.00 ± 0.0016	1.24 ± 0.0043	134	265	199.7 ± 0.3	199 ± 0.02	0.0046 ± 0.00009	7.2 ± 0.04
	$c^2 = 1.5$	1.00 ± 0.0017	1.48 ± 0.0057	139	270	200.2 ± 0.3	199 ± 0.03	0.0042 ± 0.00009	8.3 ± 0.04
	$c^2 = 2$	1.00 ± 0.0020	1.95 ± 0.0088	132	295	200.2 ± 0.4	199 ± 0.04	0.0038 ± 0.00008	10.2 ± 0.06
	$c^2 = 4$	1.01 ± 0.0029	3.76 ± 0.0223	105	299	199.9 ± 0.5	198 ± 0.07	0.0032 ± 0.00006	16.3 ± 0.10
	$c^2 = 10$	1.03 ± 0.0051	8.57 ± 0.0699	44	365	199.6 ± 0.9	194 ± 0.18	0.0030 ± 0.00006	29.4 ± 0.23
Z	–	1.00 ± 0.0014	0.95 ± 0.0112	142	252	199.9 ± 0.3	199 ± 0.03	0.0179 ± 0.00031	8.2 ± 0.09
LN	–	1.00 ± 0.0014	0.97 ± 0.0064	148	251	200.1 ± 0.3	199 ± 0.03	0.0727 ± 0.00050	7.3 ± 0.06
RRI	$p = 0.1$	1.00 ± 0.0015	0.99 ± 0.0030	144	261	199.9 ± 0.3	199 ± 0.02	0.0056 ± 0.00011	5.8 ± 0.02
	$p = 0.5$	1.01 ± 0.0025	0.98 ± 0.0045	99	287	199.6 ± 0.5	199 ± 0.02	0.0104 ± 0.00022	5.2 ± 0.02
	$p = 0.9$	1.10 ± 0.0094	0.85 ± 0.0084	19	453	201.1 ± 1.1	199 ± 0.02	0.0701 ± 0.00379	3.9 ± 0.02
$EARMA$	0.25	1.00 ± 0.0017	0.99 ± 0.0028	140	267	199.7 ± 0.3	199 ± 0.02	0.0050 ± 0.00010	5.8 ± 0.02
	0.5	1.01 ± 0.0020	0.99 ± 0.0029	128	277	199.9 ± 0.4	199 ± 0.02	0.0051 ± 0.00010	5.8 ± 0.03
	1	1.01 ± 0.0025	0.97 ± 0.0034	102	284	199.8 ± 0.5	199 ± 0.02	0.0052 ± 0.00011	5.7 ± 0.03
	3	1.03 ± 0.0040	0.95 ± 0.0049	80	366	199.9 ± 0.7	199 ± 0.02	0.0233 ± 0.00053	5.6 ± 0.03
	5.25	1.06 ± 0.0054	0.90 ± 0.0051	50	408	199.7 ± 0.9	199 ± 0.02	0.0055 ± 0.00012	5.1 ± 0.03
mH_2	$m = 2$	1.01 ± 0.0029	2.36 ± 0.0187	99	307	199.8 ± 0.5	198 ± 0.06	0.0040 ± 0.00008	13.4 ± 0.11
	$m = 5$	1.01 ± 0.0029	1.33 ± 0.0080	100	314	199.8 ± 0.5	199 ± 0.03	0.0047 ± 0.00010	8.2 ± 0.07
	$m = 10$	1.01 ± 0.0028	1.11 ± 0.0042	103	294	199.9 ± 0.5	199 ± 0.02	0.0050 ± 0.00011	6.7 ± 0.04
	$m = 20$	1.01 ± 0.0026	1.04 ± 0.0030	104	296	199.7 ± 0.5	199 ± 0.02	0.0052 ± 0.00011	6.2 ± 0.03
$RRI(H_2)$	$p = 0.1$	1.02 ± 0.0033	3.74 ± 0.0234	79	320	199.1 ± 0.6	197 ± 0.08	0.0036 ± 0.00007	16.0 ± 0.11
	$p = 0.5$	1.06 ± 0.0088	3.51 ± 0.0286	8	362	200.0 ± 0.9	198 ± 0.07	0.0092 ± 0.00466	13.9 ± 0.11
	$p = 0.9$	2.09 ± 0.0731	2.29 ± 0.0371	4	579	200.0 ± 2.0	198 ± 0.07	0.4975 ± 0.04551	9.0 ± 0.11

One can also examine the summary statistics of the transformed interarrival times, as provided in Table 6. We first consider the base case, Exp. The Conditional-Uniform test and the Lewis test transform the interarrival times to have standard uniform distributions, and hence they have average values of 0.5 and $c^2 = 0.33$. The Log test transforms the interarrival times to have exponential distribution with rate 1, so we have $E[X^{Log}] = 1$ and $c^2[X^{Log}] = 1$. Examining the rest of Table 6, we see that $E[X^{CU}]$ and $c^2[X^{CU}]$ do not provide any information. Furthermore, no information is gained by examining $E[X^{Log}]$, even though its 95% confidence intervals seem to weakly suggest the degree of departures. Also, $c^2[X^{Log}]$ values are closer to $c^2[X]$ values in Table 5. $E[X^{Lewis}]$ and $c^2[X^{Lewis}]$ provide the most information. For instance, $E[X^{Lewis}]$ and $c^2[X^{Lewis}]$ of Z and LN are now different from those of Exp, whereas they were not different in Table 5.

Table 6 Average and c^2 of transformed interarrival times on $[0, 200]$ with associated 95% confidence intervals. All results are based on 10000 replications.

$Case$	$Subcase$	$E[X^{CU}]$	$c^2[X^{CU}]$	$E[X^{Log}]$	$c^2[X^{Log}]$	$E[X^{Lewis}]$	$c^2[X^{Lewis}]$
Exp	$-$	0.50 ± 0.0004	0.34 ± 0.0007	1.00 ± 0.0014	1.00 ± 0.0028	0.50 ± 0.0004	0.34 ± 0.0007
E_k	$k = 2$	0.50 ± 0.0003	0.33 ± 0.0005	1.00 ± 0.0010	0.50 ± 0.0013	0.62 ± 0.0003	0.16 ± 0.0003
	$k = 4$	0.50 ± 0.0002	0.34 ± 0.0003	1.00 ± 0.0007	0.25 ± 0.0006	0.72 ± 0.0003	0.08 ± 0.0002
	$k = 6$	0.50 ± 0.0002	0.34 ± 0.0003	1.00 ± 0.0006	0.17 ± 0.0004	0.77 ± 0.0002	0.05 ± 0.0001
H_2	$c^2 = 1.25$	0.50 ± 0.0004	0.33 ± 0.0008	1.00 ± 0.0015	1.23 ± 0.0042	0.47 ± 0.0004	0.36 ± 0.0007
	$c^2 = 1.5$	0.50 ± 0.0005	0.34 ± 0.0008	1.00 ± 0.0017	1.46 ± 0.0056	0.45 ± 0.0005	0.38 ± 0.0008
	$c^2 = 2$	0.50 ± 0.0006	0.34 ± 0.0010	1.00 ± 0.0019	1.91 ± 0.0086	0.42 ± 0.0005	0.41 ± 0.0009
	$c^2 = 4$	0.50 ± 0.0008	0.33 ± 0.0014	1.00 ± 0.0027	3.62 ± 0.0213	0.35 ± 0.0007	0.45 ± 0.0013
	$c^2 = 10$	0.50 ± 0.0013	0.33 ± 0.0022	1.00 ± 0.0042	7.96 ± 0.0636	0.30 ± 0.0010	0.45 ± 0.0022
Z	$-$	0.50 ± 0.0004	0.33 ± 0.0007	1.00 ± 0.0014	0.92 ± 0.0101	0.58 ± 0.0006	0.20 ± 0.0004
LN	$-$	0.50 ± 0.0004	0.33 ± 0.0007	1.00 ± 0.0014	0.96 ± 0.0058	0.56 ± 0.0005	0.18 ± 0.0004
RRI	$p = 0.1$	0.50 ± 0.0004	0.33 ± 0.0008	1.00 ± 0.0015	0.99 ± 0.0029	0.50 ± 0.0004	0.34 ± 0.0008
	$p = 0.5$	0.50 ± 0.0007	0.34 ± 0.0012	1.00 ± 0.0023	0.98 ± 0.0045	0.50 ± 0.0007	0.34 ± 0.0012
	$p = 0.9$	0.50 ± 0.0016	0.34 ± 0.0028	1.00 ± 0.0050	0.84 ± 0.0089	0.55 ± 0.0018	0.32 ± 0.0031
$EARMA$	0.25	0.50 ± 0.0005	0.34 ± 0.0008	1.00 ± 0.0017	0.99 ± 0.0028	0.50 ± 0.0004	0.34 ± 0.0007
	0.5	0.50 ± 0.0006	0.33 ± 0.0010	1.00 ± 0.0019	0.98 ± 0.0029	0.50 ± 0.0004	0.33 ± 0.0007
	1	0.50 ± 0.0007	0.33 ± 0.0012	1.00 ± 0.0022	0.95 ± 0.0032	0.50 ± 0.0005	0.33 ± 0.0008
	3	0.50 ± 0.0010	0.34 ± 0.0018	1.00 ± 0.0034	0.89 ± 0.0046	0.51 ± 0.0009	0.33 ± 0.0017
	5.25	0.50 ± 0.0013	0.34 ± 0.0022	1.01 ± 0.0040	0.81 ± 0.0041	0.52 ± 0.0008	0.33 ± 0.0012
mH_2	$m = 2$	0.50 ± 0.0008	0.34 ± 0.0013	1.00 ± 0.0026	2.25 ± 0.0171	0.42 ± 0.0007	0.39 ± 0.0009
	$m = 5$	0.50 ± 0.0008	0.34 ± 0.0013	1.00 ± 0.0025	1.29 ± 0.0070	0.47 ± 0.0005	0.36 ± 0.0008
	$m = 10$	0.50 ± 0.0007	0.34 ± 0.0012	1.00 ± 0.0022	1.09 ± 0.0038	0.49 ± 0.0004	0.35 ± 0.0007
	$m = 20$	0.50 ± 0.0006	0.33 ± 0.0010	1.00 ± 0.0019	1.03 ± 0.0029	0.49 ± 0.0004	0.34 ± 0.0007
$RRI(H_2)$	$p = 0.1$	0.50 ± 0.0009	0.33 ± 0.0015	1.00 ± 0.0030	3.59 ± 0.0221	0.35 ± 0.0007	0.45 ± 0.0015
	$p = 0.5$	0.50 ± 0.0015	0.34 ± 0.0025	1.00 ± 0.0046	3.31 ± 0.0278	0.36 ± 0.0011	0.48 ± 0.0032
	$p = 0.9$	0.50 ± 0.0026	0.35 ± 0.0053	1.02 ± 0.0085	2.00 ± 0.0349	0.46 ± 0.0031	0.49 ± 0.0073

3.2 Insightful Plots A: Average Empirical CDF

As discussed in the main paper, we find that useful insight is provided by appropriate plots. Especially revealing are plots comparing the average of the ecdf's over all $10,000$ replications to the cdf associated with the null hypothesis (which depends on the transformation; recall that Standard and Log tests have exponential (rate 1) null hypotheses whereas CU and Lewis tests have standard uniform null hypothese). To make comparison easier, we have transformed the data further (when needed) to make all have (i) standard uniform null hypothesis, (ii) exponential (rate 1) null hypothesis, and (iii) standard normal null hypothesis. Figures 1-78 in this subsection are such plots for all our study cases with the three different null hypotheses. The 95% confidence interval curves are shown along with the averages in each case.

These figures show that the transformation in the Lewis KS test provides much greater separation between the average ecdf and the cdf for the renewal processes. We see that all the information we can get from Table 6 is reflected in these figures. Additional information can be gathered. For example, the plot for the Standard KS test for Z in and for LN show that the empirical CDF of the original interarrivals do differ from that of an exponential distribution with rate 1 even though their average and c^2 are the same.

3.2.1 Average Empirical CDF with Standard Uniform Null Hypothesis

Figure 1 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; Exp (base case): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

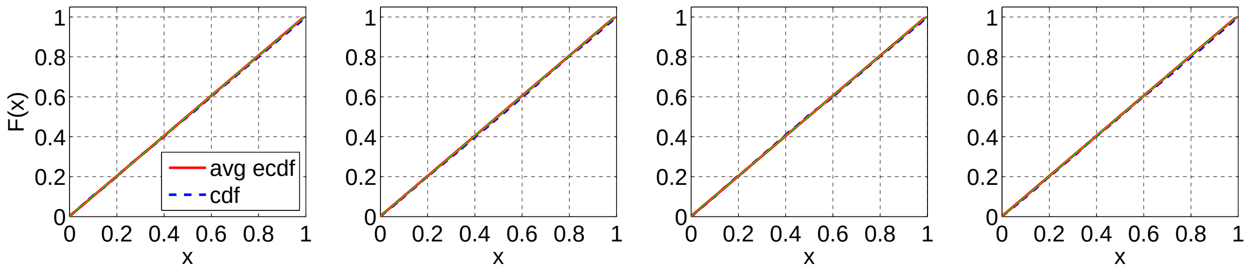


Figure 2 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; E_2: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

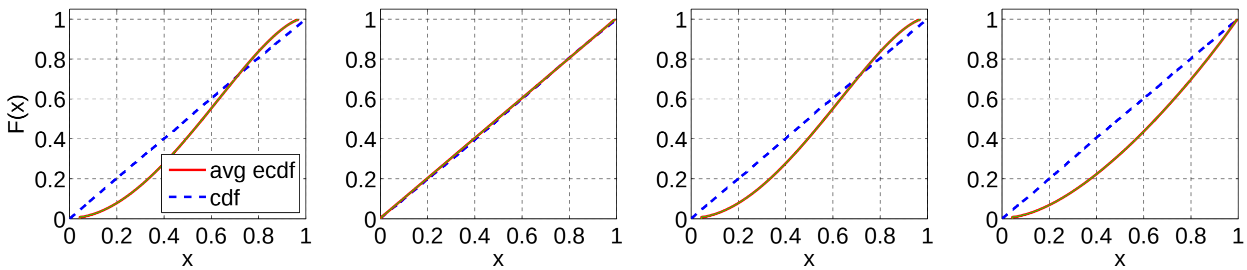


Figure 3 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; E_4: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

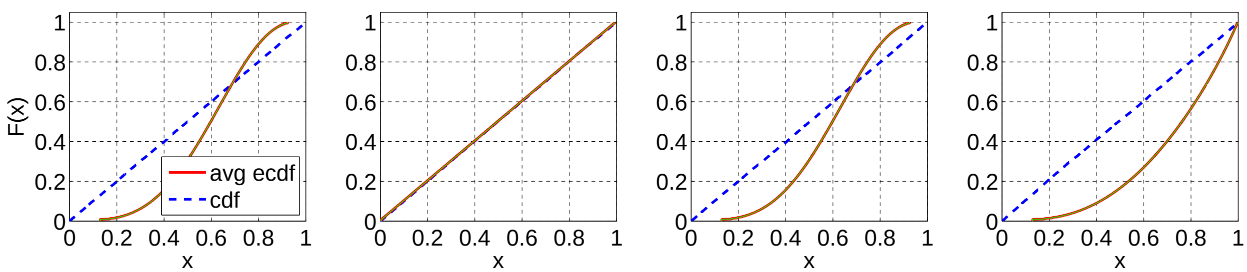


Figure 4 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; E_6: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

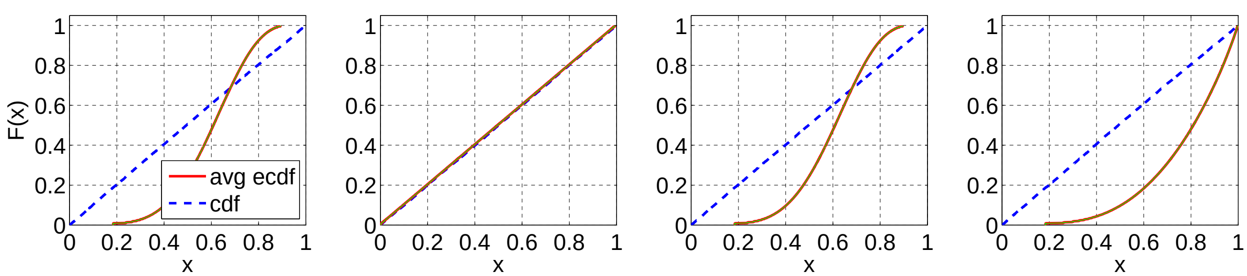


Figure 5 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; H_2 ($c^2 = 1.25$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

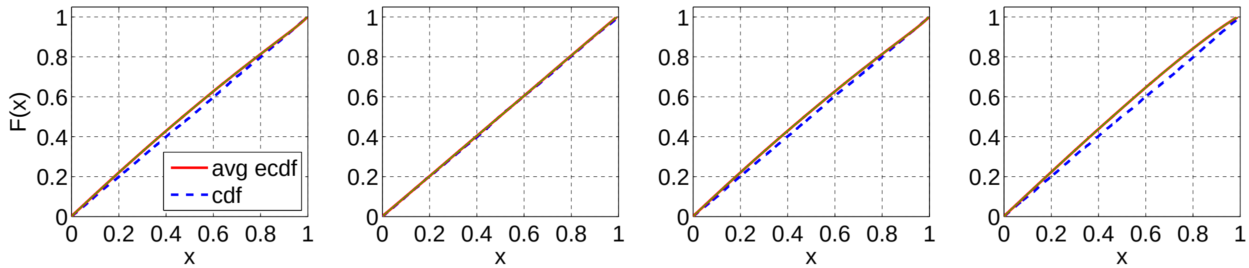


Figure 6 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; H_2 ($c^2 = 1.5$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

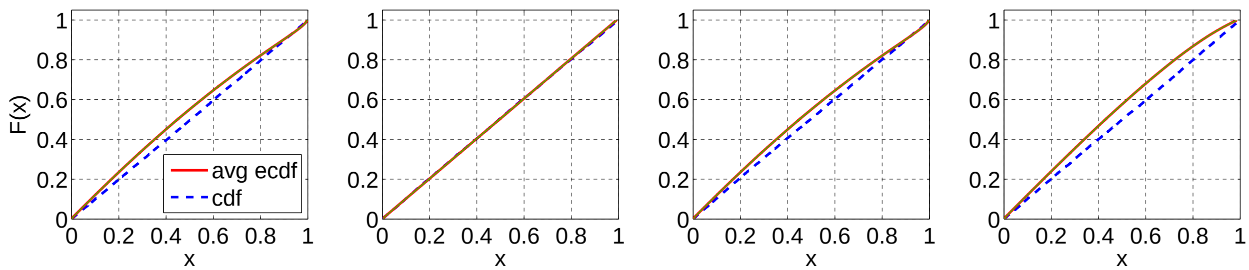


Figure 7 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; H_2 ($c^2 = 2$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

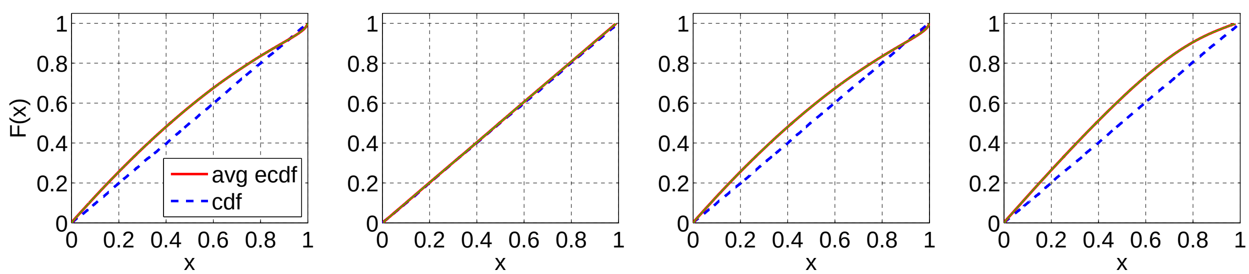


Figure 8 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; H_2 ($c^2 = 4$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

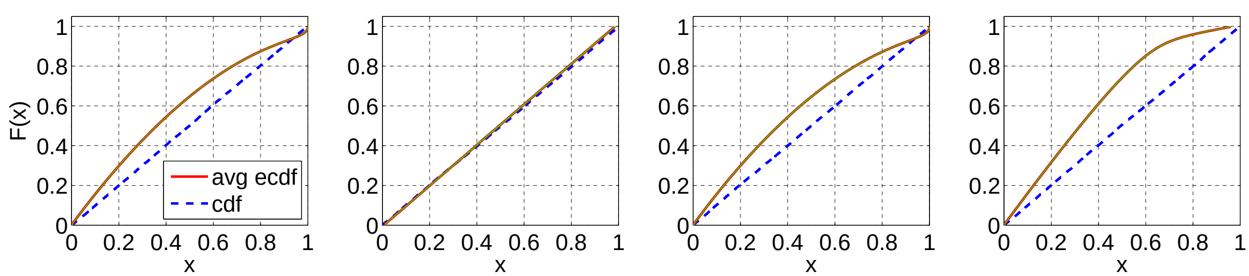


Figure 9 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; H_2 ($c^2 = 10$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

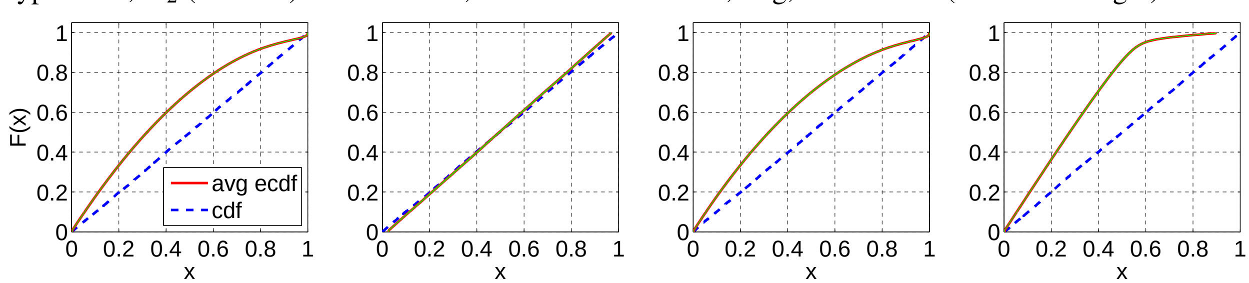


Figure 10 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; Z: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

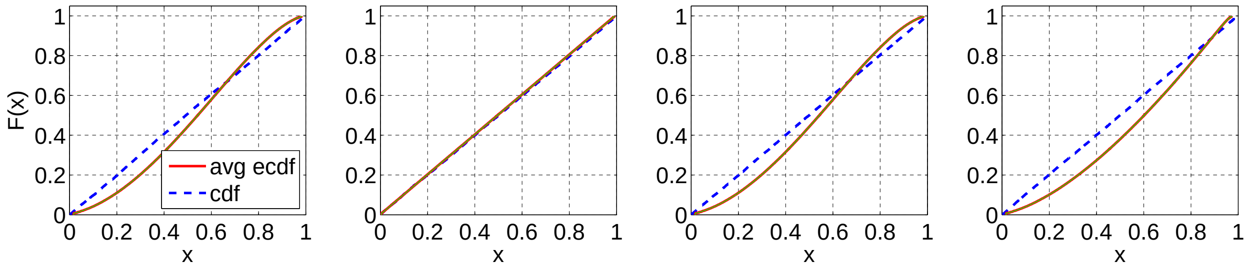


Figure 11 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; LN: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

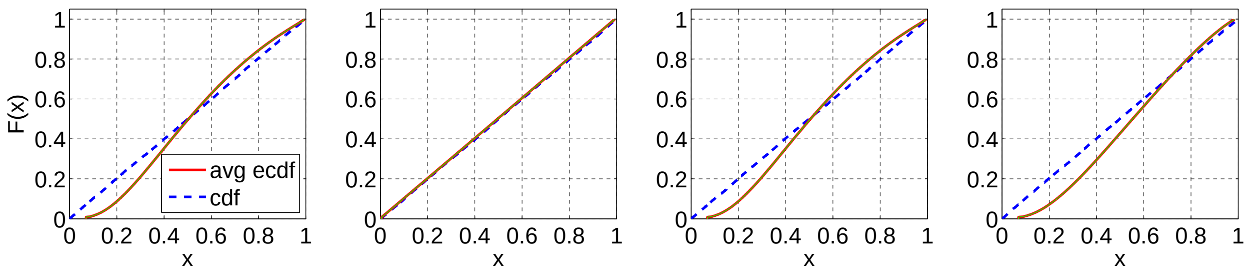


Figure 12 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; RRI $(p = 0.1)$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

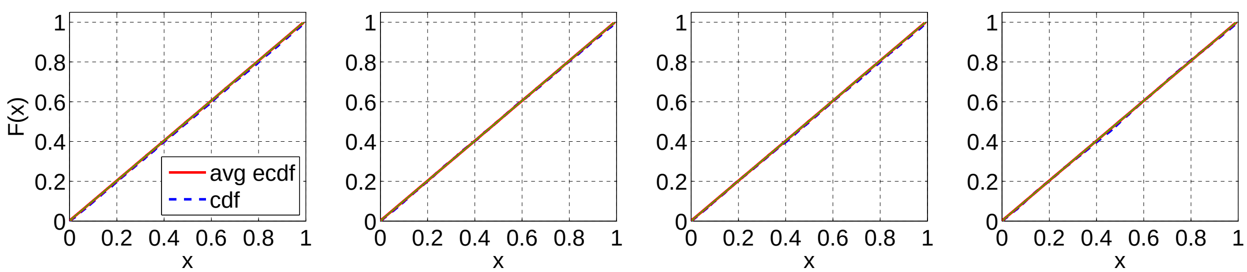


Figure 13 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; RRI $(p = 0.5)$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

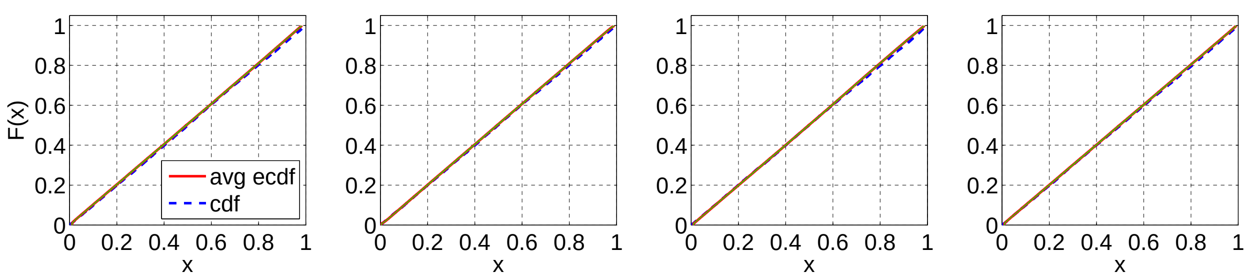


Figure 14 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; RRI $(p = 0.9)$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

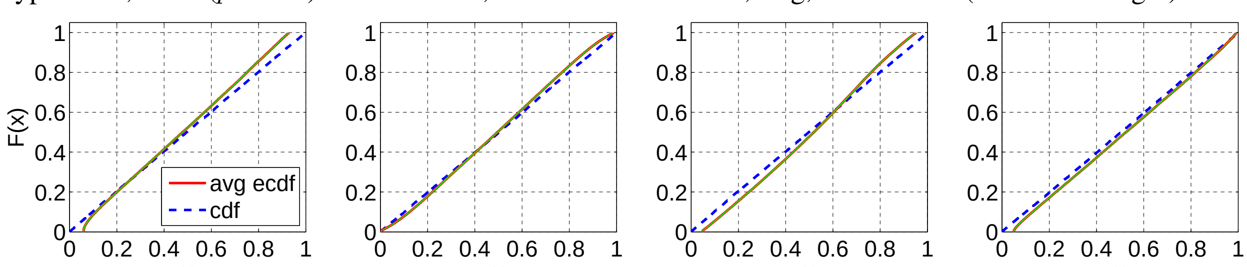


Figure 15 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $EARMA$ (0.25): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

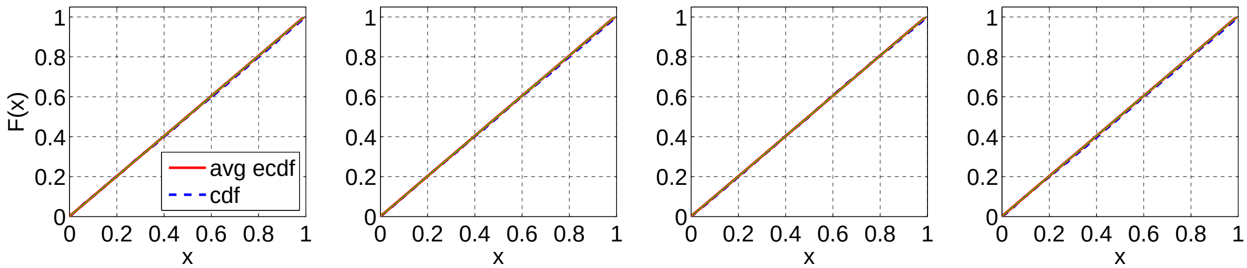


Figure 16 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $EARMA$ (0.5): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

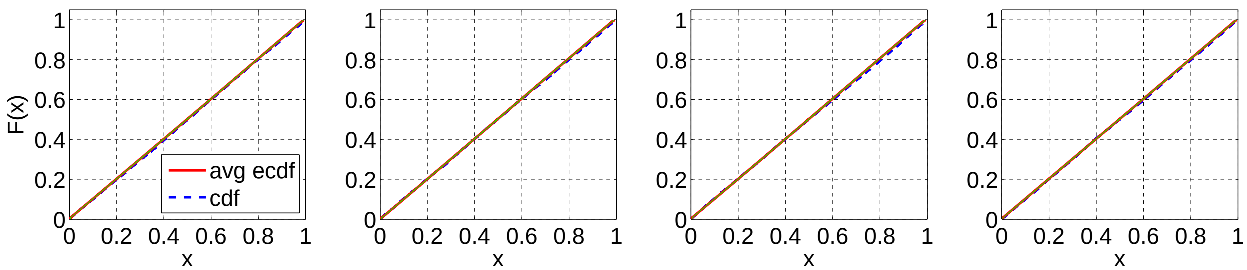


Figure 17 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $EARMA$ (1): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

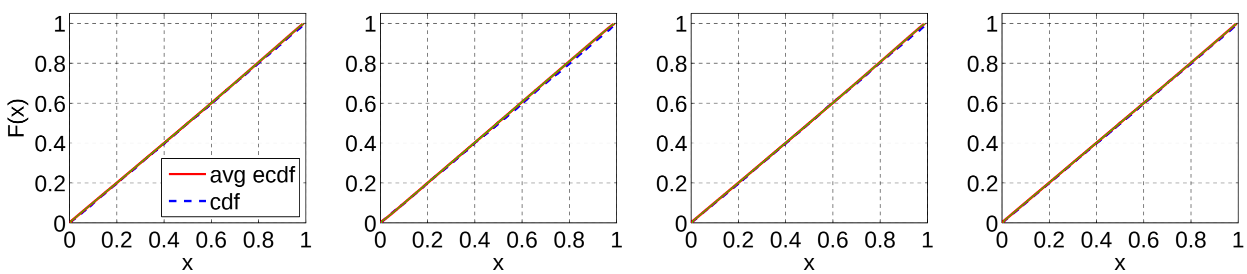


Figure 18 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $EARMA$ (3): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

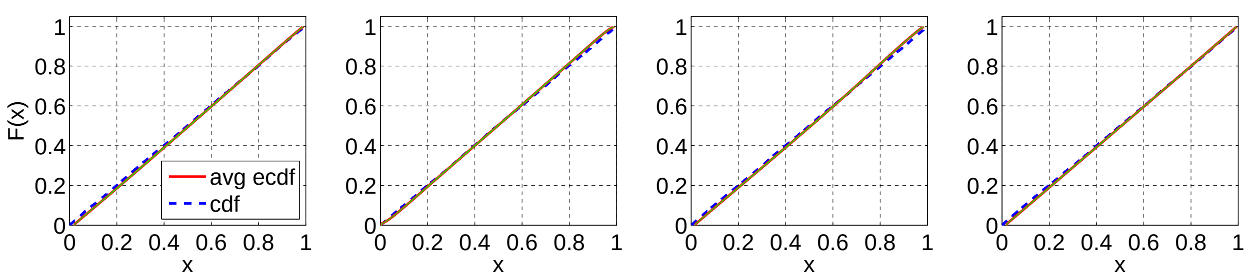


Figure 19 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $EARMA$ (5.25): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

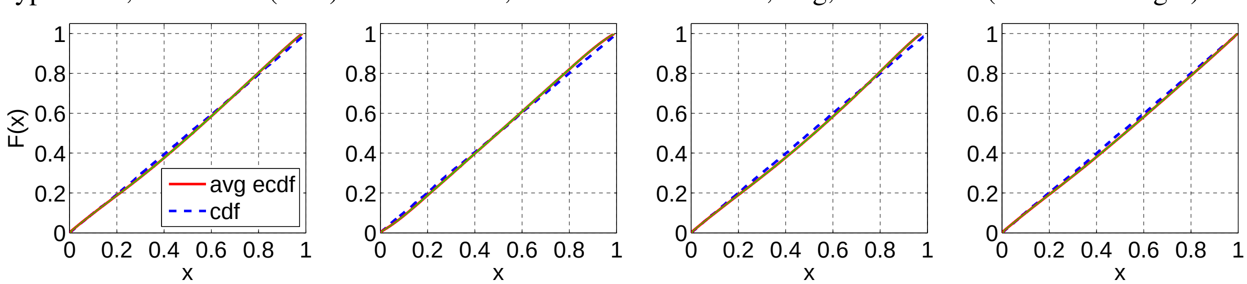


Figure 20 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $2 - H_2$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

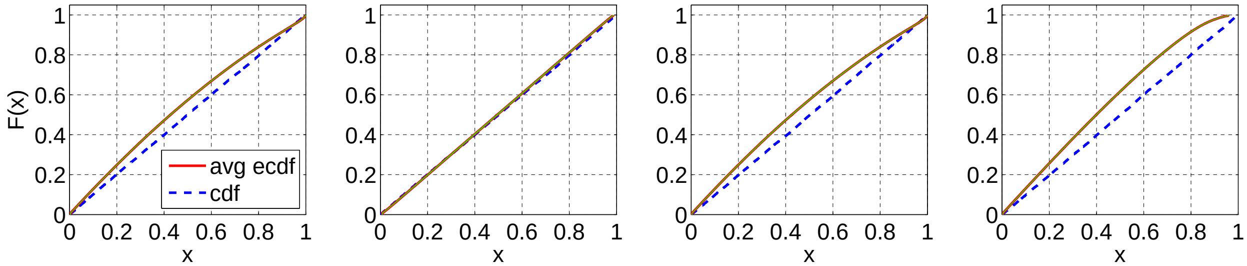


Figure 21 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $5 - H_2$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

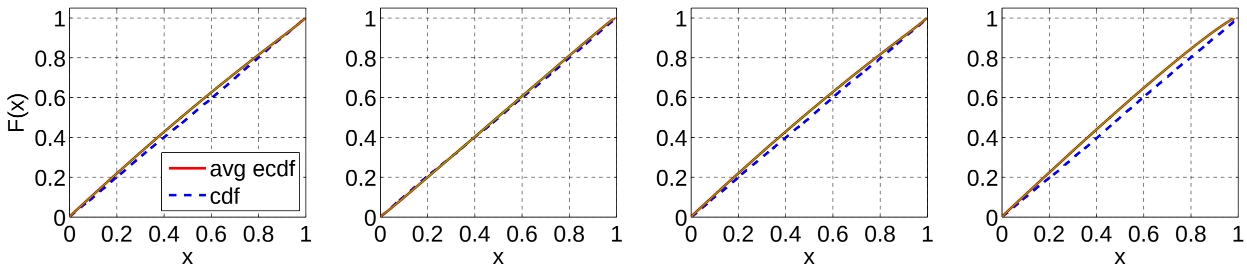


Figure 22 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $10 - H_2$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

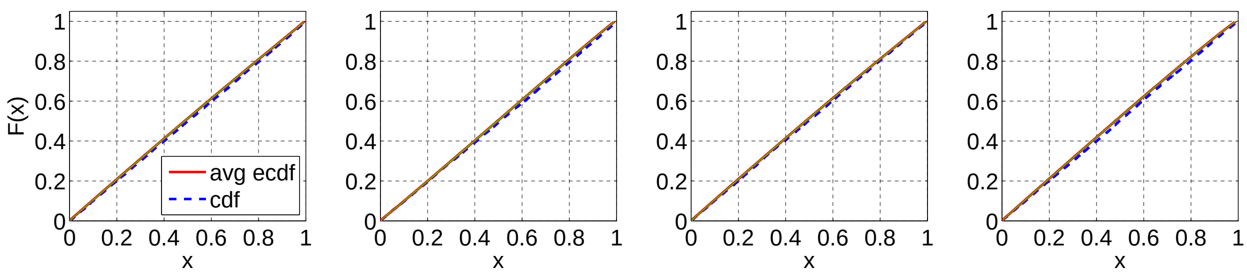


Figure 23 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $20 - H_2$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

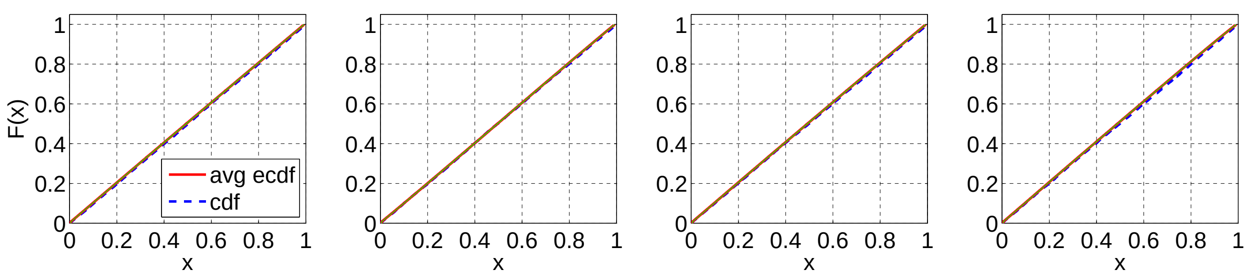


Figure 24 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; RRI (H_2, $p = 0.1$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

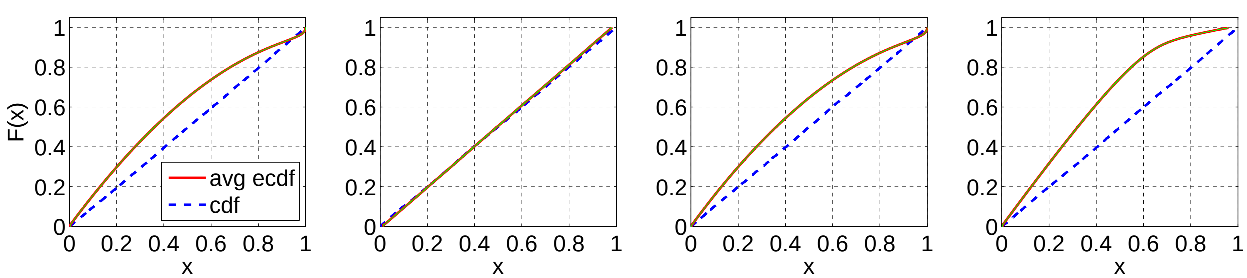


Figure 25 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; RRI (H_2, $p = 0.5$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

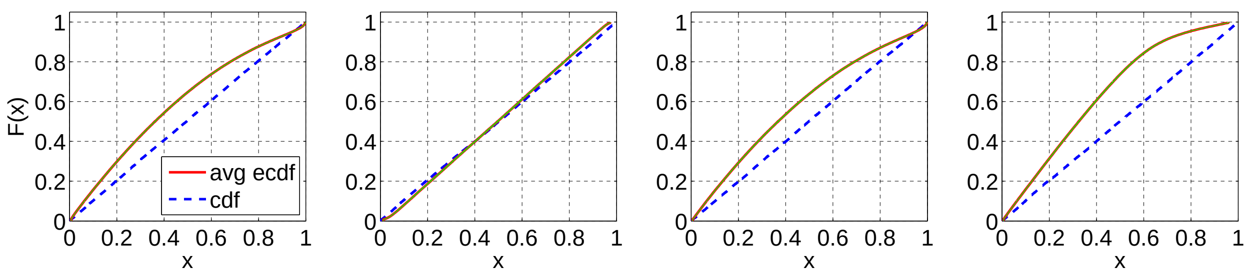
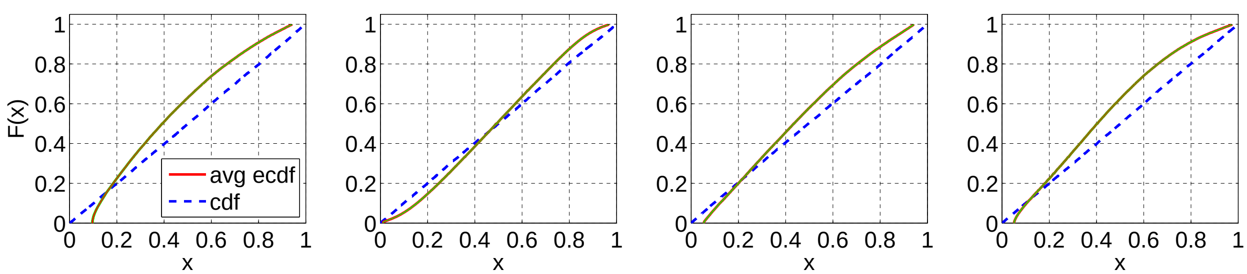


Figure 26 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; RRI (H_2, $p = 0.9$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).



3.2.2 Average Empirical CDF with Exponential (rate 1) Null Hypothesis

Figure 27 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; Exp (base case): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

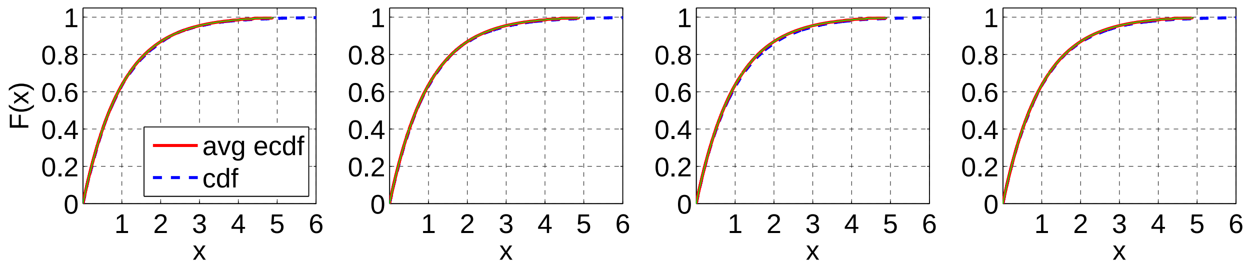


Figure 28 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; E_2: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

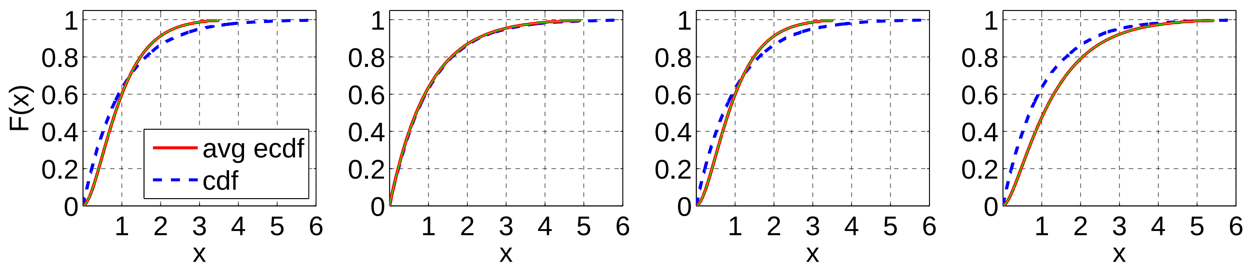


Figure 29 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; E_4: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

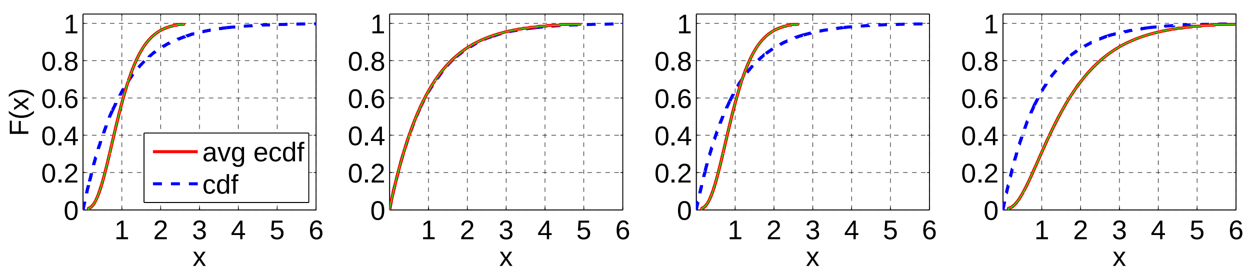


Figure 30 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; E_6: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

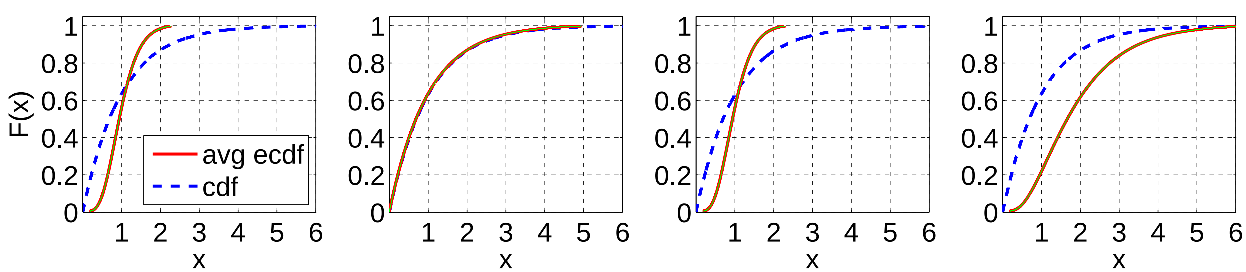


Figure 31 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; H_2 ($c^2 = 1.25$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

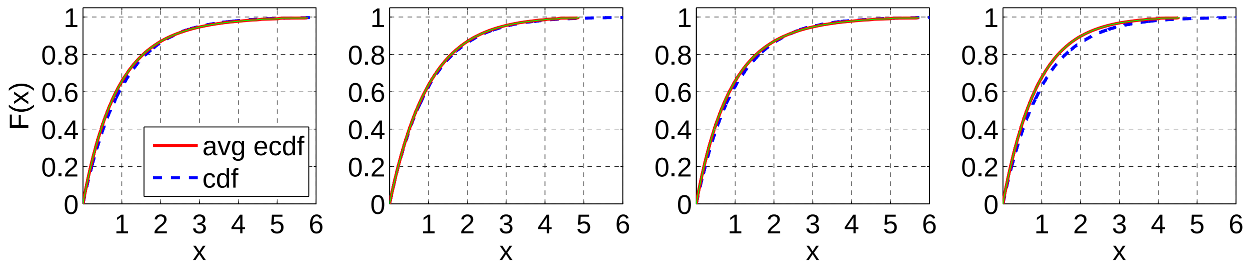


Figure 32 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; H_2 ($c^2 = 1.5$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

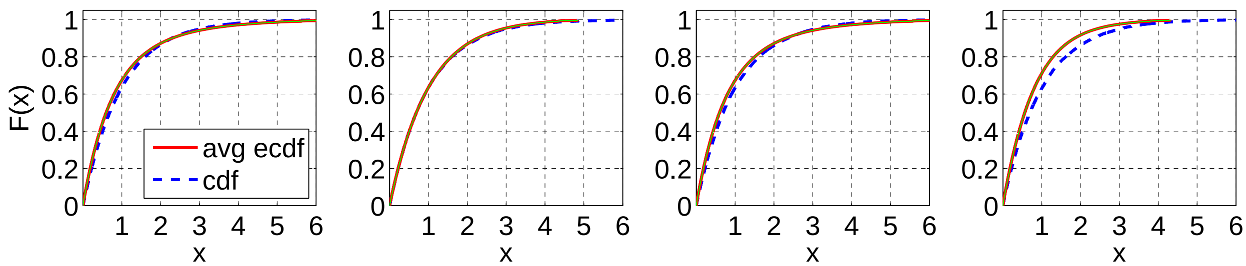


Figure 33 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; H_2 ($c^2 = 2$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

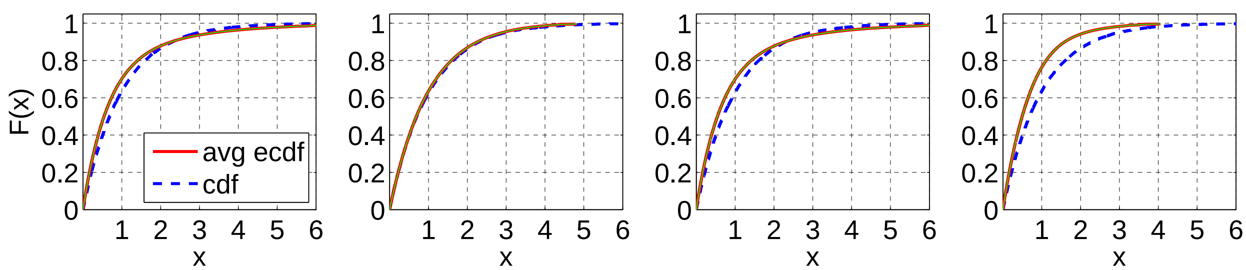


Figure 34 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; H_2 ($c^2 = 4$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

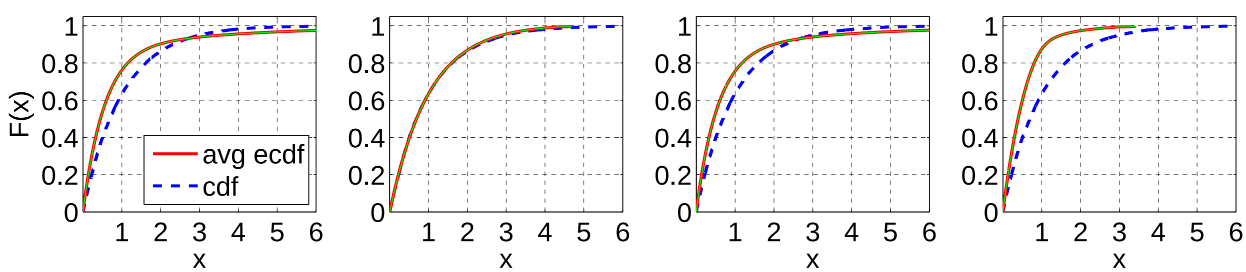


Figure 35 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; H_2 ($c^2 = 10$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

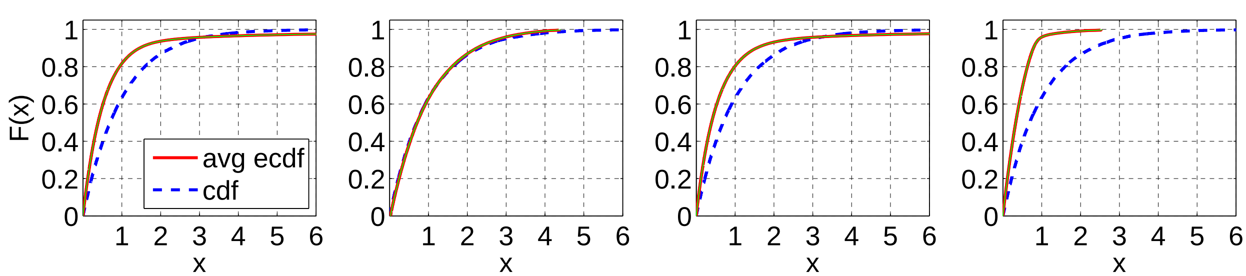


Figure 36 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; Z: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

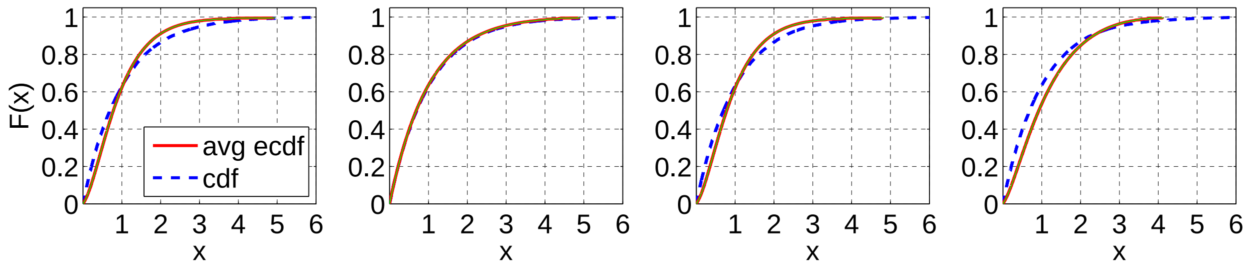


Figure 37 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; LN: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

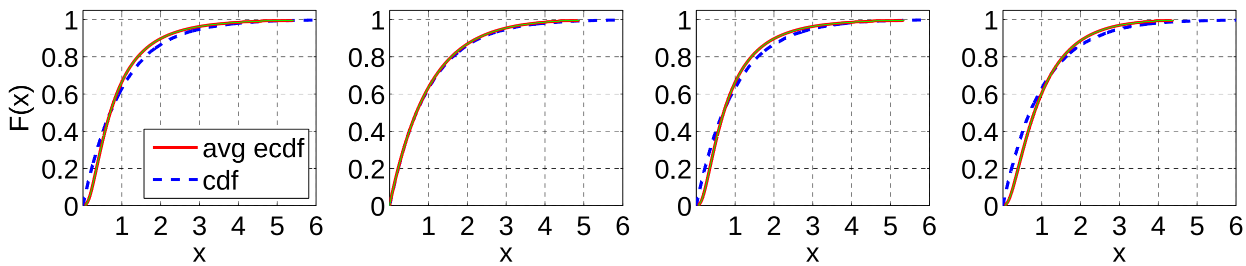


Figure 38 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; RRI $(p = 0.1)$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

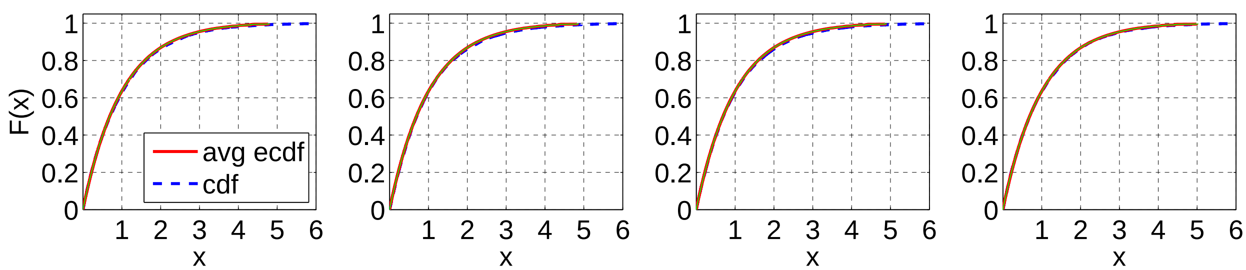


Figure 39 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; RRI $(p = 0.5)$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

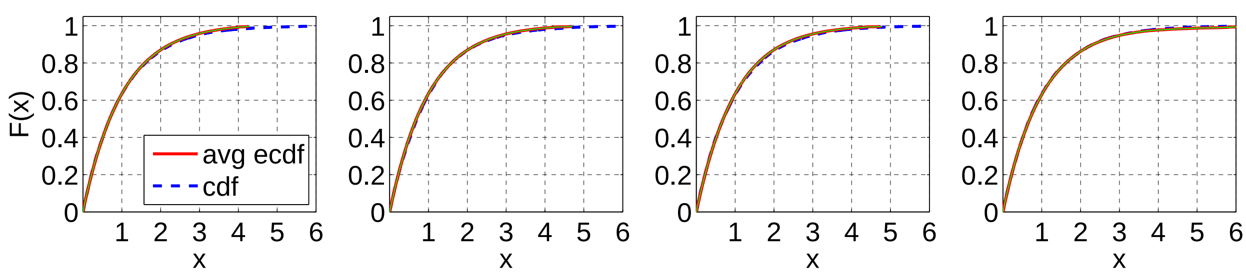


Figure 40 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; RRI $(p = 0.9)$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

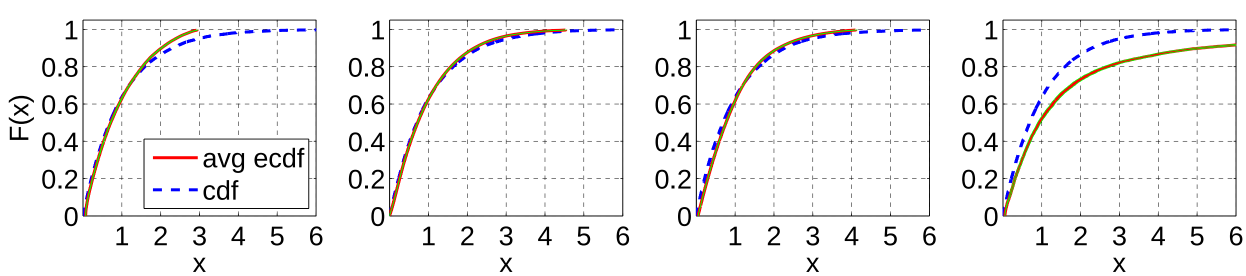


Figure 41 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $EARMA$ (0.25): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

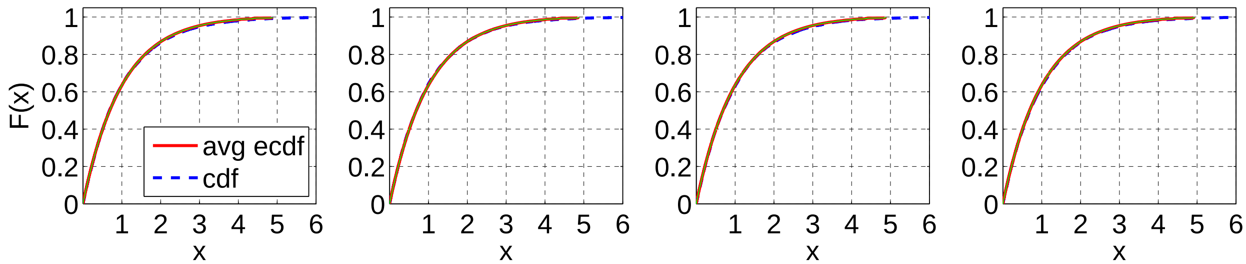


Figure 42 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $EARMA$ (0.5): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

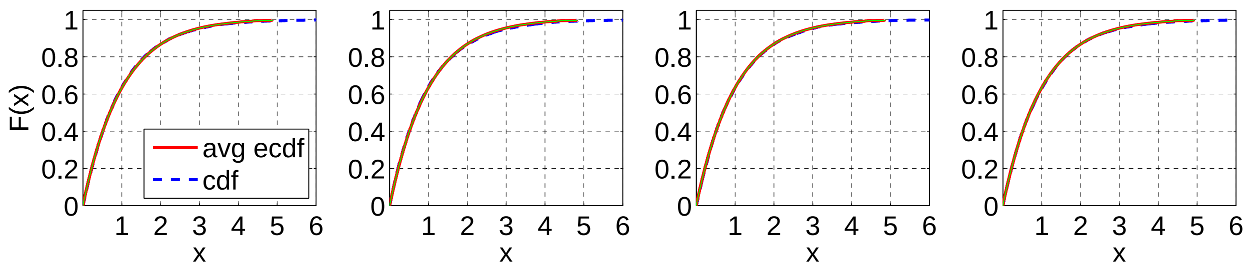


Figure 43 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $EARMA$ (1): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

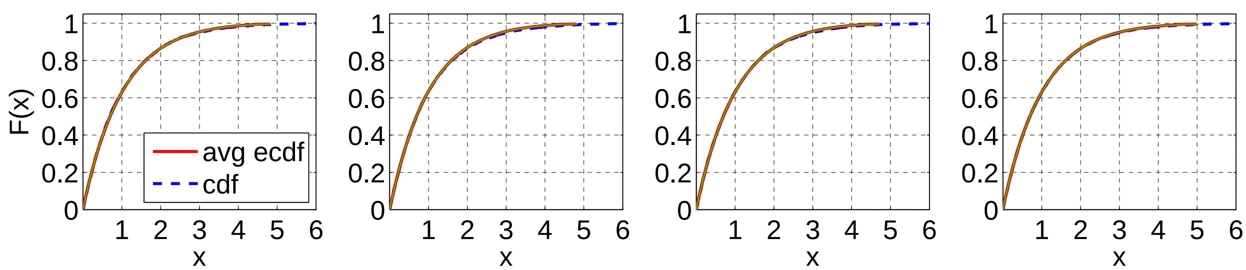


Figure 44 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $EARMA$ (3): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

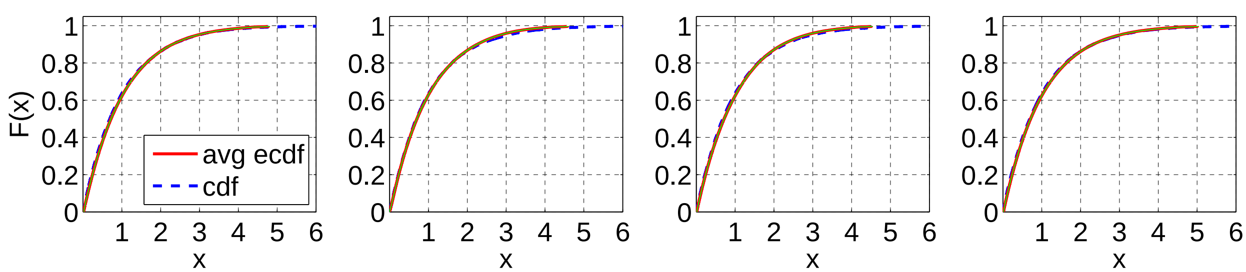


Figure 45 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $EARMA$ (5.25): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

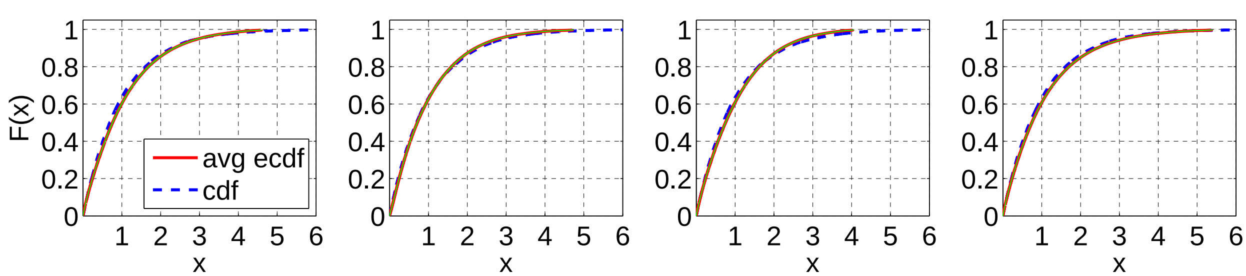


Figure 46 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $2 - H_2$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

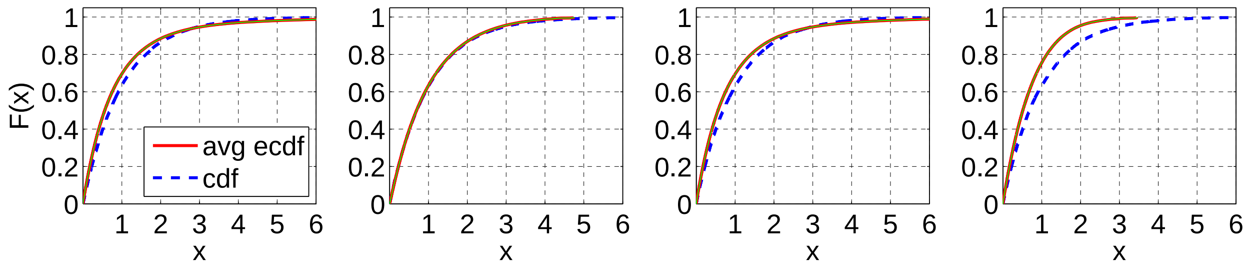


Figure 47 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $5 - H_2$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

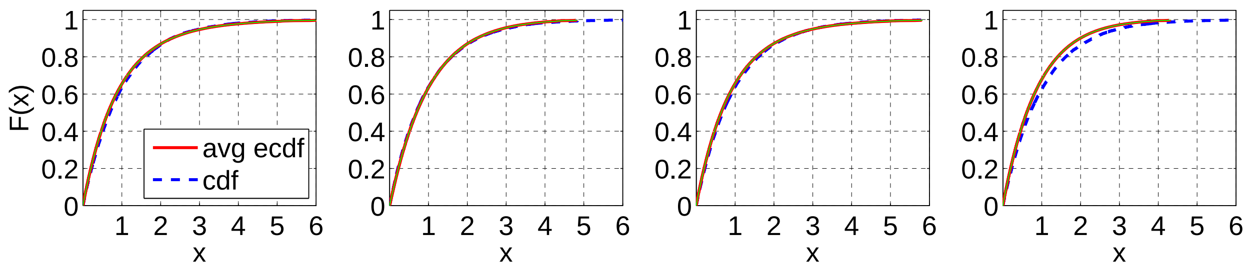


Figure 48 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $10 - H_2$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

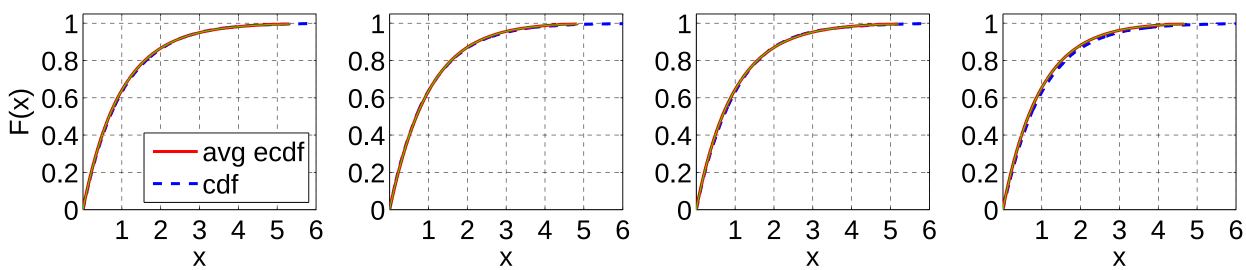


Figure 49 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $20 - H_2$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

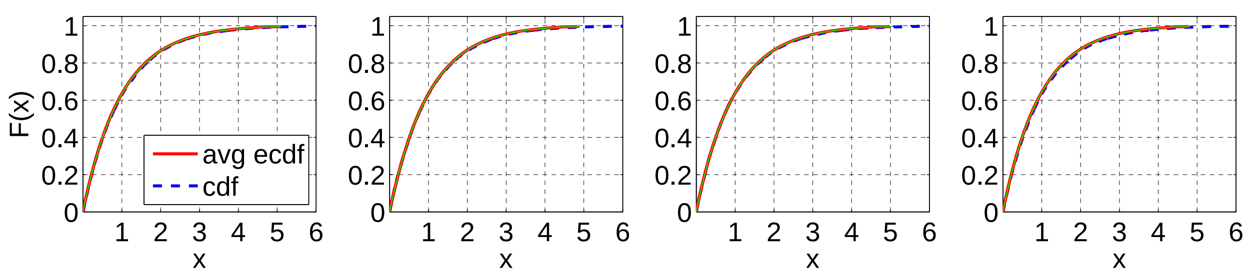


Figure 50 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; RRI ($H_2, p = 0.1$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

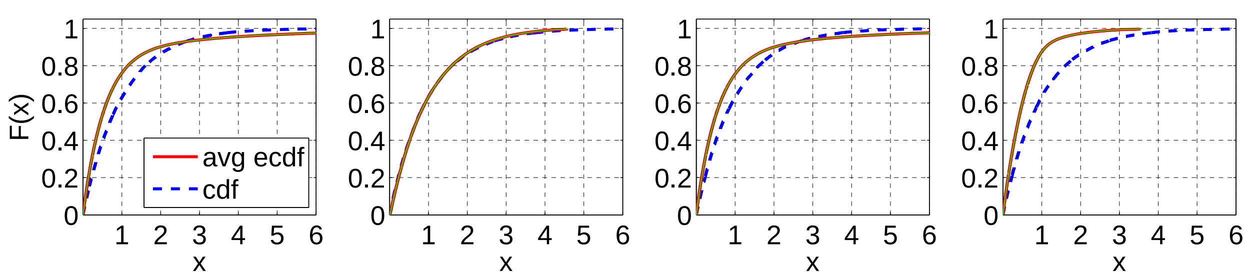


Figure 51 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; RRI (H_2, $p = 0.5$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

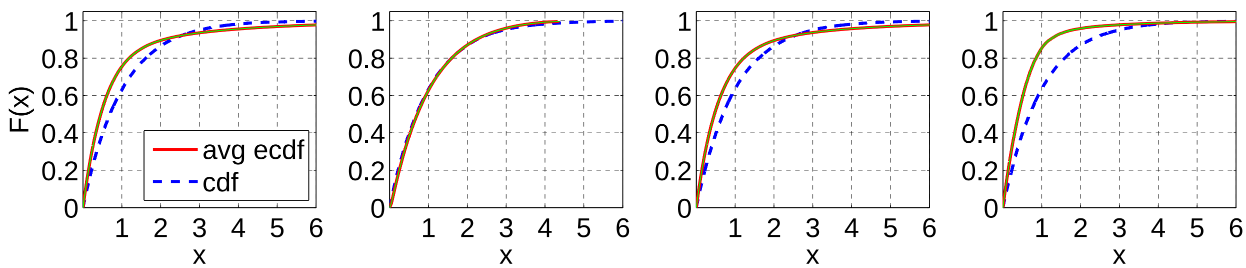
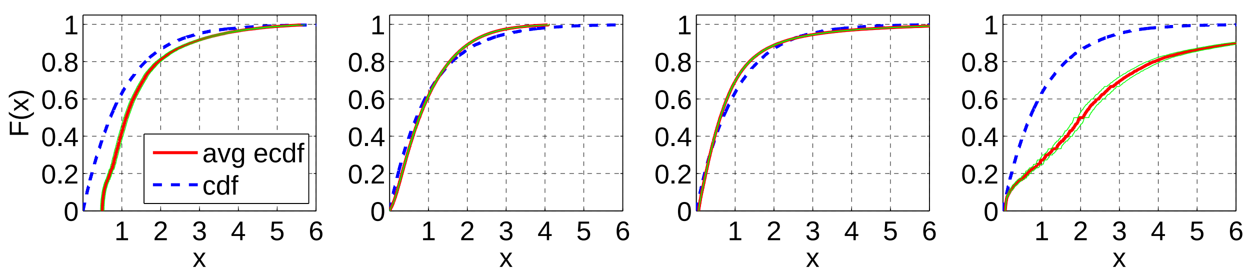


Figure 52 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; RRI (H_2, $p = 0.9$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).



3.2.3 Average Empirical CDF with Standard Normal Null Hypothesis

Figure 53 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; Exp (base case): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

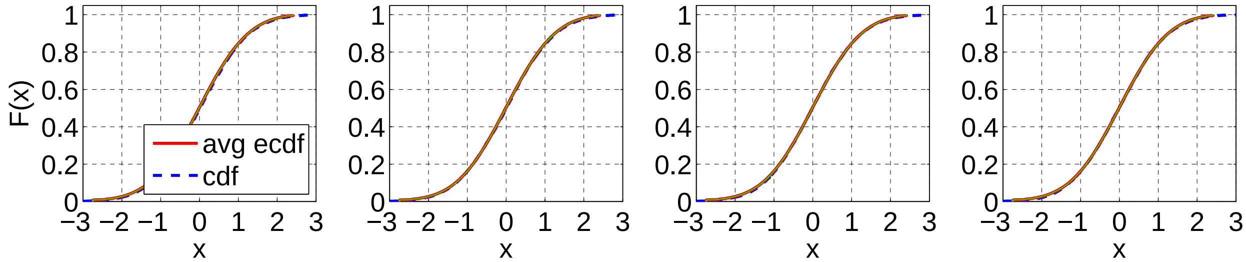


Figure 54 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; E_2: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

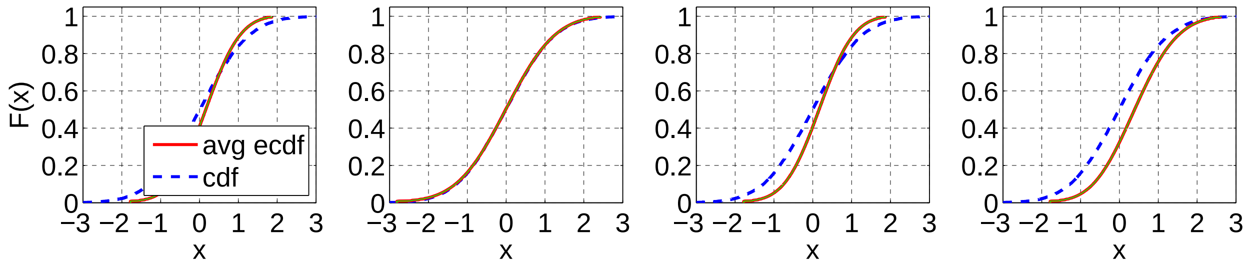


Figure 55 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; E_4: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

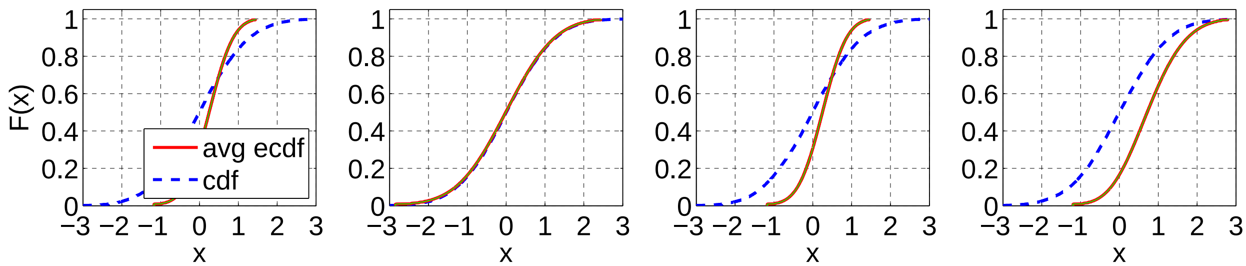


Figure 56 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; E_6: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

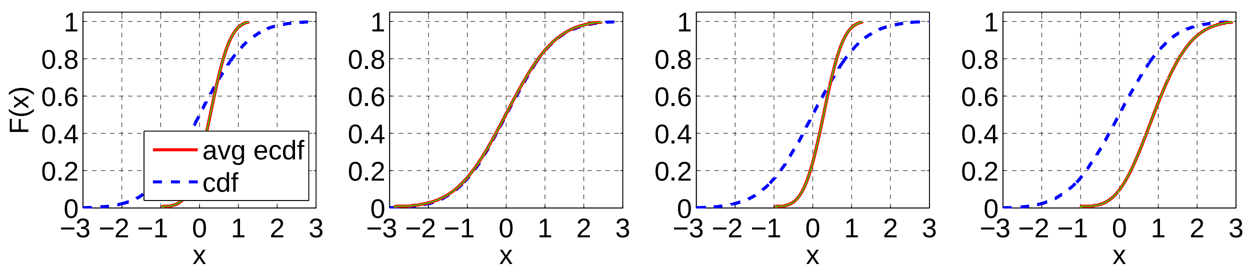


Figure 57 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; H_2 ($c^2 = 1.25$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

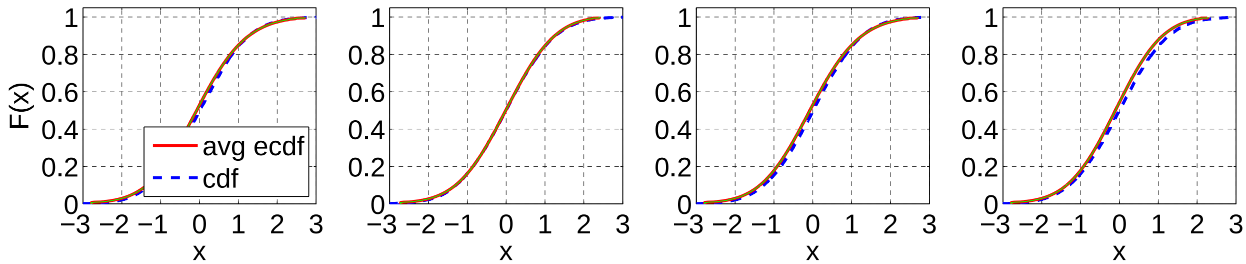


Figure 58 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; H_2 ($c^2 = 1.5$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

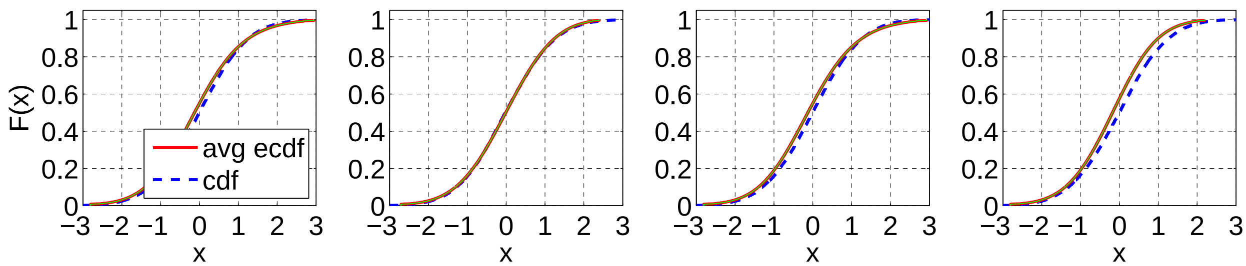


Figure 59 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; H_2 ($c^2 = 2$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

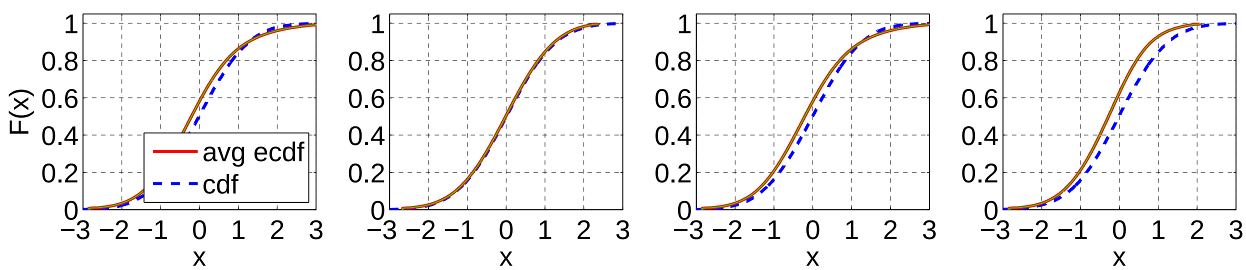


Figure 60 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; H_2 ($c^2 = 4$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

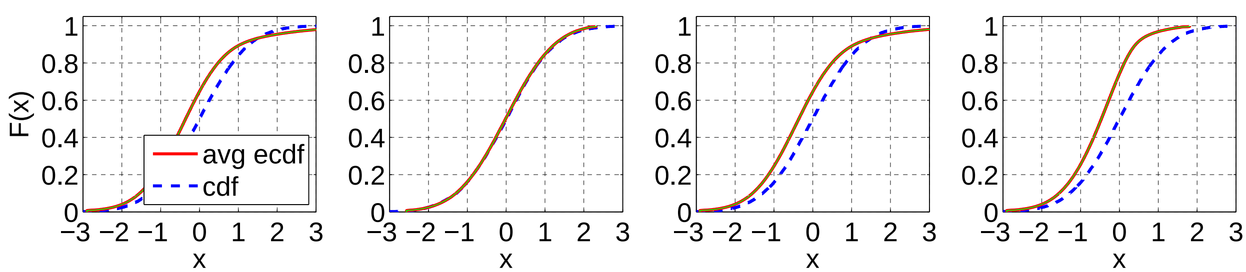


Figure 61 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; H_2 ($c^2 = 10$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

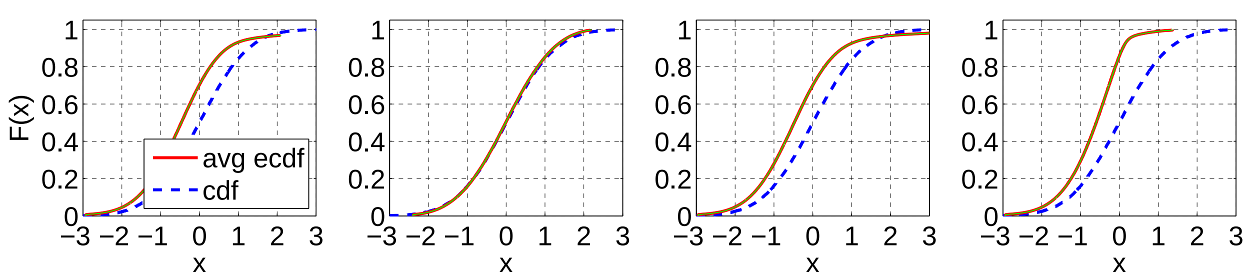


Figure 62 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; Z: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

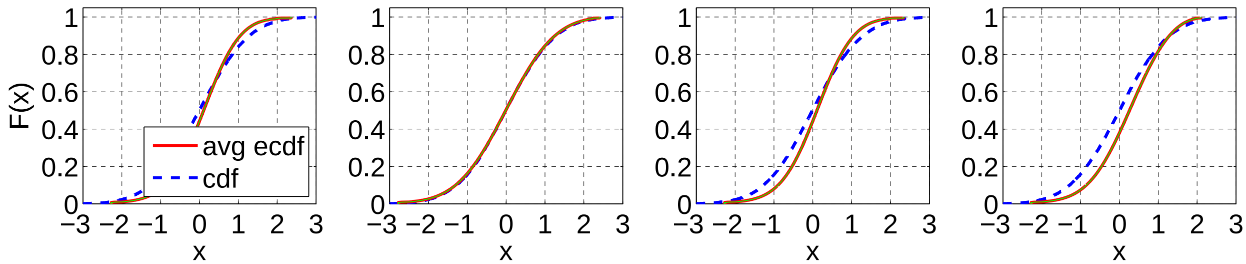


Figure 63 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; LN: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

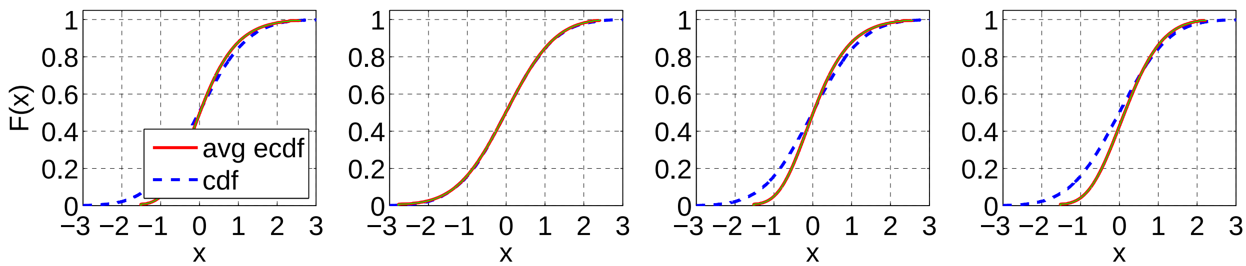


Figure 64 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; RRI ($p = 0.1$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

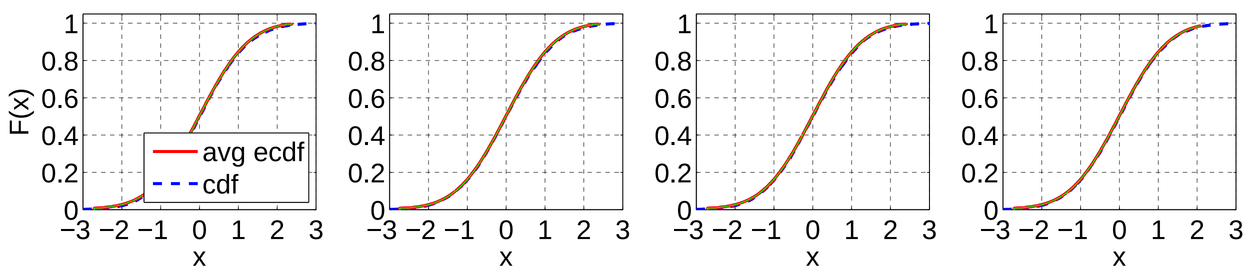


Figure 65 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; RRI ($p = 0.5$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

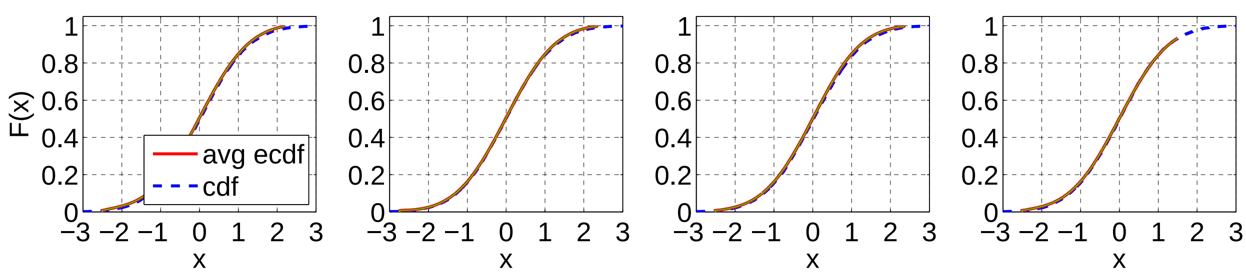


Figure 66 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; RRI ($p = 0.9$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

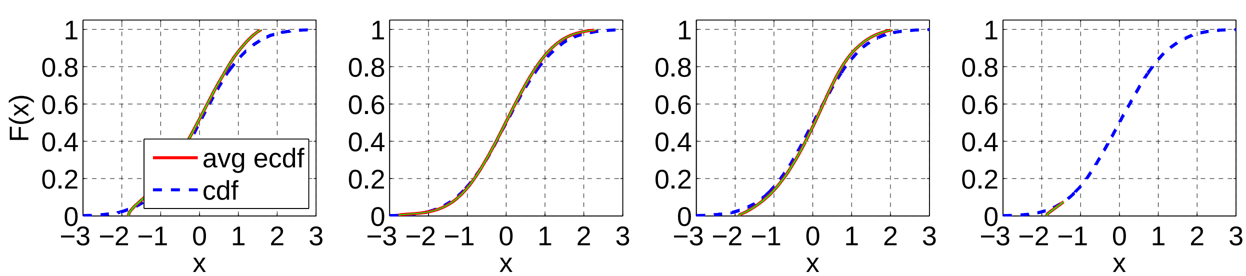


Figure 67 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $EARMA\,(0.25)$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

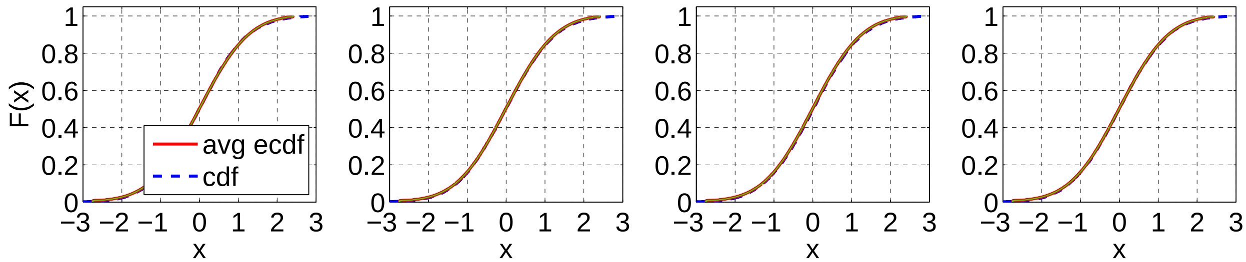


Figure 68 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $EARMA\,(0.5)$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

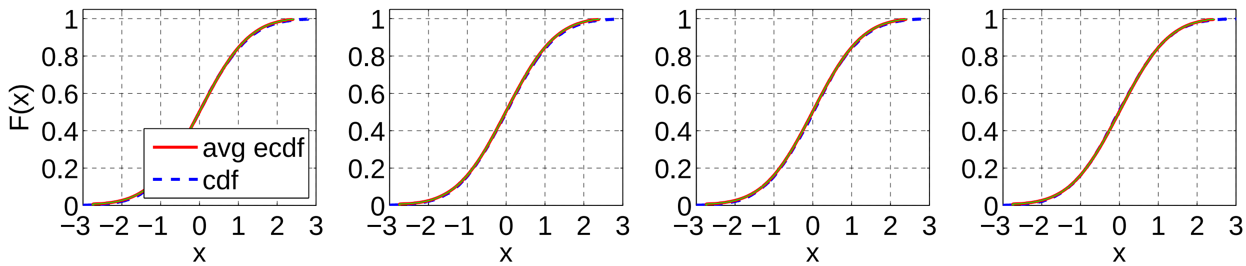


Figure 69 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $EARMA\,(1)$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

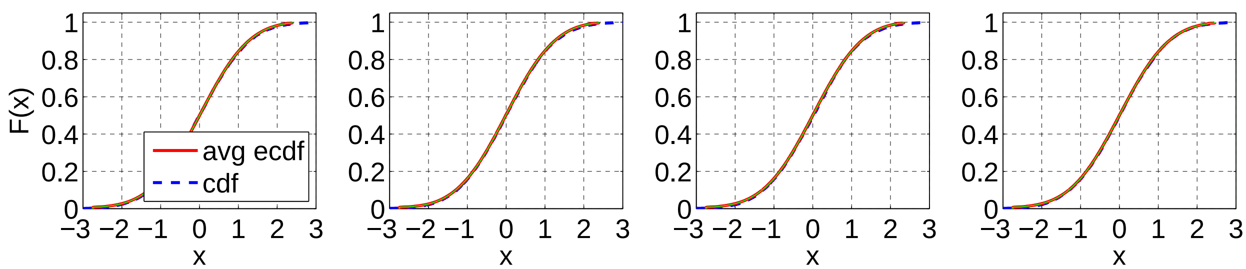


Figure 70 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $EARMA\,(3)$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

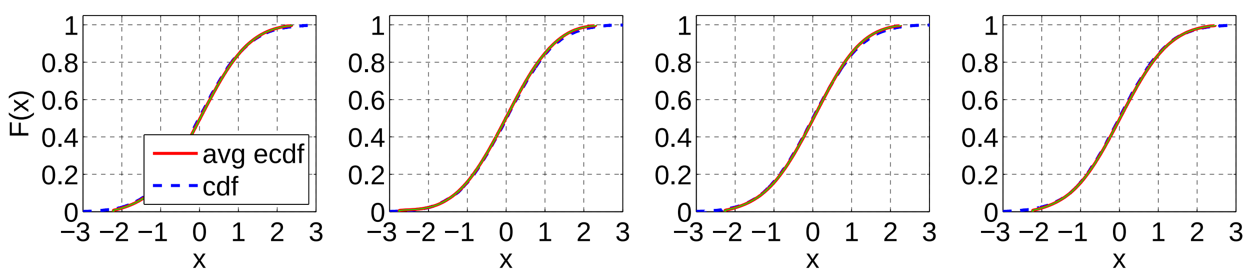


Figure 71 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $EARMA\,(5.25)$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

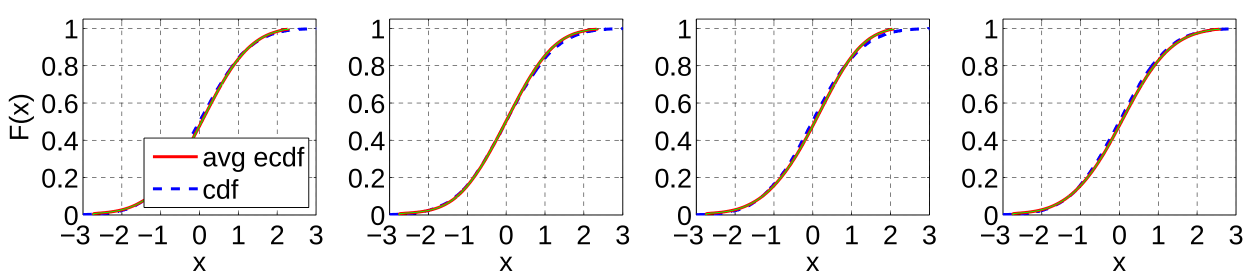


Figure 72 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $2 - H_2$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

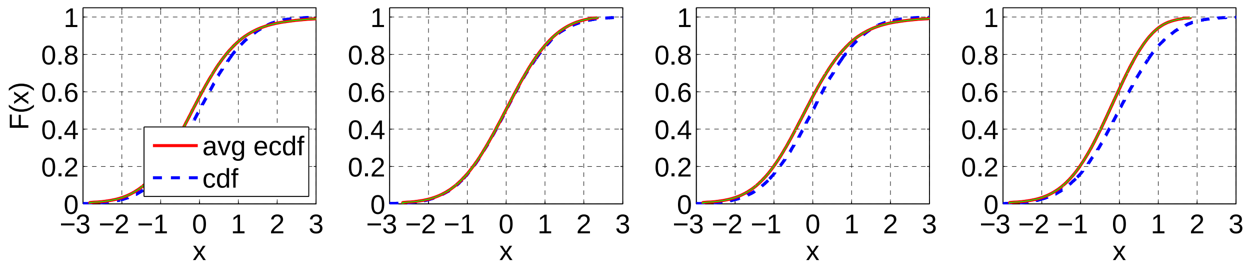


Figure 73 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $5 - H_2$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

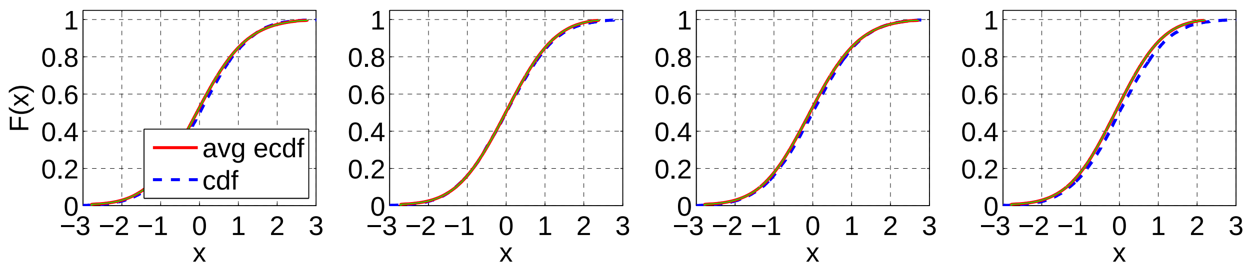


Figure 74 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $10 - H_2$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

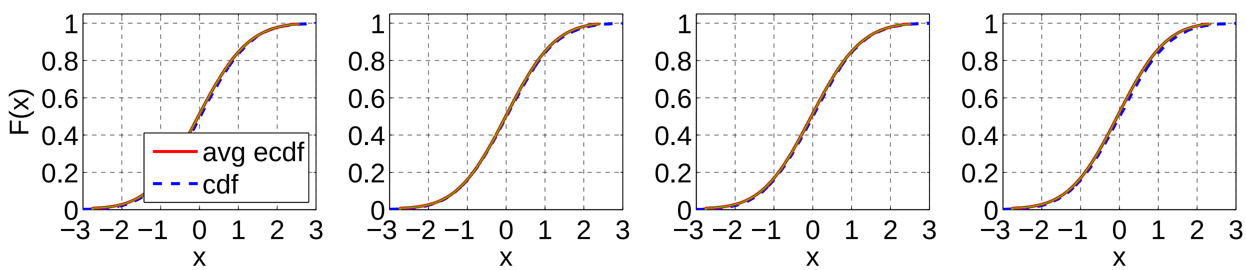


Figure 75 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; $20 - H_2$: Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

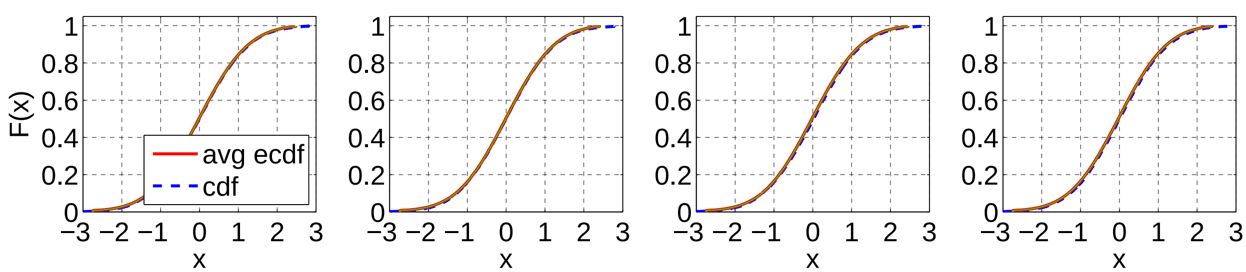


Figure 76 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; RRI ($H_2, p = 0.1$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

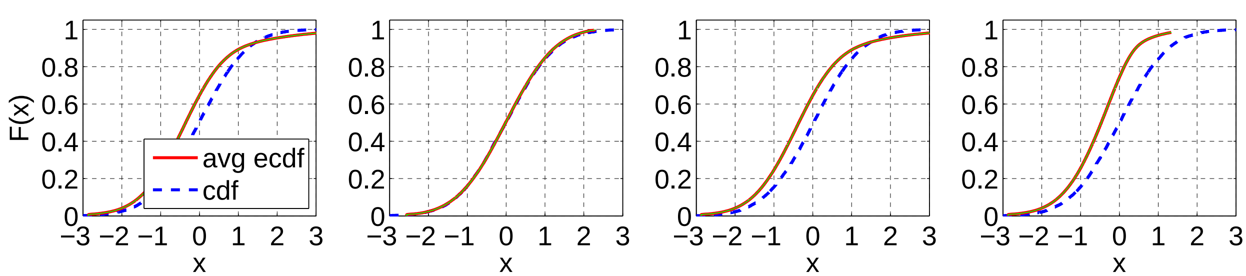


Figure 77 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; RRI (H_2, $p = 0.5$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).

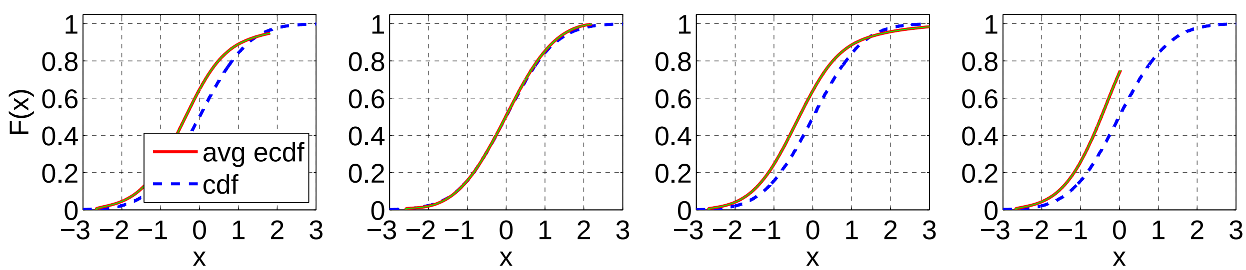
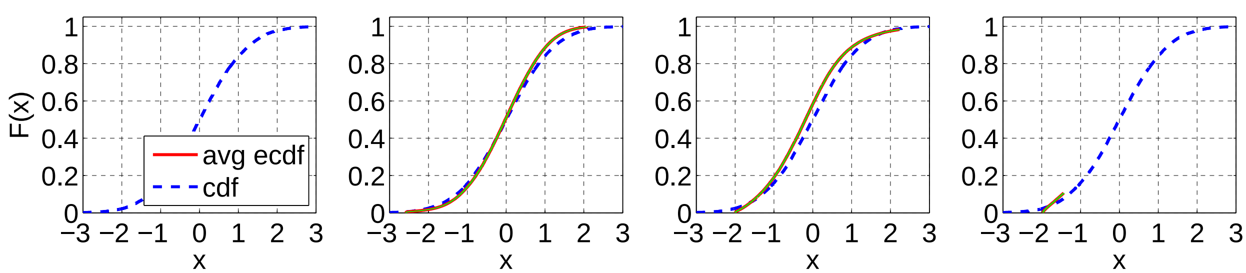


Figure 78 Comparison of the average ecdf based on 10^4 replications for $[0, 200]$ with the cdf of the null hypothesis; RRI (H_2, $p = 0.9$): Standard KS, Conditional-Uniform, Log, Lewis Tests (from left to right).



3.3 Power of Alternative K-S Tests

In this section, we describe the results of applying KS tests to the original and transformed interarrival times, with the goal of comparing the power of alternative tests. Table 7 provides the number of KS tests passed (out of 10000 replications) as well as the average p-value with associated 95% confidence intervals. We conclude that the Standard KS test, Log test and Lewis test perform reasonably well for the renewal processes (E_k, H_2, Z and LN), RRI, $RRI(H_2)$ and mH_2 with smaller values of m. Comparing the number of tests passed and average p-values, we observe that the Lewis test is the most powerful test. On the other hand, the Conditional-Uniform test performs very poorly for E_k, Z and LN. Especially, for the cases with E_k, the results suggests E_k is more like exponential than Exp is. However, the Conditional-Uniform tests performs the best for the rest of the processes, $EARMA$ and mH_2 with higher values of m. The power of the Lewis test for $EARMA$ and mH_2 with $m = 10, 20$ is weaker, revealing different tests can have advantages for different alternatives.

The power of each test is reflected in the average p-values as well. To examine the distribution of these p-values, we plot the empirical CDF of the p-values and compare them among alternative tests in Figures 79 - 104 in Section 3.4. These figures support what we see in Table 7. The line for the Lewis test is on top of other lines in the figures for E_k, H_2, Z, LN, RRI, $RRI(H_2)$ and mH_2 with smaller values of m. For $EARMA$ and mH_2 with higher values of m, the line for the Conditional-Uniform test is on top, and the line for the Lewis test is actually at the bottom.

The standard KS test used to obtain the values reported in Table 7 is actually invalid because the sample size was not specified in advance, but instead was the random number observed in $[0, t]$. Moreover, it used the true mean 1, which would not be known in application. Hence, to put the results for the standard test Table 7 in perspective, we can compare its results to the alternative results for the valid standard KS test with fixed sample size $n = 200$ in the first columns of Table 37. For the case of a specified interval $[0, t]$, we also used the known mean in the standard KS test, which would not in fact be known. Hence, in Table 8 we also give the corresponding standard KS results using the estimated mean and the Lilliefors (1969) test for the exponential cdf with unknown mean. Interestingly, none of these performance results differ greatly, so one might consider the standard KS test with estimated mean or the Lilliefors (1969) test as realizable alternatives to the standard KS test.

Table 7 Performance of alternative KS tests of a rate-1 Poisson process for the time interval $[0, 200]$: Number of KS tests passed (denoted by $\#P$) at significance level 0.05 out of $10,000$ replications and the average p-values (denoted by $E[p\text{-value}]$) with associated 95% confidence intervals.

		Standard KS		Conditional		Log		Lewis	
Case	*Subcase*	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$
Exp	$-$	9479	0.50 ± 0.006	9503	0.50 ± 0.006	9490	0.50 ± 0.006	9502	0.50 ± 0.006
E_k	$k = 2$	11	0.00 ± 0.000	9991	0.78 ± 0.005	24	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 4$	0	0.00 ± 0.000	10000	0.94 ± 0.002	0	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 6$	0	0.00 ± 0.000	10000	0.98 ± 0.001	0	0.00 ± 0.000	0	0.00 ± 0.000
H_2	$c^2 = 1.25$	8757	0.41 ± 0.006	8988	0.41 ± 0.006	8685	0.40 ± 0.006	7558	0.30 ± 0.006
	$c^2 = 1.5$	7049	0.27 ± 0.005	8428	0.33 ± 0.005	7081	0.28 ± 0.006	3990	0.11 ± 0.004
	$c^2 = 2$	3695	0.10 ± 0.003	7186	0.23 ± 0.005	4059	0.13 ± 0.004	584	0.01 ± 0.001
	$c^2 = 4$	231	0.01 ± 0.001	3551	0.08 ± 0.003	934	0.03 ± 0.002	3	0.00 ± 0.000
	$c^2 = 10$	13	0.00 ± 0.000	650	0.01 ± 0.001	608	0.02 ± 0.002	2	0.00 ± 0.000
Z	$-$	1200	0.02 ± 0.001	9412	0.57 ± 0.006	1308	0.02 ± 0.001	243	0.01 ± 0.000
LN	$-$	88	0.01 ± 0.000	9525	0.52 ± 0.006	332	0.01 ± 0.000	69	0.00 ± 0.000
RRI	$p = 0.1$	9093	0.41 ± 0.006	9044	0.42 ± 0.006	9037	0.42 ± 0.006	9080	0.41 ± 0.006
	$p = 0.5$	4631	0.11 ± 0.003	5516	0.15 ± 0.004	5204	0.13 ± 0.003	4633	0.11 ± 0.003
	$p = 0.9$	14	0.00 ± 0.000	826	0.02 ± 0.002	95	0.00 ± 0.000	12	0.00 ± 0.000
$EARMA$	0.25	9260	0.47 ± 0.006	8536	0.36 ± 0.005	9236	0.46 ± 0.006	9477	0.50 ± 0.006
	0.5	8848	0.42 ± 0.006	7433	0.26 ± 0.005	8870	0.43 ± 0.006	9406	0.49 ± 0.006
	1	8244	0.37 ± 0.006	5964	0.18 ± 0.004	8280	0.37 ± 0.006	8994	0.44 ± 0.006
	3	5196	0.21 ± 0.005	1977	0.04 ± 0.002	5725	0.22 ± 0.005	6736	0.29 ± 0.006
	5.25	4088	0.14 ± 0.004	1594	0.04 ± 0.002	4512	0.15 ± 0.004	5770	0.22 ± 0.005
mH_2	$m = 2$	4531	0.16 ± 0.005	4231	0.10 ± 0.003	4901	0.18 ± 0.005	1237	0.03 ± 0.002
	$m = 5$	7474	0.32 ± 0.006	5282	0.16 ± 0.004	7706	0.34 ± 0.006	7142	0.29 ± 0.006
	$m = 10$	7812	0.35 ± 0.006	6496	0.23 ± 0.005	8334	0.38 ± 0.006	8980	0.44 ± 0.006
	$m = 20$	8034	0.36 ± 0.006	7751	0.32 ± 0.006	8831	0.43 ± 0.006	9366	0.49 ± 0.006
$RRI(H_2)$	$p = 0.1$	322	0.01 ± 0.001	2841	0.06 ± 0.002	1012	0.03 ± 0.002	4	0.00 ± 0.000
	$p = 0.5$	478	0.01 ± 0.001	884	0.02 ± 0.001	1209	0.03 ± 0.002	69	0.00 ± 0.000
	$p = 0.9$	9	0.00 ± 0.000	772	0.06 ± 0.005	92	0.00 ± 0.000	6	0.00 ± 0.000

Table 8 Performance of Alternative Standard KS Tests for Untransformed Interarrival Times for the time interval $[0, 200]$: Number of KS tests passed (denoted by $\#P$) at significance level 0.05 out of 10000 replications and average p-values (denoted by $E[p\text{-value}]$) with associated 95% confidence intervals.

		KS with estimated mean		Lillifors test	
$Case$	$Subcase$	$\#P$	$E[p\text{-value}]$	$\#P$	$E[p\text{-value}]$
Exp	$-$	9942	0.65 ± 0.005	9482	0.37 ± 0.003
E_k	$k = 2$	8	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 4$	0	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 6$	0	0.00 ± 0.000	0	0.00 ± 0.000
H_2	$c^2 = 1.25$	9564	0.50 ± 0.006	8239	0.28 ± 0.004
	$c^2 = 1.5$	8063	0.29 ± 0.005	5379	0.14 ± 0.003
	$c^2 = 2$	3752	0.08 ± 0.003	1398	0.03 ± 0.002
	$c^2 = 4$	157	0.00 ± 0.000	23	0.00 ± 0.000
	$c^2 = 10$	76	0.00 ± 0.000	26	0.00 ± 0.000
Z	$-$	1230	0.02 ± 0.001	210	0.01 ± 0.000
LN	$-$	281	0.01 ± 0.000	9	0.00 ± 0.000
RRI	$p = 0.1$	9845	0.56 ± 0.005	8971	0.32 ± 0.003
	$p = 0.5$	6873	0.18 ± 0.004	3775	0.08 ± 0.002
	$p = 0.9$	44	0.00 ± 0.000	2	0.00 ± 0.000
$EARMA$	0.25	9942	0.65 ± 0.005	9460	0.37 ± 0.003
	0.5	9925	0.65 ± 0.005	9420	0.37 ± 0.003
	1	9845	0.60 ± 0.005	9102	0.34 ± 0.003
	3	8341	0.43 ± 0.006	6702	0.23 ± 0.004
	5.25	8015	0.35 ± 0.006	5987	0.19 ± 0.004
mH_2	$m = 2$	4949	0.14 ± 0.004	2440	0.06 ± 0.002
	$m = 5$	9414	0.49 ± 0.006	7944	0.27 ± 0.004
	$m = 10$	9874	0.61 ± 0.005	9177	0.35 ± 0.003
	$m = 20$	9935	0.65 ± 0.005	9421	0.37 ± 0.003
$RRI(H_2)$	$p = 0.1$	202	0.00 ± 0.000	41	0.00 ± 0.000
	$p = 0.5$	311	0.01 ± 0.001	94	0.00 ± 0.000
	$p = 0.9$	36	0.00 ± 0.000	1	0.00 ± 0.000

3.4 Insightful Plots B: P-value Comparisons

Figure 79 Exp - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

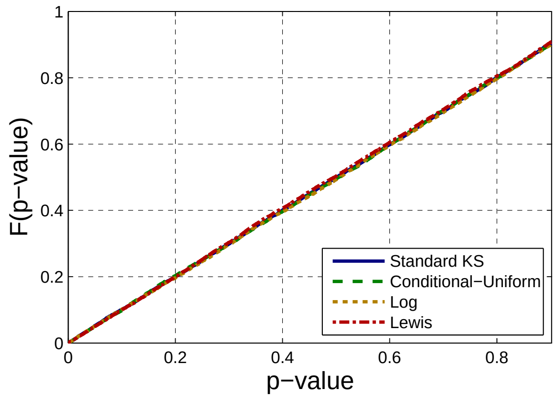


Figure 80 E_2 - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

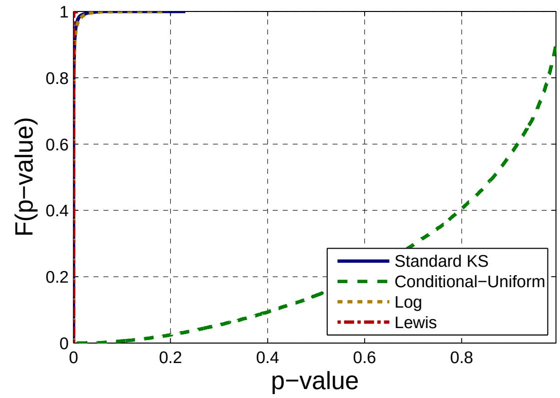


Figure 81 E_4 - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

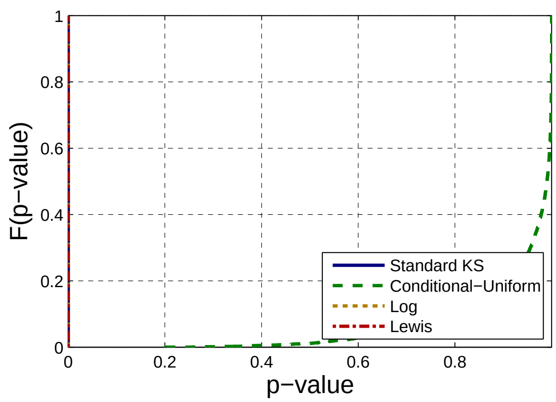


Figure 82 E_6 - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

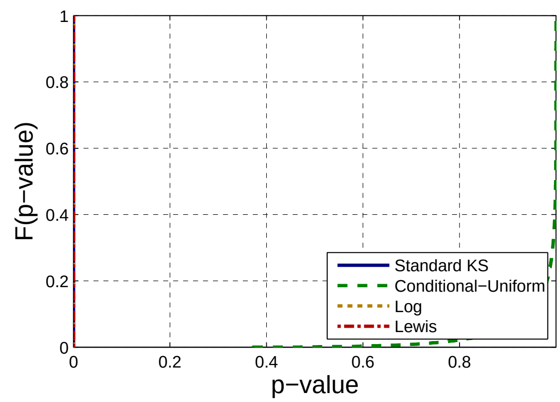


Figure 83 H_2 ($c^2 = 1.25$) - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

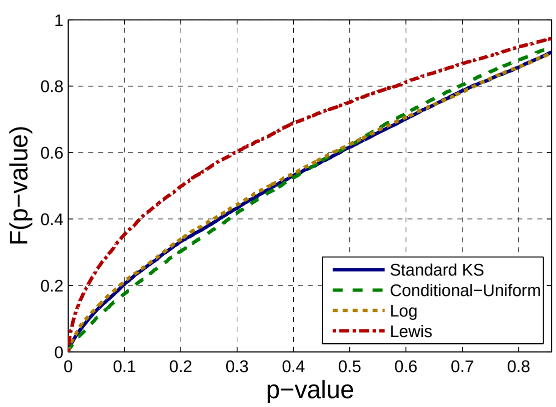


Figure 84 H_2 ($c^2 = 1.5$) - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

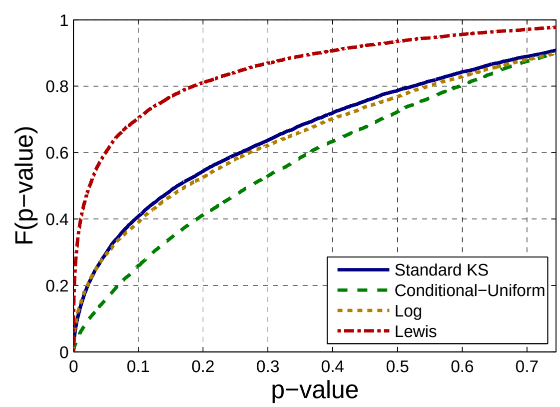


Figure 85 H_2 ($c^2 = 2$) - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

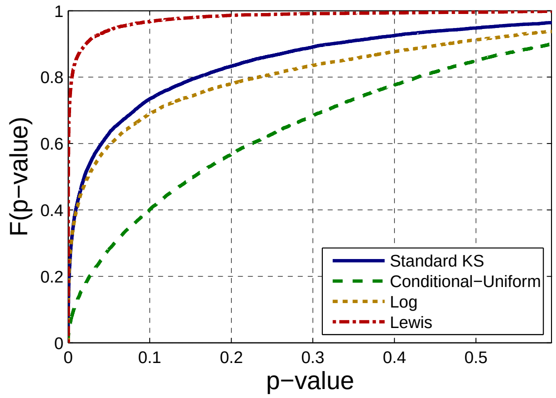


Figure 86 H_2 ($c^2 = 4$) - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

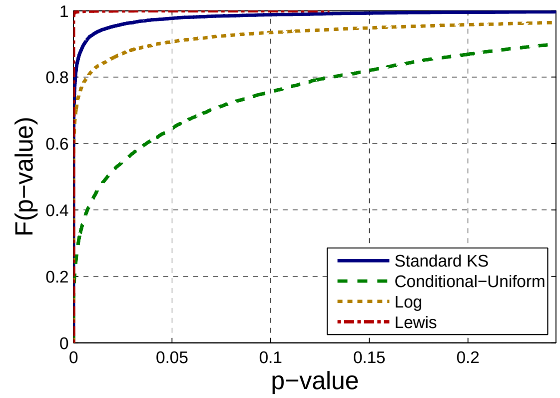


Figure 87 H_2 ($c^2 = 10$) - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

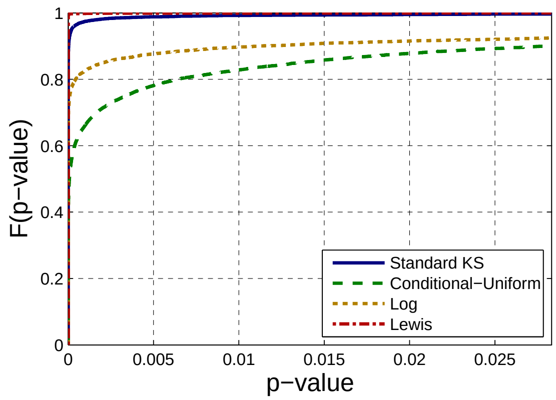


Figure 88 Z - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

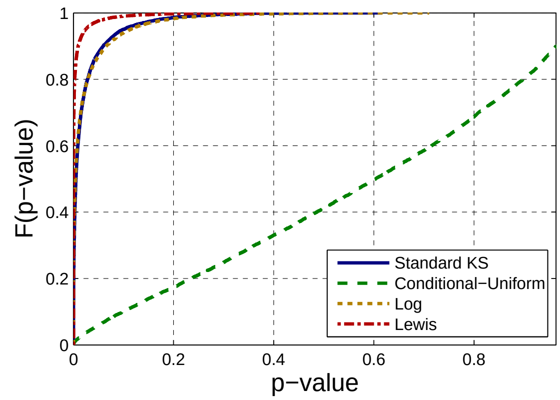


Figure 89 LN - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

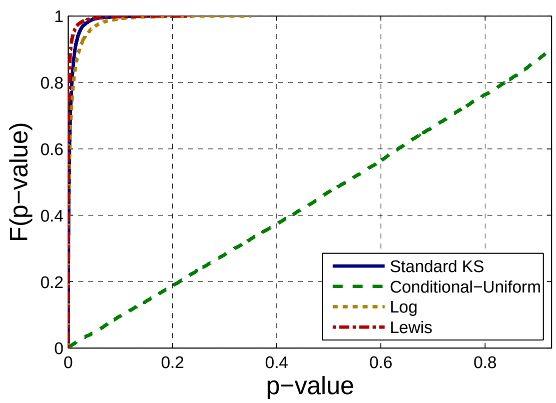


Figure 90 RRI ($p = 0.1$) - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

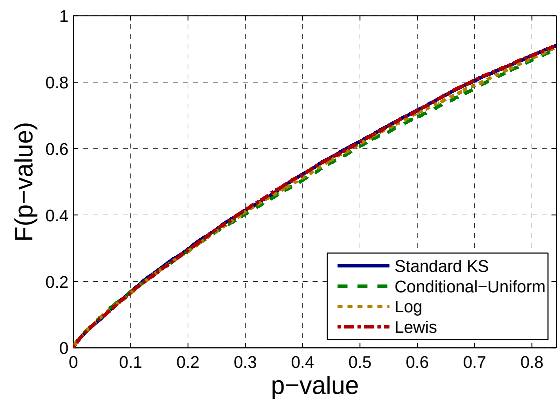


Figure 91 RRI ($p = 0.5$) - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

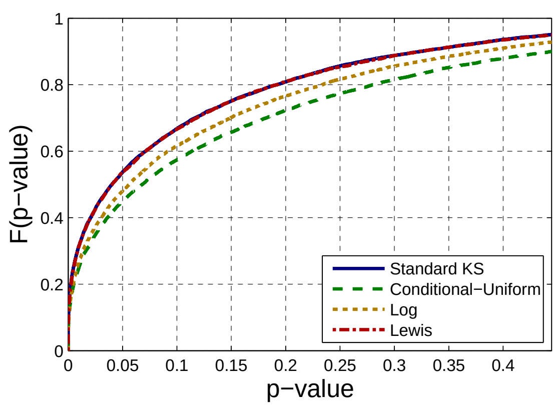


Figure 92 RRI ($p = 0.9$) - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

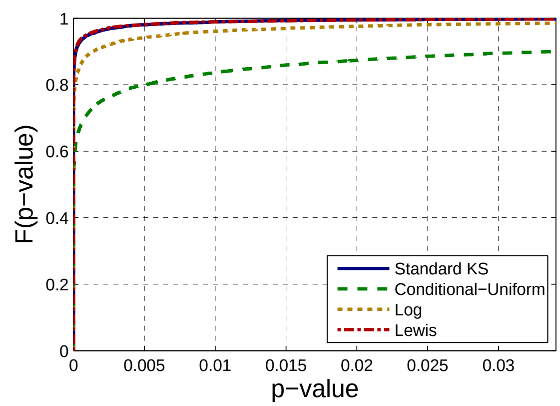


Figure 93 $EARMA$ (0.25) - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

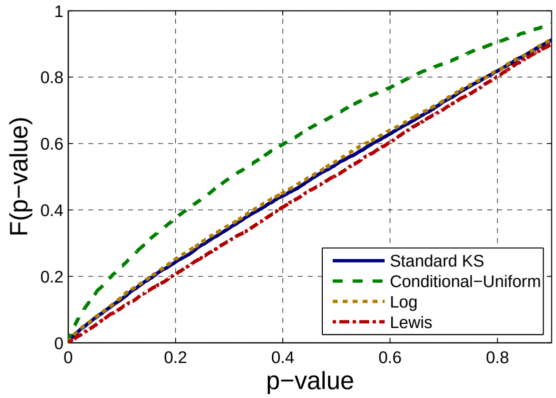


Figure 94 $EARMA$ (0.5) - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

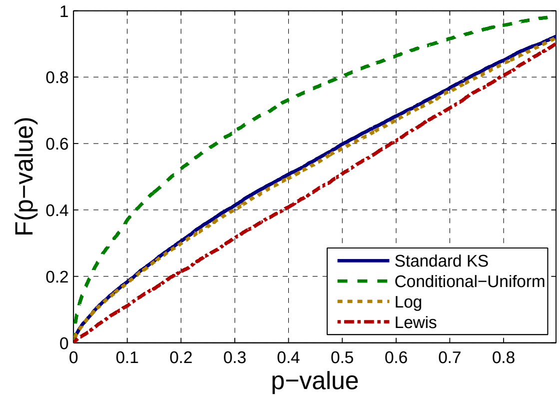


Figure 95 $EARMA$ (1) - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

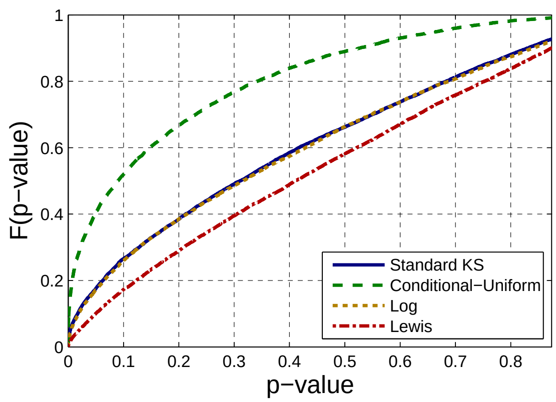


Figure 96 $EARMA$ (3) - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

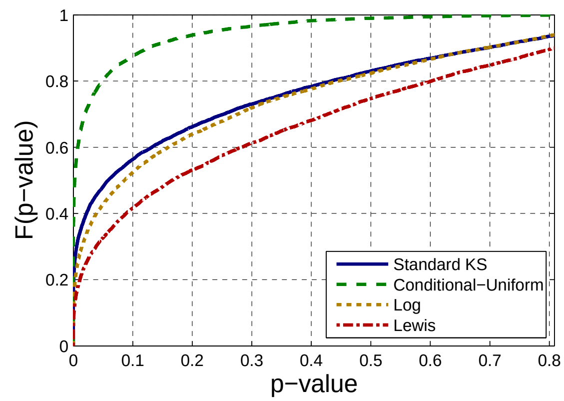


Figure 97 $EARMA$ (5.25) - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

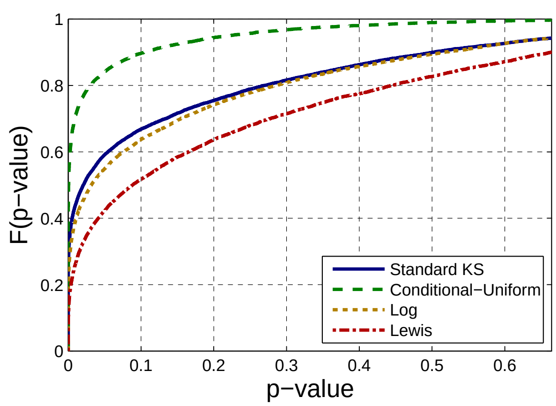


Figure 98 2 H_2 - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

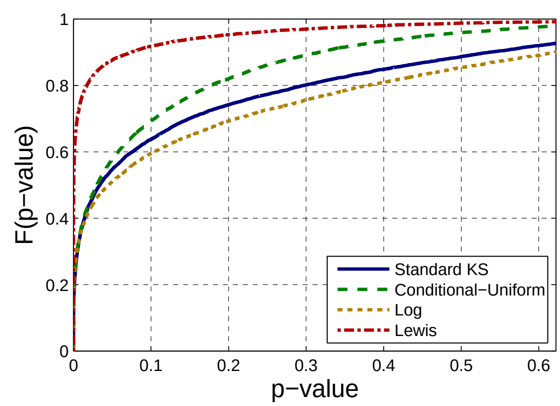


Figure 99 5 H_2 - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

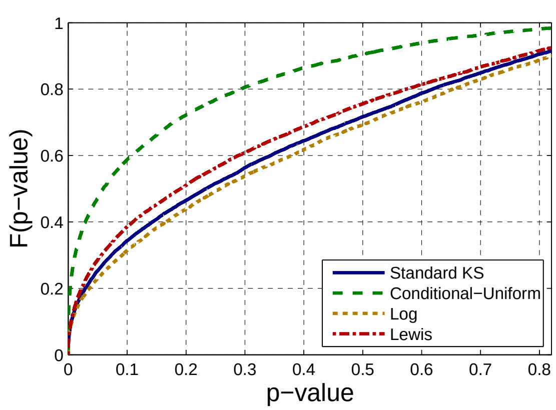


Figure 100 10 H_2 - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

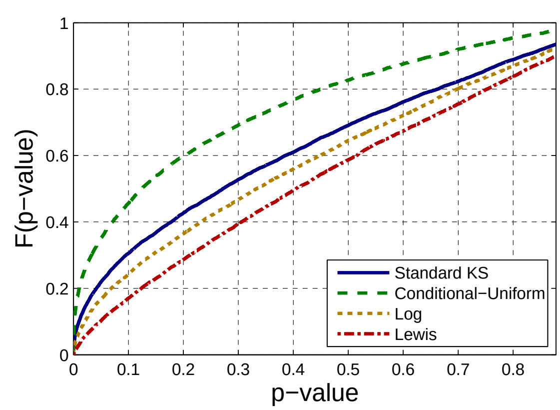


Figure 101 20 H_2 - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

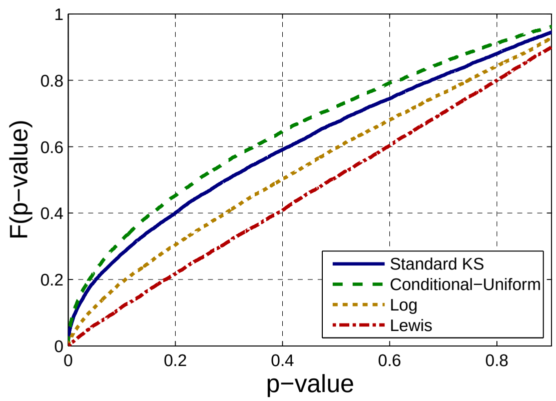


Figure 102 RRI $(H_2, p = 0.1)$ - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

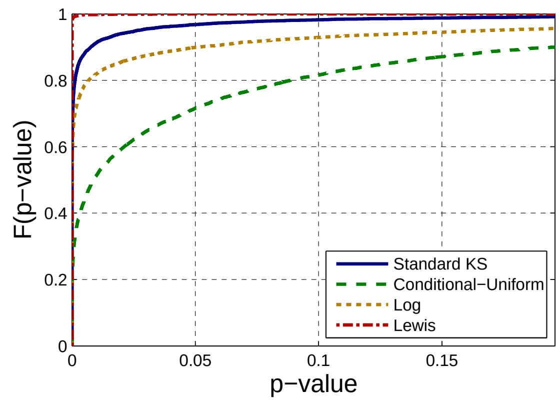


Figure 103 RRI $(H_2, p = 0.5)$ - ecdf of the p-values, based on 10^4 replications for $[0, 200]$

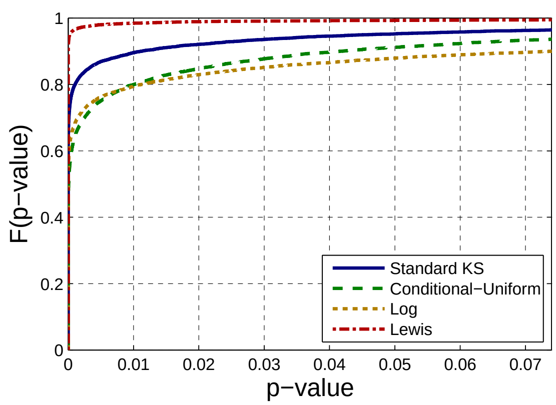
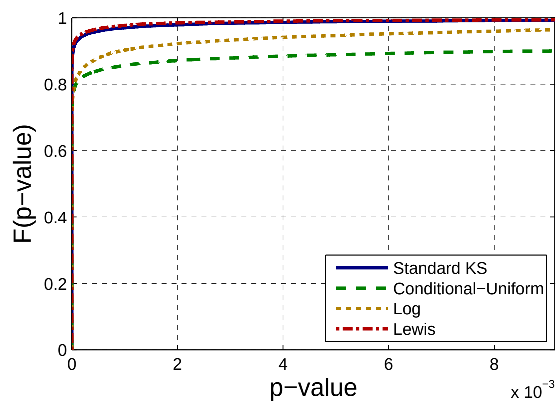


Figure 104 RRI $(H_2, p = 0.9)$ - ecdf of the p-values, based on 10^4 replications for $[0, 200]$



3.5 Power of Different Combinations of Alternative K-S Tests

Having examined the power of individual test in the previous section, we now consider the benefit of combining the results of multiple tests. Because we have observed that different tests can have advantages for different alternatives, we want to examine whether we can detect departures better when we use more than one test. Because we have basically the same hypothesis in the four alternative tests (whose results are given in Table 7), we expect to see highly-correlated p-values. Table 9 provides pairwise p-value correlations among the four alternative tests. We observe strong positive correlations between the p-values of the Standard KS test and the Log test, the Standard KS test and the Lewis test, and the Log test and the Lewis test. This suggests that including any of the three pairs in a selected combination of tests is not the best choice.

Table 9 Pairwise Correlations of p-values ($[0, 200]$): S stands for the standard test, C for the CU test, L for the Log test and Le for the Lewis test.

$Case$	$Subcase$	(S,C)	(S,L)	(S,Le)	(C,L)	(C,Le)	(L,Le)
Exp	$-$		0.30	0.48	0.29		0.43
E_k	$k = 2$	-0.05	0.49	0.26	-0.12	-0.04	0.39
	$k = 4$	-0.03	0.20	0.09	-0.07		0.22
	$k = 6$			0.06	-0.04		
H_2	$c^2 = 1.25$	0.06	0.30	0.41	0.24	0.15	0.46
	$c^2 = 1.5$	0.07	0.31	0.37	0.14	0.18	0.43
	$c^2 = 2$	0.04	0.23	0.19	0.02	0.15	0.30
	$c^2 = 4$		0.02		-0.06	0.05	0.09
	$c^2 = 10$					0.19	0.13
Z	$-$	-0.08	0.56	0.49	-0.13	-0.16	0.56
LN	$-$	-0.05	0.48	0.58	-0.06	-0.08	0.59
RRI	$p = 0.1$		0.31	0.50	0.28		0.44
	$p = 0.5$		0.28	0.45	0.16		0.39
	$p = 0.9$	-0.02	0.08	0.30	-0.02	-0.02	0.16
$EARMA$	0.25		0.22	0.41	0.31		0.37
	0.5		0.19	0.36	0.31		0.35
	1		0.17	0.32	0.28	-0.05	0.32
	3	-0.09	0.24	0.47	0.05	-0.14	0.41
	5.25	-0.11	0.13	0.33		-0.19	0.27
mH_2	$m = 2$	0.06	0.12	0.12	0.13	0.25	0.27
	$m = 5$	0.02	0.17	0.23	0.33	0.26	0.34
	$m = 10$		0.15	0.28	0.39	0.15	0.35
	$m = 20$		0.15	0.30	0.39	0.07	0.37
$RRI(H_2)$	$p = 0.1$		0.02		-0.04	0.09	0.11
	$p = 0.5$		0.06		0.05	0.10	0.14
	$p = 0.9$	-0.02	0.03	0.21	0.17	0.03	0.17

Tables 10 - 12 provide the result of applying three composite methods described Section 2.3. The results from these methods are similar, and we focus on the results from the Bonferroni method, which is the simplest method among them. Table 10 shows that these results are consistent with expectations based on the results of the individual tests in based on the results of the individual tests in Table 7. First, we do not gain power by applying the composite method when the interarrival-time distribution is non-exponential, as in the cases of renewal processes, mH_2 with $m = 2, 5$ and $RRI(H_2)$. The Lewis test alone has higher power.

that we do not gain much power by applying the composite method in the cases of renewal processes, mH_2 with $m = 2, 5$ and $RRI(H_2)$. Comparing to the results of the Lewis test in Table 7, we conclude that the Lewis test alone could have detected their departures with greater or equal power. However, for RRI the combination of the Conditional-Uniform, Log and Lewis test is the most powerful, whereas the combination of the Standard KS and Conditional-Uniform is. Given these results, our recommendation is to try the Lewis test first. If the test result is not convincing, we recommend trying the combination of the Standard KS test and the Conditional-Uniform test to detect the short-range as well as the long-range dependence.

As before, the story is more complex for the cases involving dependent exponential or near-exponential interarrival times. Table 10 shows that for RRI the combination (CU+Log+Lewis) $\equiv (0, 1, 1, 1)$ is the most powerful, whereas, for EARMA, the combination (standard+CU) $\equiv (1, 1, 0, 0)$ is most powerful. However, these composite tests are based on the standard test with known mean. We also considered the composite (Lilliefors (1969)+CU) test and found it to perform similarly to (standard+CU); see Table 13. We observe that for the dependent cases, the CU test alone has almost as much power as the best of these composite tests.

Overall, these results support using the Lewis test alone, especially to capture departures from the exponential interarrival time distribution, and then possibly the CU test to detect dependence. The composite (CU+Lewis) $\equiv (0, 1, 0, 1)$ test does consistently well, combining the advantages of both individual tests.

We also considered the composite (Lilliefors (1969)+CU) test and found it to perform similarly to (standard+CU); see Table 13. We observe that for the dependent cases, the CU test alone has almost as much power as the best of these composite tests.

Table 10 Results for all possible composite tests based on the four alternatives for the time interval ($[0, 200]$) using the Bonferroni procedure: The test order is (standard, CU, Log, Lewis) and is indicated by 1 if the test is included in the combination and 0 otherwise. Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

		2 tests						3 tests				4 tests
$Case$	$Subcase$	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
Exp	$-$	9506	9513	9564	9542	9523	9533	9494	9518	9502	9423	9542
E_k	$k = 2$	67	17	0	94	0	0	0	0	0	0	0
	$k = 4$	0	0	0	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	8755	8586	7958	8748	7934	7960	8015	8006	7947	8199	8104
	$c^2 = 1.5$	7148	6543	4536	7224	4645	4660	4903	4722	4723	5554	4962
	$c^2 = 2$	3853	2781	683	4125	831	830	980	809	815	1474	904
	$c^2 = 4$	181	81	2	449	6	5	8	2	2	3	2
	$c^2 = 10$	1	0	0	68	3	4	5	0	0	0	0
Z	$-$	2057	1164	401	2126	446	403	375	357	372	430	446
LN	$(1, 1)$	352	193	106	828	180	146	58	46	51	48	81
RRI	$p = 0.1$	8979	9034	9104	9075	8974	9088	8916	8965	8857	8767	8930
	$p = 0.5$	3726	4120	4181	4622	3786	4355	3333	3415	2931	2794	3075
	$p = 0.9$	2	5	5	14	0	12	0	0	0	0	0
$EARMA$	0.25	8725	9172	9379	8845	8893	9359	8936	9218	8873	8753	8954
	0.5	7649	8680	9084	7934	8016	9095	8150	8833	7914	7773	8068
	1	5982	7786	8316	6510	6483	8445	6717	7916	6258	6147	6458
	3	1440	4259	5250	2097	1796	5452	2000	4306	1570	1522	1727
	5.25	831	2732	3732	1383	967	3907	1044	2568	703	672	747
mH_2	$m = 2$	2973	3217	1122	3343	1300	1487	1431	1211	1078	1599	1181
	$m = 5$	5018	6811	6598	5714	5399	6978	5627	6301	5007	5042	5230
	$m = 10$	6116	7529	8006	6948	6973	8494	7160	7717	6446	6360	6684
	$m = 20$	7232	8043	8425	8118	8167	9048	8307	8268	7561	7466	7782
$RRI(H_2)$	$p = 0.1$	208	81	1	395	10	10	13	1	1	7	1
	$p = 0.5$	98	177	11	267	51	61	45	9	6	9	5
	$p = 0.9$	0	2	4	73	4	9	3	1	0	0	1

Table 11 Results for all possible composite tests based on the four alternatives for the time interval ($[0, 200]$) using Holm's procedure: The test order is (standard, CU, Log, Lewis) and is indicated by 1 if the test is included in the combination and 0 otherwise. Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

		2 tests						3 tests				4 tests
$Case$	$Subcase$	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
Exp	$-$	9501	9501	9541	9523	9512	9511	9451	9468	9470	9409	9521
E_k	$k = 2$	67	9	0	94	0	0	0	0	0	0	0
	$k = 4$	0	0	0	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	8730	8549	7891	8717	7888	7901	7922	7887	7846	8144	8044
	$c^2 = 1.5$	7102	6469	4467	7192	4581	4578	4750	4571	4594	5446	4881
	$c^2 = 2$	3754	2672	659	4070	816	797	934	753	765	1391	877
	$c^2 = 4$	160	68	1	419	6	4	6	1	2	3	2
	$c^2 = 10$	1	0	0	63	3	4	4	0	0	0	0
Z	$-$	2039	1004	347	2101	435	354	314	280	314	397	398
LN	$(1, 1)$	342	128	70	816	176	115	56	38	46	41	71
RRI	$p = 0.1$	8961	9002	9066	9042	8956	9042	8854	8890	8799	8743	8897
	$p = 0.5$	3624	4010	4029	4507	3651	4217	3134	3231	2747	2648	2965
	$p = 0.9$	2	4	4	11	0	9	0	0	0	0	0
$EARMA$	0.25	8709	9155	9354	8803	8881	9341	8895	9173	8834	8725	8941
	0.5	7608	8648	9065	7897	7993	9069	8101	8799	7871	7740	8041
	1	5943	7754	8285	6444	6451	8406	6657	7848	6203	6113	6423
	3	1379	4206	5188	2051	1746	5392	1926	4240	1500	1473	1688
	5.25	791	2679	3673	1328	916	3849	961	2479	648	631	713
mH_2	$m = 2$	2880	3150	1078	3262	1253	1450	1365	1151	1002	1517	1145
	$m = 5$	4962	6768	6535	5648	5332	6913	5515	6204	4907	4973	5173
	$m = 10$	6065	7486	7970	6889	6946	8453	7103	7654	6390	6313	6654
	$m = 20$	7181	8005	8395	8063	8140	9017	8250	8217	7516	7420	7747
$RRI(H_2)$	$p = 0.1$	189	75	1	369	10	9	13	1	1	7	1
	$p = 0.5$	86	166	10	242	48	59	38	9	6	9	5
	$p = 0.9$	0	2	3	71	4	8	2	1	0	0	0

Table 12 Results for all possible composite tests based on the four alternatives for the time interval ($[0, 200]$) using Simes' procedure: The test order is (standard, CU, Log, Lewis) and is indicated by 1 if the test is included in the combination and 0 otherwise. Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

		2 tests						3 tests				4 tests
$Case$	$Subcase$	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
Exp	$-$	9501	9501	9541	9523	9512	9511	9419	9437	9438	9382	9479
E_k	$k = 2$	67	9	0	94	0	0	0	0	0	0	0
	$k = 4$	0	0	0	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	8730	8549	7891	8717	7888	7901	7846	7820	7760	8086	7917
	$c^2 = 1.5$	7102	6469	4467	7192	4581	4578	4647	4482	4498	5341	4704
	$c^2 = 2$	3754	2672	659	4070	816	797	894	728	736	1343	816
	$c^2 = 4$	160	68	1	419	6	4	5	1	1	3	2
	$c^2 = 10$	1	0	0	63	3	4	4	0	0	0	0
Z	$-$	2039	1004	347	2101	435	354	278	250	277	375	330
LN	$(1, 1)$	342	128	70	816	176	115	53	34	38	36	56
RRI	$p = 0.1$	8961	9002	9066	9042	8956	9042	8799	8841	8748	8706	8816
	$p = 0.5$	3624	4010	4029	4507	3651	4217	3024	3124	2641	2544	2794
	$p = 0.9$	2	4	4	11	0	9	0	0	0	0	0
$EARMA$	0.25	8709	9155	9354	8803	8881	9341	8859	9148	8806	8676	8885
	0.5	7608	8648	9065	7897	7993	9069	8055	8769	7841	7704	7967
	1	5943	7754	8285	6444	6451	8406	6583	7797	6136	6041	6336
	3	1379	4206	5188	2051	1746	5392	1881	4182	1452	1428	1608
	5.25	791	2679	3673	1328	916	3849	915	2417	611	599	669
mH_2	$m = 2$	2880	3150	1078	3262	1253	1450	1309	1116	965	1451	1071
	$m = 5$	4962	6768	6535	5648	5332	6913	5448	6137	4849	4911	5053
	$m = 10$	6065	7486	7970	6889	6946	8453	7054	7616	6350	6252	6561
	$m = 20$	7181	8005	8395	8063	8140	9017	8192	8181	7472	7364	7664
$RRI(H_2)$	$p = 0.1$	189	75	1	369	10	9	13	1	1	7	1
	$p = 0.5$	86	166	10	242	48	59	38	8	6	7	4
	$p = 0.9$	0	2	3	71	4	8	2	1	0	0	0

Table 13 Results for individual and composite tests based on the Standard KS, CU, and Lilliefors tests for the time interval ($[0, 200]$) using the Bonferroni procedure: Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

$Case$	$Subcase$	Standard	Lilliefors	CU	(Standard+CU)	(Lilliefors+CU)
Exp	$-$	9479	9482	9503	9506	9532
E_k	$k=2$	11	0	9991	67	0
	$k=4$	0	0	10000	0	0
	$k=6$	0	0	10000	0	0
H_2	$c^2=1.25$	8757	8239	8988	8755	8423
	$c^2=1.5$	7049	5379	8428	7148	5940
	$c^2=2$	3695	1398	7186	3853	1860
	$c^2=4$	231	23	3551	181	33
	$c^2=10$	13	26	650	1	12
Z	$-$	1200	210	9412	2057	365
LN	$(1,1)$	88	9	9525	352	42
RRI	$p=0.1$	9093	8971	9044	8979	8921
	$p=0.5$	4631	3775	5516	3726	3298
	$p=0.9$	14	2	826	2	0
$EARMA$	0.25	9260	9460	8536	8725	8873
	0.5	8848	9420	7433	7649	7990
	1	8244	9102	5964	5982	6509
	3	5196	6702	1977	1440	1788
	5.25	4088	5987	1594	831	1041
mH_2	$m=2$	4531	2440	4231	2973	2214
	$m=5$	7474	7944	5282	5018	5658
	$m=10$	7812	9177	6496	6116	7027
	$m=20$	8034	9421	7751	7232	8174
$RRI(H_2)$	$p=0.1$	322	41	2841	208	50
	$p=0.5$	478	94	884	98	78
	$p=0.9$	9	1	772	0	2

3.6 Power of Alternative Tests Other than the Two-sided K-S Tests

Given transformed interarrival times, many standard statistical tests can be applied to see whether they are from a PP. Brown et al. (2005) elected to apply the standard *Kolmogorov-Smirnov* (KS) test, so we have focused on the KS test in this work. There are other tests of fit that are based on the ecdf defined in equation (1). Some of the well-known ones are the one-sided Kolmogorov-Smirnov test, Kuiper test, Cramer-von Mises test, Anderson-Darling test and Watson test, all of which are based on the vertical differences between $F_n(x)$ and $F(x)$ (D'Agostino and Stephens 1986). In this section, we provide results of two of these tests: the one-sided KS tests and the Anderson-Darling test, e.g., see Stephens (1974).

3.6.1 One-Sided K-S Tests

The one-sided KS tests measures the largest vertical difference when $F_n(x)$ is greater/smaller than $F(x)$ and are defined as follows:

$$D_n^+ \equiv \sup_x \{F_n(x) - F(x)\}, \quad D_n^- \equiv \sup_x \{F(x) - F_n(x)\}. \tag{8}$$

We note that with these definitions, D_n in (9) can be rewritten as $D_n = \max\{D_n^+, D_n^-\}$.

Tables 14 and 15 provide the results of applying one-sided KS tests. The plots of the ecdf and cdf in Section 3.2 suggest that these alternative tests will perform very well, especially for the Lewis transformation. That is, we expect that the correct one-sided Lewis KS test will have even greater power than the Lewis KS test itself, whereas the wrong one-sided Lewis KS test will have essentially no power at all, like the CU KS test. That is what we observe in these two tables. However, since different one-sided KS tests are needed for these two different alternative hypotheses and since there is not likely to be such a narrow range of alternative applications in applications, the standard KS test or possibly the (also symmetric) Anderson-Darling test seems more appropriate.

Table 14 Performance of alternative KS tests of a rate-1 Poisson process for the time interval $[0, 200]$: Number of One-sided KS tests passed (denoted by $\#P$) at significance level 0.05 out of $10,000$ replications and the average p-values (denoted by $E[p\text{-value}]$) with associated 95% confidence intervals.

		Standard-smaller		Standard-larger		CU-smaller		CU-larger	
$Case$	$Subcase$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$
Exp	$-$	9510	0.51 ± 0.0057	9479	0.49 ± 0.0056	9505	0.50 ± 0.0057	9515	0.50 ± 0.0056
E_k	$k = 2$	4	0.00 ± 0.0000	9081	0.26 ± 0.0036	9973	0.65 ± 0.0047	9964	0.65 ± 0.0046
	$k = 4$	0	0.00 ± 0.0000	886	0.02 ± 0.0005	10000	0.78 ± 0.0033	10000	0.78 ± 0.0033
	$k = 6$	0	0.00 ± 0.0000	1	0.00 ± 0.0001	10000	0.83 ± 0.0026	10000	0.83 ± 0.0026
H_2	$c^2 = 1.25$	9872	0.63 ± 0.0050	8091	0.32 ± 0.0054	9127	0.45 ± 0.0059	9125	0.45 ± 0.0058
	$c^2 = 1.5$	9972	0.67 ± 0.0043	5950	0.18 ± 0.0043	8703	0.42 ± 0.0060	8698	0.41 ± 0.0060
	$c^2 = 2$	9989	0.64 ± 0.0040	2667	0.06 ± 0.0023	7979	0.36 ± 0.0061	7977	0.36 ± 0.0061
	$c^2 = 4$	9994	0.58 ± 0.0037	125	0.00 ± 0.0003	5995	0.26 ± 0.0061	5903	0.26 ± 0.0060
	$c^2 = 10$	10000	0.71 ± 0.0030	8	0.00 ± 0.0001	3854	0.18 ± 0.0058	3901	0.18 ± 0.0058
Z	$-$	606	0.01 ± 0.0006	8873	0.28 ± 0.0042	9470	0.54 ± 0.0057	9520	0.55 ± 0.0057
LN	$(1, 1)$	19	0.00 ± 0.0001	8035	0.28 ± 0.0051	9471	0.51 ± 0.0057	9562	0.52 ± 0.0056
RRI	$p = 0.1$	9191	0.46 ± 0.0058	9128	0.45 ± 0.0058	9147	0.46 ± 0.0059	9168	0.46 ± 0.0059
	$p = 0.5$	6445	0.26 ± 0.0057	6606	0.28 ± 0.0059	7121	0.33 ± 0.0063	6989	0.31 ± 0.0062
	$p = 0.9$	1882	0.08 ± 0.0039	2617	0.12 ± 0.0050	4109	0.23 ± 0.0068	4087	0.23 ± 0.0068
$EARMA$	0.25	9338	0.50 ± 0.0059	9347	0.50 ± 0.0059	8876	0.43 ± 0.0060	8816	0.44 ± 0.0061
	0.5	9093	0.50 ± 0.0063	9076	0.49 ± 0.0063	8076	0.38 ± 0.0062	8214	0.39 ± 0.0062
	1	8594	0.47 ± 0.0065	8754	0.48 ± 0.0065	7399	0.35 ± 0.0065	7318	0.34 ± 0.0064
	3	6938	0.41 ± 0.0075	7400	0.48 ± 0.0078	5235	0.25 ± 0.0064	4834	0.22 ± 0.0061
	5.25	6265	0.37 ± 0.0074	7008	0.44 ± 0.0076	5124	0.28 ± 0.0071	4685	0.25 ± 0.0067
mH_2	$m = 2$	9953	0.72 ± 0.0043	3672	0.11 ± 0.0038	6359	0.29 ± 0.0063	6334	0.29 ± 0.0063
	$m = 5$	9393	0.61 ± 0.0062	7175	0.34 ± 0.0065	7062	0.35 ± 0.0067	6942	0.35 ± 0.0067
	$m = 10$	8933	0.55 ± 0.0067	8017	0.43 ± 0.0068	7730	0.40 ± 0.0068	7673	0.40 ± 0.0067
	$m = 20$	8792	0.52 ± 0.0068	8436	0.47 ± 0.0069	8381	0.44 ± 0.0065	8432	0.45 ± 0.0065
$RRI(H_2)$	$p = 0.1$	9977	0.57 ± 0.0041	188	0.00 ± 0.0005	5500	0.24 ± 0.0060	5496	0.24 ± 0.0060
	$p = 0.5$	9360	0.49 ± 0.0055	510	0.01 ± 0.0014	4157	0.20 ± 0.0062	4134	0.20 ± 0.0061
	$p = 0.9$	4321	0.20 ± 0.0058	1682	0.12 ± 0.0060	3763	0.26 ± 0.0077	3801	0.26 ± 0.0076

Table 15 Performance of alternative KS tests of a rate-1 Poisson process for the time interval $[0, 200]$: Number of One-sided KS tests passed (denoted by $\#P$) at significance level 0.05 out of $10,000$ replications and the average p-values (denoted by $E[p\text{-value}]$) with associated 95% confidence intervals.

		Log-smaller		Log-larger		Lewis-smaller		Lewis-larger	
$Case$	$Subcase$	$\#P$	$E[p\text{-value}]$	$\#P$	$E[p\text{-value}]$	$\#P$	$E[p\text{-value}]$	$\#P$	$E[p\text{-value}]$
Exp	$-$	9482	0.50 ± 0.0056	9507	0.50 ± 0.0057	9504	0.50 ± 0.0057	9505	0.50 ± 0.0057
E_k	$k = 2$	7	0.00 ± 0.0001	9211	0.26 ± 0.0035	0	0.00 ± 0.0000	10000	0.98 ± 0.0007
	$k = 4$	0	0.00 ± 0.0000	881	0.02 ± 0.0005	0	0.00 ± 0.0000	10000	1.00 ± 0.0001
	$k = 6$	0	0.00 ± 0.0000	1	0.00 ± 0.0001	0	0.00 ± 0.0000	10000	1.00 ± 0.0001
H_2	$c^2 = 1.25$	9850	0.63 ± 0.0051	8022	0.32 ± 0.0056	9950	0.77 ± 0.0046	6512	0.19 ± 0.0043
	$c^2 = 1.5$	9943	0.65 ± 0.0045	6161	0.20 ± 0.0048	9994	0.88 ± 0.0031	2972	0.06 ± 0.0023
	$c^2 = 2$	9982	0.63 ± 0.0040	3126	0.09 ± 0.0032	10000	0.94 ± 0.0017	325	0.01 ± 0.0006
	$c^2 = 4$	9997	0.58 ± 0.0034	659	0.02 ± 0.0013	10000	0.98 ± 0.0009	1	0.00 ± 0.0000
	$c^2 = 10$	10000	0.71 ± 0.0028	418	0.01 ± 0.0013	10000	0.98 ± 0.0007	0	0.00 ± 0.0000
Z	$-$	1107	0.02 ± 0.0013	8762	0.30 ± 0.0044	241	0.00 ± 0.0005	9238	0.67 ± 0.0067
LN	$(1, 1)$	249	0.01 ± 0.0004	8219	0.30 ± 0.0052	22	0.00 ± 0.0001	9316	0.53 ± 0.0061
RRI	$p = 0.1$	9189	0.46 ± 0.0058	9162	0.46 ± 0.0058	9154	0.45 ± 0.0058	9186	0.46 ± 0.0058
	$p = 0.5$	6713	0.28 ± 0.0058	6898	0.29 ± 0.0058	6081	0.22 ± 0.0051	7017	0.32 ± 0.0063
	$p = 0.9$	2205	0.09 ± 0.0042	2768	0.12 ± 0.0051	753	0.02 ± 0.0015	4076	0.25 ± 0.0080
$EARMA$	0.25	9320	0.49 ± 0.0059	9294	0.51 ± 0.0060	9453	0.50 ± 0.0057	9493	0.51 ± 0.0058
	0.5	9108	0.49 ± 0.0063	9100	0.51 ± 0.0063	9422	0.49 ± 0.0058	9467	0.52 ± 0.0058
	1	8588	0.46 ± 0.0066	8816	0.50 ± 0.0065	8995	0.45 ± 0.0060	9275	0.53 ± 0.0062
	3	7003	0.39 ± 0.0072	7675	0.45 ± 0.0073	7496	0.41 ± 0.0071	8336	0.56 ± 0.0075
	5.25	6111	0.36 ± 0.0075	7453	0.44 ± 0.0073	6338	0.29 ± 0.0063	8472	0.55 ± 0.0071
mH_2	$m = 2$	9899	0.70 ± 0.0045	4150	0.14 ± 0.0046	10000	0.93 ± 0.0022	818	0.02 ± 0.0012
	$m = 5$	9446	0.60 ± 0.0062	7418	0.35 ± 0.0065	9941	0.77 ± 0.0047	6207	0.20 ± 0.0045
	$m = 10$	9285	0.55 ± 0.0064	8295	0.43 ± 0.0066	9775	0.63 ± 0.0056	8522	0.35 ± 0.0055
	$m = 20$	9227	0.52 ± 0.0062	8859	0.48 ± 0.0063	9659	0.56 ± 0.0056	9169	0.44 ± 0.0056
$RRI(H_2)$	$p = 0.1$	9979	0.58 ± 0.0038	724	0.02 ± 0.0015	10000	0.97 ± 0.0012	3	0.00 ± 0.0000
	$p = 0.5$	9514	0.51 ± 0.0055	1123	0.04 ± 0.0025	9940	0.84 ± 0.0040	59	0.00 ± 0.0004
	$p = 0.9$	4545	0.24 ± 0.0067	1846	0.09 ± 0.0045	3880	0.16 ± 0.0050	1391	0.08 ± 0.0010

3.6.2 Anderson-Darling Tests

The Anderson-Darling (AD) statistic is defined as

$$A^2 \equiv n \int_{-\infty}^{\infty} \{F_n(x) - F(x)\}^2 \psi(x) dF(x), \tag{9}$$

with the weight function $\psi(x) \equiv [\{F(x)\}\{1 - F(x)\}]^{-1}$. That is, the AD distance places more weight on observations in the tails of the distribution. Like all other ecdf tests, the AD test makes use of the fact that, when given a hypothesized underlying distribution F and assuming the data does arise from this distribution, the data can be transformed to a uniform distribution by $Z = F(X)$. Furthermore, as shown in Proposition 1 of the main paper and claimed on Section 4.2.3 of D'Agostino and Stephens (1986), the corresponding vertical differences in the ecdf does not change when such data transformation takes place.

Table 16 shows the results of applying the alternative AD tests. The results for the AD test are very similar to the results for the KS test. In fact, the power of the AD test against non-exponential interarrival times tends to slightly higher than that of the KS test in Table 7 for all except the powerless CU test. The power of the AD Lewis test is still greater than the power of the AD Log test for these cases.

Table 16 Performance of alternative AD tests of a rate-1 Poisson process for the time interval $[0, 200]$: Number of AD tests passed (denoted by $\#P$) at significance level 0.05 out of $10,000$ replications and the average p-values (denoted by $E[p\text{-value}]$) with associated 95% confidence intervals.

		Standard		CU		Log		Lewis	
$Case$	$Subcase$	$\#P$	$E[p\text{-value}]$	$\#P$	$E[p\text{-value}]$	$\#P$	$E[p\text{-value}]$	$\#P$	$E[p\text{-value}]$
Exp	$-$	9496	0.50 ± 0.0057	9517	0.50 ± 0.0056	9493	0.50 ± 0.0057	9515	0.50 ± 0.0057
E_k	$k = 2$	0	0.00 ± 0.0000	9978	0.76 ± 0.0048	1	0.00 ± 0.0000	0	0.00 ± 0.0000
	$k = 4$	0	0.00 ± 0.0000	10000	0.93 ± 0.0026	0	0.00 ± 0.0000	0	0.00 ± 0.0000
	$k = 6$	0	0.00 ± 0.0000	10000	0.97 ± 0.0015	0	0.00 ± 0.0000	0	0.00 ± 0.0000
H_2	$c^2 = 1.25$	8633	0.37 ± 0.0056	9091	0.41 ± 0.0055	8554	0.38 ± 0.0057	7125	0.28 ± 0.0056
	$c^2 = 1.5$	6671	0.22 ± 0.0047	8594	0.34 ± 0.0052	6782	0.23 ± 0.0048	3362	0.10 ± 0.0036
	$c^2 = 2$	2659	0.05 ± 0.0021	7630	0.25 ± 0.0045	3266	0.08 ± 0.0027	371	0.01 ± 0.0010
	$c^2 = 4$	9	0.00 ± 0.0001	4191	0.09 ± 0.0026	366	0.01 ± 0.0006	2	0.00 ± 0.0000
	$c^2 = 10$	0	0.00 ± 0.0000	865	0.02 ± 0.0009	160	0.00 ± 0.0005	0	0.00 ± 0.0000
Z	$-$	325	0.01 ± 0.0004	9402	0.56 ± 0.0062	393	0.01 ± 0.0005	59	0.00 ± 0.0002
LN	$(1, 1)$	7	0.00 ± 0.0001	9488	0.52 ± 0.0058	49	0.00 ± 0.0002	7	0.00 ± 0.0001
RRI	$p = 0.1$	9161	0.42 ± 0.0055	9105	0.43 ± 0.0056	9125	0.42 ± 0.0055	8889	0.40 ± 0.0055
	$p = 0.5$	5358	0.12 ± 0.0031	6039	0.17 ± 0.0039	5831	0.14 ± 0.0033	4068	0.09 ± 0.0026
	$p = 0.9$	24	0.00 ± 0.0001	1012	0.02 ± 0.0017	136	0.00 ± 0.0002	3	0.00 ± 0.0000
$EARMA$	0.25	9126	0.46 ± 0.0058	8645	0.36 ± 0.0054	9151	0.46 ± 0.0059	9461	0.50 ± 0.0057
	0.5	8678	0.41 ± 0.0059	7748	0.27 ± 0.0048	8693	0.42 ± 0.0060	9392	0.49 ± 0.0058
	1	7879	0.34 ± 0.0058	6303	0.19 ± 0.0042	7971	0.35 ± 0.0059	8888	0.43 ± 0.0059
	3	4050	0.14 ± 0.0045	2320	0.05 ± 0.0018	4399	0.14 ± 0.0045	5553	0.21 ± 0.0054
	5.25	3822	0.12 ± 0.0041	1732	0.04 ± 0.0018	4134	0.12 ± 0.0039	5488	0.20 ± 0.0050
mH_2	$m = 2$	3642	0.09 ± 0.0030	4775	0.11 ± 0.0031	4253	0.12 ± 0.0037	848	0.02 ± 0.0017
	$m = 5$	6980	0.27 ± 0.0055	5620	0.16 ± 0.0040	7368	0.30 ± 0.0057	6689	0.27 ± 0.0056
	$m = 10$	7374	0.32 ± 0.0058	6668	0.23 ± 0.0050	8038	0.36 ± 0.0059	8850	0.43 ± 0.0059
	$m = 20$	7665	0.34 ± 0.0060	7871	0.32 ± 0.0056	8632	0.42 ± 0.0060	9352	0.49 ± 0.0058
$RRI(H_2)$	$p = 0.1$	30	0.00 ± 0.0001	3458	0.07 ± 0.0022	419	0.01 ± 0.0007	1	0.00 ± 0.0000
	$p = 0.5$	118	0.00 ± 0.0003	1179	0.02 ± 0.0012	599	0.01 ± 0.0008	59	0.00 ± 0.0003
	$p = 0.9$	6	0.00 ± 0.0001	792	0.06 ± 0.0045	99	0.00 ± 0.0003	4	0.00 ± 0.0000

4 Extending Section 3: The Effect of Longer Intervals and Subintervals

4.1 Experiments with the Interval $[0, 2000]$

In Section 3, we compared the power of alternative test on the fixed interval $[0, 200]$. In this section, we examine the effect of a larger sample size by considering the fixed interval $[0, 2000]$. The summary statistics of the untransformed and transformed arrivals in Tables 17 and 18 are similar to those in 5 and 6. We note that the high value of $E[X]$ for $RRI(H_2)$ with $p = 0.9$ in Table 5 now looks more normal.

Table 17 Summary statistics of different arrival processes on $[0, 2000]$ with associated 95% confidence intervals. All results are based on 10000 replications. $\{X_k : k \geq 1\}$ are the interarrival times where n is the number of arrivals in $[0, 2000]$.

$Case$	$Subcase$	$E[X]$	$c^2[X]$	$Min[n]$	$Max[n]$	$E[n]$	t_n	$Min[X]$	$Max[X]$
Exp	$-$	1.00 ± 0.0004	1.00 ± 0.0009	1835	2186	1999.4 ± 0.9	1999 ± 0.02	0.0005 ± 0.00001	8.2 ± 0.03
E_k	$k = 2$	1.00 ± 0.0003	0.50 ± 0.0004	1871	2118	1999.8 ± 0.6	1999 ± 0.01	0.0139 ± 0.00015	5.5 ± 0.02
	$k = 4$	1.00 ± 0.0002	0.25 ± 0.0002	1908	2091	2001.0 ± 0.4	1999 ± 0.01	0.0770 ± 0.00051	3.7 ± 0.01
	$k = 6$	1.00 ± 0.0001	0.17 ± 0.0001	1937	2066	2000.0 ± 0.4	1999 ± 0.01	0.1373 ± 0.00084	3.0 ± 0.01
H_2	$c^2 = 1.25$	1.00 ± 0.0005	1.25 ± 0.0014	1830	2183	2000.3 ± 1.0	1999 ± 0.02	0.0004 ± 0.00001	10.6 ± 0.04
	$c^2 = 1.5$	1.00 ± 0.0005	1.50 ± 0.0019	1795	2231	2000.6 ± 1.1	1999 ± 0.03	0.0004 ± 0.00001	12.5 ± 0.05
	$c^2 = 2$	1.00 ± 0.0006	1.99 ± 0.0029	1719	2225	2000.1 ± 1.3	1999 ± 0.04	0.0004 ± 0.00001	15.6 ± 0.06
	$c^2 = 4$	1.00 ± 0.0009	3.98 ± 0.0077	1642	2324	2000.0 ± 1.8	1998 ± 0.07	0.0003 ± 0.00001	26.6 ± 0.11
	$c^2 = 10$	1.00 ± 0.0014	9.87 ± 0.0289	1416	2497	1999.5 ± 2.8	1995 ± 0.17	0.0003 ± 0.00001	53.9 ± 0.26
Z	$-$	1.00 ± 0.0004	0.99 ± 0.0042	1814	2165	1999.0 ± 0.9	1999 ± 0.03	0.0021 ± 0.00004	17.9 ± 0.11
LN	$-$	1.00 ± 0.0004	1.00 ± 0.0022	1815	2179	2000.4 ± 0.9	1999 ± 0.02	0.0411 ± 0.00023	12.9 ± 0.08
RRI	$p = 0.1$	1.00 ± 0.0005	1.00 ± 0.0010	1821	2203	2000.1 ± 1.0	1999 ± 0.02	0.0005 ± 0.00001	8.1 ± 0.02
	$p = 0.5$	1.00 ± 0.0008	1.00 ± 0.0015	1708	2277	1999.6 ± 1.5	1999 ± 0.02	0.0010 ± 0.00002	7.5 ± 0.02
	$p = 0.9$	1.01 ± 0.0020	0.98 ± 0.0035	1321	2734	2000.9 ± 3.8	1999 ± 0.02	0.0051 ± 0.00010	5.9 ± 0.02
$EARMA$	0.25	1.00 ± 0.0005	1.00 ± 0.0010	1806	2184	1999.6 ± 1.1	1999 ± 0.02	0.0005 ± 0.00001	8.1 ± 0.03
	0.5	1.00 ± 0.0006	1.00 ± 0.0010	1761	2255	2000.0 ± 1.2	1999 ± 0.02	0.0005 ± 0.00001	8.1 ± 0.03
	1	1.00 ± 0.0008	1.00 ± 0.0011	1698	2277	2000.0 ± 1.5	1999 ± 0.02	0.0005 ± 0.00001	8.0 ± 0.03
	3	1.00 ± 0.0012	0.99 ± 0.0017	1482	2450	2001.0 ± 2.3	1999 ± 0.02	0.0020 ± 0.00004	8.0 ± 0.03
	5.25	1.01 ± 0.0015	0.99 ± 0.0019	1420	2530	2000.8 ± 3.0	1999 ± 0.02	0.0005 ± 0.00001	7.6 ± 0.03
mH_2	$m = 2$	1.00 ± 0.0009	2.49 ± 0.0067	1658	2341	1999.8 ± 1.7	1998 ± 0.06	0.0004 ± 0.00001	23.6 ± 0.11
	$m = 5$	1.00 ± 0.0009	1.38 ± 0.0031	1686	2345	1999.3 ± 1.7	1999 ± 0.03	0.0004 ± 0.00001	15.1 ± 0.10
	$m = 10$	1.00 ± 0.0009	1.14 ± 0.0015	1610	2353	1999.7 ± 1.7	1999 ± 0.02	0.0005 ± 0.00001	10.5 ± 0.05
	$m = 20$	1.00 ± 0.0009	1.06 ± 0.0011	1638	2337	2000.4 ± 1.7	1999 ± 0.02	0.0005 ± 0.00001	9.1 ± 0.03
$RRI(H_2)$	$p = 0.1$	1.00 ± 0.0010	3.98 ± 0.0086	1644	2327	1997.9 ± 2.0	1997 ± 0.07	0.0004 ± 0.00001	26.2 ± 0.11
	$p = 0.5$	1.00 ± 0.0016	3.94 ± 0.0129	1429	2614	2000.4 ± 3.0	1998 ± 0.07	0.0006 ± 0.00001	23.6 ± 0.11
	$p = 0.9$	1.05 ± 0.0070	3.69 ± 0.0257	84	3164	1997.4 ± 7.5	1998 ± 0.07	0.0052 ± 0.00363	17.0 ± 0.11

Table 19 provides the number of KS tests passed (out of 10000 replications) as well as the average p-value with associated 95% confidence intervals. Comparing to the results given in Table 7, we see that the power of all of the tests increases in the sample size. For instance, there are sample paths of H_2 with $c^2 > 1.25$, Z and LN passing the Lewis test in Table 7, even though none passed the Lewis test in Table 19. The observations we draw in Section 3 still applies: the Lewis test is the most powerful test for the arrival processes with E_k, H_2, Z, LN, RRI, $RRI(H_2)$ and mH_2 with smaller values of m, whereas the Conditional-Uniform test performs the best for the rest of the arrival processes. Also, we comment that Table 20 shows that the Lillifors test still outperforms the Standard KS test and the KS test with estimated

Table 18 Average and c^2 of transformed interarrival times on $[0, 2000]$ with associated 95% confidence intervals. All results are based on 10000 replications.

$Case$	$Subcase$	$E[X^{CU}]$	$c^2[X^{CU}]$	$E[X^{Log}]$	$c^2[X^{Log}]$	$E[X^{Lewis}]$	$c^2[X^{Lewis}]$
Exp	$-$	0.50 ± 0.0001	0.33 ± 0.0002	1.00 ± 0.0004	1.00 ± 0.0009	0.50 ± 0.0001	0.33 ± 0.0002
E_k	$k = 2$	0.50 ± 0.0001	0.33 ± 0.0001	1.00 ± 0.0003	0.50 ± 0.0004	0.62 ± 0.0001	0.16 ± 0.0001
	$k = 4$	0.50 ± 0.0001	0.33 ± 0.0001	1.00 ± 0.0002	0.25 ± 0.0002	0.73 ± 0.0001	0.08 ± 0.0001
	$k = 6$	0.50 ± 0.0001	0.33 ± 0.0001	1.00 ± 0.0002	0.17 ± 0.0000	0.77 ± 0.0001	0.05 ± 0.0001
H_2	$c^2 = 1.25$	0.50 ± 0.0001	0.33 ± 0.0002	1.00 ± 0.0005	1.25 ± 0.0014	0.47 ± 0.0001	0.36 ± 0.0002
	$c^2 = 1.5$	0.50 ± 0.0002	0.33 ± 0.0003	1.00 ± 0.0005	1.49 ± 0.0019	0.45 ± 0.0001	0.38 ± 0.0002
	$c^2 = 2$	0.50 ± 0.0002	0.33 ± 0.0003	1.00 ± 0.0006	1.99 ± 0.0029	0.42 ± 0.0002	0.40 ± 0.0003
	$c^2 = 4$	0.50 ± 0.0003	0.33 ± 0.0004	1.00 ± 0.0009	3.95 ± 0.0076	0.35 ± 0.0002	0.44 ± 0.0004
	$c^2 = 10$	0.50 ± 0.0004	0.33 ± 0.0007	1.00 ± 0.0014	9.70 ± 0.0283	0.30 ± 0.0003	0.43 ± 0.0005
Z	$-$	0.50 ± 0.0001	0.33 ± 0.0002	1.00 ± 0.0004	0.99 ± 0.0041	0.58 ± 0.0002	0.19 ± 0.0001
LN	$-$	0.50 ± 0.0001	0.33 ± 0.0002	1.00 ± 0.0004	0.99 ± 0.0022	0.56 ± 0.0002	0.18 ± 0.0001
RRI	$p = 0.1$	0.50 ± 0.0001	0.33 ± 0.0002	1.00 ± 0.0005	1.00 ± 0.0010	0.50 ± 0.0001	0.33 ± 0.0002
	$p = 0.5$	0.50 ± 0.0002	0.33 ± 0.0004	1.00 ± 0.0007	1.00 ± 0.0015	0.50 ± 0.0002	0.33 ± 0.0004
	$p = 0.9$	0.50 ± 0.0005	0.33 ± 0.0009	1.00 ± 0.0018	0.98 ± 0.0036	0.50 ± 0.0005	0.33 ± 0.0009
$EARMA$	0.25	0.50 ± 0.0002	0.33 ± 0.0003	1.00 ± 0.0005	1.00 ± 0.0010	0.50 ± 0.0001	0.33 ± 0.0002
	0.5	0.50 ± 0.0002	0.33 ± 0.0003	1.00 ± 0.0006	1.00 ± 0.0010	0.50 ± 0.0001	0.33 ± 0.0002
	1	0.50 ± 0.0002	0.33 ± 0.0004	1.00 ± 0.0007	0.99 ± 0.0011	0.50 ± 0.0002	0.33 ± 0.0002
	3	0.50 ± 0.0003	0.33 ± 0.0006	1.00 ± 0.0012	0.99 ± 0.0017	0.50 ± 0.0003	0.33 ± 0.0005
	5.25	0.50 ± 0.0004	0.33 ± 0.0007	1.00 ± 0.0014	0.97 ± 0.0019	0.50 ± 0.0003	0.33 ± 0.0004
mH_2	$m = 2$	0.50 ± 0.0003	0.33 ± 0.0004	1.00 ± 0.0009	2.47 ± 0.0066	0.42 ± 0.0002	0.38 ± 0.0003
	$m = 5$	0.50 ± 0.0002	0.33 ± 0.0004	1.00 ± 0.0008	1.38 ± 0.0030	0.47 ± 0.0002	0.36 ± 0.0002
	$m = 10$	0.50 ± 0.0003	0.33 ± 0.0004	1.00 ± 0.0008	1.14 ± 0.0015	0.48 ± 0.0001	0.35 ± 0.0002
	$m = 20$	0.50 ± 0.0002	0.33 ± 0.0004	1.00 ± 0.0008	1.06 ± 0.0011	0.49 ± 0.0001	0.34 ± 0.0002
$RRI(H_2)$	$p = 0.1$	0.50 ± 0.0003	0.33 ± 0.0005	1.00 ± 0.0010	3.94 ± 0.0085	0.35 ± 0.0002	0.44 ± 0.0004
	$p = 0.5$	0.50 ± 0.0004	0.33 ± 0.0007	1.00 ± 0.0015	3.89 ± 0.0126	0.35 ± 0.0004	0.44 ± 0.0007
	$p = 0.9$	0.50 ± 0.0012	0.33 ± 0.0020	1.00 ± 0.0037	3.54 ± 0.0250	0.36 ± 0.0009	0.47 ± 0.0024

mean.

The correlation results in Table 21 are similar to what we observe in Table 9: there are strong positive correlations between the p-values of the Standard KS test and the Log test, the Standard KS test and the Lewis test, and the Log test and the Lewis test. Tables 10 - 12 provide the result of applying three composite methods. As in previous section with larger sample size, we again recommend trying the Lewis test first and then trying the combination of the Standard KS and the Conditional-Uniform tests in the case the result is not convincing.

Table 19 Performance of Alternative Tests on $[0, 2000]$: Number of KS tests passed (denoted by $\#P$) at significance level 0.05 out of 10,000 replications and the average p-values (denoted by $E[p\text{-value}]$) with associated 95% confidence intervals.

		Standard KS		Conditional		Log		Lewis	
Case	*Subcase*	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$
Exp	$-$	9525	0.50 ± 0.006	9491	0.50 ± 0.006	9513	0.50 ± 0.006	9476	0.50 ± 0.006
E_k	$k = 2$	0	0.00 ± 0.000	9983	0.79 ± 0.004	0	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 4$	0	0.00 ± 0.000	10000	0.95 ± 0.002	0	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 6$	0	0.00 ± 0.000	10000	0.98 ± 0.001	0	0.00 ± 0.000	0	0.00 ± 0.000
H_2	$c^2 = 1.25$	3402	0.08 ± 0.003	8945	0.40 ± 0.005	3627	0.10 ± 0.003	263	0.01 ± 0.001
	$c^2 = 1.5$	86	0.00 ± 0.000	8311	0.32 ± 0.005	200	0.00 ± 0.000	0	0.00 ± 0.000
	$c^2 = 2$	0	0.00 ± 0.000	6878	0.21 ± 0.004	0	0.00 ± 0.000	0	0.00 ± 0.000
	$c^2 = 4$	0	0.00 ± 0.000	2804	0.06 ± 0.002	0	0.00 ± 0.000	0	0.00 ± 0.000
	$c^2 = 10$	0	0.00 ± 0.000	174	0.00 ± 0.000	0	0.00 ± 0.000	0	0.00 ± 0.000
Z	$-$	0	0.00 ± 0.000	9491	0.52 ± 0.006	0	0.00 ± 0.000	0	0.00 ± 0.000
LN	$-$	0	0.00 ± 0.000	9519	0.51 ± 0.006	0	0.00 ± 0.000	0	0.00 ± 0.000
RRI	$p = 0.1$	9010	0.41 ± 0.005	9119	0.41 ± 0.005	9027	0.41 ± 0.005	9009	0.40 ± 0.005
	$p = 0.5$	4449	0.10 ± 0.003	4667	0.11 ± 0.003	4693	0.11 ± 0.003	4502	0.10 ± 0.003
	$p = 0.9$	0	0.00 ± 0.000	21	0.00 ± 0.000	6	0.00 ± 0.000	1	0.00 ± 0.000
$EARMA$	0.25	9338	0.47 ± 0.006	8318	0.33 ± 0.005	9152	0.46 ± 0.006	9429	0.49 ± 0.006
	0.5	8808	0.42 ± 0.006	7029	0.22 ± 0.004	8870	0.42 ± 0.006	9392	0.49 ± 0.006
	1	8182	0.37 ± 0.006	4749	0.12 ± 0.003	8017	0.36 ± 0.006	8894	0.43 ± 0.006
	3	5233	0.20 ± 0.005	805	0.01 ± 0.001	5465	0.21 ± 0.005	6660	0.29 ± 0.006
	5.25	4111	0.14 ± 0.004	179	0.00 ± 0.000	4308	0.14 ± 0.004	5783	0.21 ± 0.005
mH_2	$m = 2$	2	0.00 ± 0.000	3001	0.06 ± 0.002	21	0.00 ± 0.000	0	0.00 ± 0.000
	$m = 5$	3217	0.09 ± 0.003	3376	0.07 ± 0.002	3403	0.10 ± 0.004	139	0.00 ± 0.000
	$m = 10$	6413	0.25 ± 0.005	3752	0.09 ± 0.003	6571	0.26 ± 0.005	4406	0.13 ± 0.004
	$m = 20$	7360	0.31 ± 0.006	4343	0.11 ± 0.003	7655	0.33 ± 0.006	8147	0.35 ± 0.006
$RRI(H_2)$	$p = 0.1$	0	0.00 ± 0.000	1960	0.04 ± 0.001	0	0.00 ± 0.000	0	0.00 ± 0.000
	$p = 0.5$	0	0.00 ± 0.000	154	0.00 ± 0.000	0	0.00 ± 0.000	0	0.00 ± 0.000
	$p = 0.9$	0	0.00 ± 0.000	1	0.00 ± 0.000	0	0.00 ± 0.000	0	0.00 ± 0.000

Table 20 Performance of Alternative Standard KS Tests for Untransformed Interarrival Times for the time interval $[0, 2000]$: Number of KS tests passed (denoted by $\#P$) at significance level 0.05 out of $10,000$ replications and the average p-values (denoted by $E[p\text{-value}]$) with associated 95% confidence intervals.

		KS with estimated mean		Lillifors test	
$Case$	$Subcase$	$\#P$	$E[p\text{-value}]$	$\#P$	$E[p\text{-value}]$
Exp	$-$	9954	0.66 ± 0.005	9508	0.37 ± 0.003
E_k	$k = 2$	0	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 4$	0	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 6$	0	0.00 ± 0.000	0	0.00 ± 0.000
H_2	$c^2 = 1.25$	3106	0.06 ± 0.002	822	0.02 ± 0.001
	$c^2 = 1.5$	3	0.00 ± 0.000	0	0.00 ± 0.000
	$c^2 = 2$	0	0.00 ± 0.000	0	0.00 ± 0.000
	$c^2 = 4$	0	0.00 ± 0.000	0	0.00 ± 0.000
	$c^2 = 10$	0	0.00 ± 0.000	0	0.00 ± 0.000
Z	$-$	0	0.00 ± 0.000	0	0.00 ± 0.000
LN	$-$	0	0.00 ± 0.000	0	0.00 ± 0.000
RRI	$p = 0.1$	9831	0.56 ± 0.005	8923	0.31 ± 0.004
	$p = 0.5$	6733	0.17 ± 0.003	3588	0.07 ± 0.002
	$p = 0.9$	2	0.00 ± 0.000	0	0.00 ± 0.000
$EARMA$	0.25	9932	0.65 ± 0.005	9486	0.37 ± 0.003
	0.5	9933	0.65 ± 0.005	9418	0.37 ± 0.003
	1	9837	0.60 ± 0.005	8987	0.34 ± 0.004
	3	8304	0.43 ± 0.007	6679	0.23 ± 0.004
	5.25	8084	0.35 ± 0.006	6045	0.18 ± 0.004
mH_2	$m = 2$	0	0.00 ± 0.000	0	0.00 ± 0.000
	$m = 5$	2324	0.04 ± 0.002	581	0.01 ± 0.001
	$m = 10$	8372	0.31 ± 0.005	5727	0.16 ± 0.003
	$m = 20$	9694	0.54 ± 0.006	8557	0.30 ± 0.004
$RRI(H_2)$	$p = 0.1$	0	0.00 ± 0.000	0	0.00 ± 0.000
	$p = 0.5$	0	0.00 ± 0.000	0	0.00 ± 0.000
	$p = 0.9$	0	0.00 ± 0.000	0	0.00 ± 0.000

Table 21 Pairwise Correlations of p-values ($[0, 2000]$): S stands for the standard test, C for the CU test, L for the Log test and Le for the Lewis test.

$Case$	$Subcase$	(S, C)	(S, L)	(S, Le)	(C, L)	(C, Le)	(L, Le)
Exp	$-$		0.34	0.50	0.29		0.49
E_k	$k = 2$				-0.02		
	$k = 4$		0.05	0.50	-0.02		
	$k = 6$				-0.05	-0.09	
H_2	$c^2 = 1.25$	0.04	0.34	0.28	-0.08	0.03	0.31
	$c^2 = 1.5$		0.09	0.03	-0.11	0.03	0.09
	$c^2 = 2$				-0.03		0.21
	$c^2 = 4$						
	$c^2 = 10$						
Z	$-$		0.90	0.96	-0.02		0.95
LN	$-$		0.11	0.05	-0.02		0.42
RRI	$p = 0.1$		0.34	0.51	0.27		0.47
	$p = 0.5$		0.32	0.45	0.15		0.40
	$p = 0.9$		0.06	0.10			0.11
$EARMA$	0.25		0.26	0.39	0.30		0.40
	0.5		0.19	0.35	0.31		0.35
	1		0.17	0.33	0.29		0.33
	3		0.25	0.48	0.14		0.40
	5.25		0.13	0.31	0.05	-0.05	0.28
mH_2	$m = 2$				-0.03		
	$m = 5$	0.03	0.19	0.15	-0.03	0.07	0.18
	$m = 10$	0.04	0.20	0.28	0.20	0.14	0.34
	$m = 20$		0.16	0.28	0.28	0.10	0.32
$RRI(H_2)$	$p = 0.1$						
	$p = 0.5$						
	$p = 0.9$						

Table 22 Results for all possible composite tests based on the four alternatives for the time interval ($[0, 2000]$) using the Bonferroni procedure: The test order is (standard, CU, Log, Lewis) and is indicated by 1 if the test is included in the combination and 0 otherwise. Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

		2 tests						3 tests				4 tests
$Case$	$Subcase$	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
Exp	$-$	9532	9560	9583	9542	9489	9547	9531	9552	9521	9482	9566
E_k	$k=2$	0	0	0	0	0	0	0	0	0	0	0
	$k=4$	0	0	0	0	0	0	0	0	0	0	0
	$k=6$	0	0	0	0	0	0	0	0	0	0	0
H_2	$c^2=1.25$	4292	2844	432	4391	462	451	584	547	562	1370	668
	$c^2=1.5$	153	31	0	249	0	0	0	0	0	0	0
	$c^2=2$	0	0	0	0	0	0	0	0	0	0	0
	$c^2=4$	0	0	0	0	0	0	0	0	0	0	0
	$c^2=10$	0	0	0	0	0	0	0	0	0	0	0
Z	$-$	0	0	0	0	0	0	0	0	0	0	0
LN	$(1,1)$	0	0	0	0	0	0	0	0	0	0	0
RRI	$p=0.1$	8958	8965	9056	9046	8946	9032	8886	8935	8846	8741	8920
	$p=0.5$	3181	3689	3955	3838	3237	4092	2823	3066	2402	2265	2555
	$p=0.9$	0	0	0	0	0	0	0	0	0	0	0
$EARMA$	0.25	8629	9174	9402	8636	8657	9262	8752	9248	8759	8659	8847
	0.5	7336	8665	9048	7631	7665	9078	7878	8806	7661	7541	7857
	1	5086	7589	8277	5469	5414	8172	5770	7718	5435	5343	5717
	3	797	4071	5242	1115	998	5276	1198	4106	939	911	1060
	5.25	171	2576	3571	265	206	3777	282	2385	185	189	217
mH_2	$m=2$	2	0	0	0	0	0	0	0	0	0	0
	$m=5$	1752	1936	187	1856	179	232	224	221	172	466	207
	$m=10$	3400	5395	4346	3979	2894	4580	3244	4139	2804	3104	3053
	$m=20$	4282	6734	7249	4944	4820	7525	5133	6706	4538	4510	4805
$RRI(H_2)$	$p=0.1$	0	0	0	0	0	0	0	0	0	0	0
	$p=0.5$	0	0	0	0	0	0	0	0	0	0	0
	$p=0.9$	0	0	0	0	0	0	0	0	0	0	0

Table 23 Results for all possible composite tests based on the four alternatives for the time interval ($[0, 2000]$) using Holm's procedure: The test order is (standard, CU, Log, Lewis) and is indicated by 1 if the test is included in the combination and 0 otherwise. Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

		2 tests						3 tests				4 tests
$Case$	$Subcase$	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
Exp	$-$	9526	9548	9555	9526	9480	9526	9502	9507	9487	9469	9552
E_k	$k = 2$	0	0	0	0	0	0	0	0	0	0	0
	$k = 4$	0	0	0	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	4243	2705	412	4360	453	441	541	498	518	1308	639
	$c^2 = 1.5$	143	26	0	229	0	0	0	0	0	0	0
	$c^2 = 2$	0	0	0	0	0	0	0	0	0	0	0
	$c^2 = 4$	0	0	0	0	0	0	0	0	0	0	0
	$c^2 = 10$	0	0	0	0	0	0	0	0	0	0	0
Z	$-$	0	0	0	0	0	0	0	0	0	0	0
LN	$(1, 1)$	0	0	0	0	0	0	0	0	0	0	0
RRI	$p = 0.1$	8941	8933	9004	9011	8925	8985	8815	8832	8767	8698	8880
	$p = 0.5$	3075	3551	3812	3707	3108	3926	2640	2862	2222	2137	2458
	$p = 0.9$	0	0	0	0	0	0	0	0	0	0	0
$EARMA$	0.25	8609	9161	9382	8592	8631	9241	8702	9202	8716	8631	8816
	0.5	7293	8641	9015	7585	7637	9059	7825	8745	7589	7493	7832
	1	5015	7556	8239	5414	5355	8137	5679	7649	5336	5268	5669
	3	769	4022	5183	1074	955	5204	1138	4021	896	877	1036
	5.25	154	2534	3526	243	184	3712	256	2307	163	170	208
mH_2	$m = 2$	2	0	0	0	0	0	0	0	0	0	0
	$m = 5$	1668	1872	177	1771	164	223	208	203	149	414	190
	$m = 10$	3335	5332	4265	3901	2778	4501	3087	3998	2667	3012	2964
	$m = 20$	4217	6690	7190	4888	4760	7465	5052	6614	4459	4453	4748
$RRI(H_2)$	$p = 0.1$	0	0	0	0	0	0	0	0	0	0	0
	$p = 0.5$	0	0	0	0	0	0	0	0	0	0	0
	$p = 0.9$	0	0	0	0	0	0	0	0	0	0	0

Table 24 Results for all possible composite tests based on the four alternatives for the time interval ($[0, 2000]$) using Simes' procedure: The test order is (standard, CU, Log, Lewis) and is indicated by 1 if the test is included in the combination and 0 otherwise. Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

		2 tests						3 tests				4 tests
$Case$	$Subcase$	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
Exp	–	9526	9548	9555	9526	9480	9526	9463	9472	9446	9449	9512
E_k	$k = 2$	0	0	0	0	0	0	0	0	0	0	0
	$k = 4$	0	0	0	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	4243	2705	412	4360	453	441	504	468	486	1261	580
	$c^2 = 1.5$	143	26	0	229	0	0	0	0	0	0	0
	$c^2 = 2$	0	0	0	0	0	0	0	0	0	0	0
	$c^2 = 4$	0	0	0	0	0	0	0	0	0	0	0
	$c^2 = 10$	0	0	0	0	0	0	0	0	0	0	0
Z	–	0	0	0	0	0	0	0	0	0	0	0
LN	$(1, 1)$	0	0	0	0	0	0	0	0	0	0	0
RRI	$p = 0.1$	8941	8933	9004	9011	8925	8985	8765	8788	8720	8650	8789
	$p = 0.5$	3075	3551	3812	3707	3108	3926	2535	2752	2120	2046	2269
	$p = 0.9$	0	0	0	0	0	0	0	0	0	0	0
$EARMA$	0.25	8609	9161	9382	8592	8631	9241	8658	9161	8676	8597	8749
	0.5	7293	8641	9015	7585	7637	9059	7777	8716	7547	7439	7740
	1	5015	7556	8239	5414	5355	8137	5615	7595	5288	5205	5548
	3	769	4022	5183	1074	955	5204	1110	3971	856	840	990
	5.25	154	2534	3526	243	184	3712	250	2261	156	159	187
mH_2	$m = 2$	2	0	0	0	0	0	0	0	0	0	0
	$m = 5$	1668	1872	177	1771	164	223	196	192	141	395	168
	$m = 10$	3335	5332	4265	3901	2778	4501	2986	3907	2571	2924	2800
	$m = 20$	4217	6690	7190	4888	4760	7465	4989	6558	4405	4388	4659
$RRI(H_2)$	$p = 0.1$	0	0	0	0	0	0	0	0	0	0	0
	$p = 0.5$	0	0	0	0	0	0	0	0	0	0	0
	$p = 0.9$	0	0	0	0	0	0	0	0	0	0	0

Table 25 Results for individual and composite tests based on the Standard KS, CU, and Lilliefors tests for the time interval ($[0, 2000]$) using the Bonferroni procedure: Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

$Case$	$Subcase$	Standard	Lilliefors	CU	(Standard+CU)	(Lilliefors+CU)
Exp	$-$	9525	9508	9491	9532	9522
E_k	$k = 2$	0	0	9983	0	0
	$k = 4$	0	0	10000	0	0
	$k = 6$	0	0	10000	0	0
H_2	$c^2 = 1.25$	3402	822	8945	4292	1354
	$c^2 = 1.5$	86	0	8311	153	0
	$c^2 = 2$	0	0	6878	0	0
	$c^2 = 4$	0	0	2804	0	0
	$c^2 = 10$	0	0	174	0	0
Z	$-$	0	0	9491	0	0
LN	$(1, 1)$	0	0	9519	0	0
RRI	$p = 0.1$	9010	8923	9119	8958	8908
	$p = 0.5$	4449	3588	4667	3181	2736
	$p = 0.9$	0	0	21	0	0
$EARMA$	0.25	9338	9486	8318	8629	8683
	0.5	8808	9418	7029	7336	7688
	1	8182	8987	4749	5086	5476
	3	5233	6679	805	797	999
	5.25	4111	6045	179	171	223
mH_2	$m = 2$	2	0	3001	2	0
	$m = 5$	3217	581	3376	1752	581
	$m = 10$	6413	5727	3752	3400	3444
	$m = 20$	7360	8557	4343	4282	4930
$RRI(H_2)$	$p = 0.1$	0	0	1960	0	0
	$p = 0.5$	0	0	154	0	0
	$p = 0.9$	0	0	1	0	0

4.2 Experiments with the Interval $[0, 200]$ with 10 Subintervals

In this section, we examine the effect of introducing subintervals on the power of alternative tests. We use the same interval $[0, 200]$ discussed in Section 3 and divide it into 10 equally sized subintervals, $[0, 10]$, ..., $[190, 200]$. In doing so, there are interarrival times that extend over two consecutive subintervals. We treat them as lost in their first subinterval, and as starting at the beginning of their second subinterval. Even though Table 26 shows that the average values of the interarrival times decreases slightly due to this truncation, the results in Tables 26 and 27 are similar to the results in Tables 5 and 6.

Table 28 illustrates that the power of test decreases when we introduce subintervals, suggesting it is better to avoid introducing subintervals when possible. Also, the Lewis test still performs the best for E_k, H_2, Z, LN and RRI, but the Conditional-Uniform test no longer performs the best for $EARMA$ and mH_2. Instead, the performance of the Lewis test is significantly improved for $EARMA$ with the highest dependence. This suggests that when we break the interval into subintervals, part of the local dependence is lost, and it becomes harder for the Conditional-Uniform test to detect it. We instead observe that the Standard KS test now performs the best for the rest, most of $EARMA$, mH_2 and $RRI(H_2)$. In addition, Table 8 suggests that the Standard KS test now outperforms the Lillifors test and the KS test with estimated mean.

The correlation results in Table 30 are similar to what we observe in Tables 21 and 9. The power of different combinations of alternative tests in Tables 31 - 33 gives slightly different results, as expected from the results in Table 28. Since the Conditional-Uniform test is no longer powerful at detecting dependence, we now advocate using the combination of the Lewis test and the Standard KS test, when the result from the Lewis test is not sufficient to make a conclusion.

Table 26 Summary statistics of the arrival processes on $[0, 200]$ (with 10 equally sized subintervals) with associated 95% confidence intervals. All results are based on 10000 replications. $\{X_n : n \geq 1\}$ are the interarrival times where n is the total number of arrivals in the 10 subintervals.

$Case$	$Subcase$	$E[U]$	$c^2[U]$	$Min[n]$	$Max[n]$	$E[n]$	t_n	$Min[U]$	$Max[U]$
Exp	–	0.95 ± 0.0013	0.99 ± 0.0027	146	257	200.0 ± 0.3	190 ± 0.06	0.0049 ± 0.00010	5.6 ± 0.02
E_k	$k = 2$	0.96 ± 0.0009	0.51 ± 0.0013	165	245	200.0 ± 0.2	192 ± 0.04	0.0353 ± 0.00045	3.9 ± 0.01
	$k = 4$	0.97 ± 0.0007	0.27 ± 0.0006	175	224	200.2 ± 0.1	194 ± 0.03	0.0740 ± 0.00107	2.9 ± 0.01
	$k = 6$	0.97 ± 0.0006	0.18 ± 0.0004	179	222	200.0 ± 0.1	194 ± 0.03	0.0846 ± 0.00135	2.5 ± 0.01
H_2	$c^2 = 1.25$	0.95 ± 0.0015	1.20 ± 0.0040	134	265	199.7 ± 0.3	189 ± 0.08	0.0044 ± 0.00009	6.7 ± 0.03
	$c^2 = 1.5$	0.94 ± 0.0016	1.40 ± 0.0052	139	270	200.2 ± 0.3	187 ± 0.09	0.0041 ± 0.00008	7.5 ± 0.04
	$c^2 = 2$	0.93 ± 0.0018	1.78 ± 0.0073	132	295	200.2 ± 0.4	185 ± 0.12	0.0036 ± 0.00007	8.9 ± 0.05
	$c^2 = 4$	0.89 ± 0.0022	2.92 ± 0.0139	105	299	199.9 ± 0.5	175 ± 0.22	0.0031 ± 0.00006	12.0 ± 0.06
	$c^2 = 10$	0.79 ± 0.0025	4.27 ± 0.0245	44	365	199.6 ± 0.9	153 ± 0.41	0.0029 ± 0.00006	14.3 ± 0.07
Z	–	0.95 ± 0.0012	0.80 ± 0.0063	142	252	199.9 ± 0.3	190 ± 0.10	0.0157 ± 0.00027	6.7 ± 0.06
LN	–	0.95 ± 0.0013	0.91 ± 0.0045	148	251	200.1 ± 0.3	190 ± 0.08	0.0512 ± 0.00059	6.5 ± 0.04
RRI	$p = 0.1$	0.96 ± 0.0014	0.99 ± 0.0029	144	261	199.9 ± 0.3	190 ± 0.06	0.0053 ± 0.00011	5.5 ± 0.02
	$p = 0.5$	0.97 ± 0.0023	0.98 ± 0.0042	99	287	199.6 ± 0.5	190 ± 0.06	0.0095 ± 0.00020	5.1 ± 0.02
	$p = 0.9$	1.05 ± 0.0082	0.87 ± 0.0081	19	453	201.1 ± 1.1	190 ± 0.09	0.0406 ± 0.00107	3.8 ± 0.02
$EARMA$	0.25	0.96 ± 0.0016	0.99 ± 0.0027	140	267	199.7 ± 0.3	190 ± 0.06	0.0048 ± 0.00010	5.5 ± 0.02
	0.5	0.96 ± 0.0018	0.98 ± 0.0028	128	277	199.9 ± 0.4	190 ± 0.06	0.0048 ± 0.00010	5.5 ± 0.02
	1	0.96 ± 0.0023	0.97 ± 0.0032	102	284	199.8 ± 0.5	190 ± 0.06	0.0049 ± 0.00010	5.4 ± 0.02
	3	0.98 ± 0.0036	0.95 ± 0.0047	80	366	199.9 ± 0.7	190 ± 0.06	0.0194 ± 0.00043	5.3 ± 0.02
	5.25	1.01 ± 0.0048	0.91 ± 0.0048	50	408	199.7 ± 0.9	190 ± 0.07	0.0053 ± 0.00012	5.0 ± 0.03
mH_2	$m = 2$	0.93 ± 0.0023	1.87 ± 0.0109	99	307	199.8 ± 0.5	182 ± 0.17	0.0039 ± 0.00008	10.1 ± 0.07
	$m = 5$	0.96 ± 0.0025	1.24 ± 0.0055	100	314	199.8 ± 0.5	188 ± 0.10	0.0045 ± 0.00009	7.2 ± 0.05
	$m = 10$	0.96 ± 0.0025	1.08 ± 0.0037	103	294	199.9 ± 0.5	189 ± 0.08	0.0048 ± 0.00010	6.2 ± 0.03
	$m = 20$	0.96 ± 0.0024	1.03 ± 0.0030	104	296	199.7 ± 0.5	190 ± 0.07	0.0050 ± 0.00010	5.8 ± 0.03
$RRI(H_2)$	$p = 0.1$	0.90 ± 0.0024	2.89 ± 0.0144	79	320	199.1 ± 0.6	175 ± 0.23	0.0035 ± 0.00007	11.9 ± 0.06
	$p = 0.5$	0.93 ± 0.0052	2.82 ± 0.0180	8	362	200.0 ± 0.9	175 ± 0.29	0.0068 ± 0.00055	11.3 ± 0.06
	$p = 0.9$	1.55 ± 0.0388	2.07 ± 0.0282	4	579	200.0 ± 2.0	176 ± 0.45	0.1241 ± 0.01332	8.3 ± 0.08

Table 27 Average and c^2 of transformed interarrival times on $[0, 200]$ (with 10 equally sized subintervals) with associated 95% confidence intervals. All results are based on 10000 replications.

$Case$	$Subcase$	$E[X^{CU}]$	$c^2[X^{CU}]$	$E[X^{Log}]$	$c^2[X^{Log}]$	$E[X^{Lewis}]$	$c^2[X^{Lewis}]$
Exp	–	0.50 ± 0.0004	0.34 ± 0.0007	1.00 ± 0.0014	1.00 ± 0.0027	0.50 ± 0.0004	0.34 ± 0.0007
E_k	$k = 2$	0.50 ± 0.0003	0.33 ± 0.0005	1.00 ± 0.0011	0.53 ± 0.0014	0.62 ± 0.0003	0.17 ± 0.0004
	$k = 4$	0.50 ± 0.0002	0.34 ± 0.0004	1.00 ± 0.0009	0.29 ± 0.0009	0.71 ± 0.0003	0.10 ± 0.0002
	$k = 6$	0.50 ± 0.0002	0.33 ± 0.0003	1.00 ± 0.0008	0.22 ± 0.0008	0.75 ± 0.0002	0.07 ± 0.0002
H_2	$c^2 = 1.25$	0.50 ± 0.0004	0.34 ± 0.0007	1.00 ± 0.0015	1.16 ± 0.0035	0.48 ± 0.0004	0.36 ± 0.0007
	$c^2 = 1.5$	0.50 ± 0.0005	0.33 ± 0.0008	1.00 ± 0.0016	1.31 ± 0.0044	0.46 ± 0.0004	0.38 ± 0.0008
	$c^2 = 2$	0.50 ± 0.0005	0.33 ± 0.0009	1.00 ± 0.0017	1.57 ± 0.0058	0.43 ± 0.0005	0.40 ± 0.0009
	$c^2 = 4$	0.50 ± 0.0006	0.33 ± 0.0011	1.00 ± 0.0020	2.26 ± 0.0105	0.39 ± 0.0006	0.45 ± 0.0011
	$c^2 = 10$	0.50 ± 0.0007	0.33 ± 0.0012	1.00 ± 0.0022	2.82 ± 0.0173	0.39 ± 0.0008	0.47 ± 0.0014
Z	–	0.50 ± 0.0003	0.33 ± 0.0006	1.00 ± 0.0012	0.70 ± 0.0030	0.58 ± 0.0004	0.21 ± 0.0005
LN	–	0.50 ± 0.0004	0.33 ± 0.0006	1.00 ± 0.0013	0.81 ± 0.0029	0.56 ± 0.0004	0.19 ± 0.0004
RRI	$p = 0.1$	0.50 ± 0.0004	0.34 ± 0.0007	1.00 ± 0.0014	0.99 ± 0.0030	0.50 ± 0.0004	0.33 ± 0.0008
	$p = 0.5$	0.50 ± 0.0005	0.34 ± 0.0009	1.00 ± 0.0017	0.99 ± 0.0049	0.54 ± 0.0007	0.32 ± 0.0011
	$p = 0.9$	0.50 ± 0.0007	0.34 ± 0.0013	1.00 ± 0.0023	0.87 ± 0.0134	0.67 ± 0.0015	0.23 ± 0.0020
$EARMA$	0.25	0.50 ± 0.0004	0.34 ± 0.0007	1.00 ± 0.0014	0.98 ± 0.0027	0.51 ± 0.0004	0.33 ± 0.0007
	0.5	0.50 ± 0.0005	0.33 ± 0.0008	1.00 ± 0.0015	0.95 ± 0.0028	0.51 ± 0.0004	0.33 ± 0.0007
	1	0.50 ± 0.0004	0.33 ± 0.0008	1.00 ± 0.0015	0.87 ± 0.0025	0.52 ± 0.0004	0.32 ± 0.0007
	3	0.50 ± 0.0007	0.34 ± 0.0012	1.00 ± 0.0022	0.68 ± 0.0037	0.55 ± 0.0008	0.27 ± 0.0014
	5.25	0.50 ± 0.0005	0.34 ± 0.0008	1.00 ± 0.0015	0.58 ± 0.0020	0.59 ± 0.0005	0.27 ± 0.0007
mH_2	$m = 2$	0.50 ± 0.0005	0.33 ± 0.0009	1.00 ± 0.0017	1.44 ± 0.0063	0.45 ± 0.0005	0.38 ± 0.0009
	$m = 5$	0.50 ± 0.0004	0.33 ± 0.0007	1.00 ± 0.0015	1.08 ± 0.0033	0.49 ± 0.0004	0.35 ± 0.0007
	$m = 10$	0.50 ± 0.0004	0.33 ± 0.0007	1.00 ± 0.0014	1.01 ± 0.0028	0.50 ± 0.0004	0.34 ± 0.0007
	$m = 20$	0.50 ± 0.0004	0.33 ± 0.0007	1.00 ± 0.0014	1.00 ± 0.0028	0.50 ± 0.0004	0.34 ± 0.0007
$RRI(H_2)$	$p = 0.1$	0.50 ± 0.0006	0.34 ± 0.0011	1.00 ± 0.0020	2.23 ± 0.0112	0.40 ± 0.0006	0.45 ± 0.0012
	$p = 0.5$	0.50 ± 0.0007	0.34 ± 0.0013	1.00 ± 0.0023	2.05 ± 0.0156	0.45 ± 0.0008	0.42 ± 0.0016
	$p = 0.9$	0.50 ± 0.0011	0.34 ± 0.0019	1.00 ± 0.0035	1.56 ± 0.0342	0.60 ± 0.0018	0.29 ± 0.0028

Table 28 Performance of Alternative Tests on $[0, 200]$ (with 10 equally sized subintervals): Number of KS tests passed (denoted by $\#P$) at significance level 0.05 out of $10,000$ replications and the average p-values (denoted by $E[p\text{-value}]$) with associated 95% confidence intervals.

		Standard KS		Conditional		Log		Lewis	
$Case$	$Subcase$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$
Exp	$-$	9206	0.45 ± 0.006	9544	0.50 ± 0.006	9533	0.50 ± 0.006	9495	0.50 ± 0.006
E_k	$k = 2$	87	0.00 ± 0.000	9979	0.74 ± 0.005	79	0.00 ± 0.000	2	0.00 ± 0.000
	$k = 4$	0	0.00 ± 0.000	10000	0.90 ± 0.003	0	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 6$	0	0.00 ± 0.000	10000	0.95 ± 0.002	0	0.00 ± 0.000	0	0.00 ± 0.000
H_2	$c^2 = 1.25$	7687	0.32 ± 0.006	9147	0.45 ± 0.006	8993	0.43 ± 0.006	8160	0.35 ± 0.006
	$c^2 = 1.5$	5546	0.18 ± 0.005	8848	0.40 ± 0.006	7848	0.33 ± 0.006	5459	0.18 ± 0.005
	$c^2 = 2$	2441	0.06 ± 0.003	8214	0.34 ± 0.006	5537	0.19 ± 0.005	1783	0.04 ± 0.002
	$c^2 = 4$	118	0.00 ± 0.000	6790	0.25 ± 0.005	2178	0.06 ± 0.003	138	0.00 ± 0.000
	$c^2 = 10$	11	0.00 ± 0.000	6323	0.23 ± 0.005	2177	0.06 ± 0.003	260	0.01 ± 0.001
Z	$-$	2274	0.04 ± 0.001	9882	0.64 ± 0.005	2014	0.04 ± 0.002	499	0.01 ± 0.001
LN	$-$	328	0.01 ± 0.000	9730	0.59 ± 0.006	1057	0.02 ± 0.001	175	0.00 ± 0.000
RRI	$p = 0.1$	8768	0.38 ± 0.006	9311	0.46 ± 0.006	9232	0.44 ± 0.006	9054	0.42 ± 0.006
	$p = 0.5$	4824	0.12 ± 0.003	7992	0.33 ± 0.006	6076	0.18 ± 0.004	3199	0.07 ± 0.002
	$p = 0.9$	21	0.00 ± 0.000	5890	0.25 ± 0.006	93	0.00 ± 0.000	1	0.00 ± 0.000
$EARMA$	0.25	9013	0.44 ± 0.006	9314	0.48 ± 0.006	9453	0.49 ± 0.006	9487	0.50 ± 0.006
	0.5	8589	0.41 ± 0.006	8923	0.44 ± 0.006	9295	0.47 ± 0.006	9244	0.47 ± 0.006
	1	8084	0.36 ± 0.006	9238	0.48 ± 0.006	8926	0.42 ± 0.006	8117	0.34 ± 0.006
	3	5227	0.21 ± 0.005	5869	0.20 ± 0.005	4729	0.14 ± 0.004	4556	0.17 ± 0.005
	5.25	4345	0.15 ± 0.005	9154	0.49 ± 0.006	2562	0.06 ± 0.002	352	0.01 ± 0.001
mH_2	$m = 2$	3401	0.11 ± 0.004	8249	0.35 ± 0.006	7250	0.29 ± 0.006	4611	0.15 ± 0.004
	$m = 5$	6747	0.28 ± 0.006	9291	0.46 ± 0.006	9336	0.48 ± 0.006	9182	0.46 ± 0.006
	$m = 10$	7394	0.32 ± 0.006	9465	0.49 ± 0.006	9462	0.50 ± 0.006	9453	0.49 ± 0.006
	$m = 20$	7774	0.34 ± 0.006	9459	0.49 ± 0.006	9505	0.50 ± 0.006	9491	0.50 ± 0.006
$RRI(H_2)$	$p = 0.1$	191	0.00 ± 0.000	6712	0.24 ± 0.005	2500	0.07 ± 0.003	267	0.01 ± 0.001
	$p = 0.5$	435	0.01 ± 0.001	5764	0.19 ± 0.005	4070	0.11 ± 0.004	2151	0.05 ± 0.002
	$p = 0.9$	15	0.00 ± 0.000	4431	0.19 ± 0.006	849	0.04 ± 0.003	497	0.03 ± 0.003

Table 29 Performance of Alternative Tests for Untransformed Interarrival Times on $[0, 200]$ (with 10 equally sized subintervals): Number of KS tests passed (denoted by $\#P$) at significance level 0.05 out of $10,000$ replications and the average p-values (denoted by $E[p\text{-value}]$) with associated 95% confidence intervals.

		KS with estimated mean		Lillifors test	
$Case$	$Subcase$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$
Exp	$-$	9949	0.65 ± 0.005	9493	0.37 ± 0.003
E_k	$k = 2$	20	0.00 ± 0.000	1	0.00 ± 0.000
	$k = 4$	0	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 6$	0	0.00 ± 0.000	0	0.00 ± 0.000
H_2	$c^2 = 1.25$	9681	0.54 ± 0.006	8550	0.30 ± 0.004
	$c^2 = 1.5$	8601	0.35 ± 0.005	6257	0.18 ± 0.004
	$c^2 = 2$	5290	0.13 ± 0.004	2397	0.05 ± 0.002
	$c^2 = 4$	908	0.02 ± 0.001	183	0.01 ± 0.001
	$c^2 = 10$	971	0.02 ± 0.001	324	0.01 ± 0.001
Z	$-$	1276	0.02 ± 0.001	233	0.01 ± 0.000
LN	$-$	297	0.01 ± 0.000	15	0.00 ± 0.000
RRI	$p = 0.1$	9847	0.57 ± 0.005	9031	0.32 ± 0.003
	$p = 0.5$	7235	0.20 ± 0.004	4224	0.09 ± 0.002
	$p = 0.9$	69	0.00 ± 0.000	7	0.00 ± 0.000
$EARMA$	0.25	9949	0.66 ± 0.005	9485	0.38 ± 0.003
	0.5	9930	0.65 ± 0.005	9439	0.37 ± 0.003
	1	9878	0.61 ± 0.005	9142	0.35 ± 0.003
	3	8432	0.44 ± 0.006	6795	0.24 ± 0.004
	5.25	8324	0.37 ± 0.006	6294	0.20 ± 0.004
mH_2	$m = 2$	6883	0.22 ± 0.005	4023	0.10 ± 0.003
	$m = 5$	9645	0.53 ± 0.006	8462	0.30 ± 0.004
	$m = 10$	9905	0.63 ± 0.005	9284	0.36 ± 0.003
	$m = 20$	9940	0.65 ± 0.005	9446	0.37 ± 0.003
$RRI(H_2)$	$p = 0.1$	873	0.02 ± 0.001	220	0.01 ± 0.001
	$p = 0.5$	715	0.01 ± 0.001	200	0.00 ± 0.001
	$p = 0.9$	503	0.02 ± 0.002	314	0.01 ± 0.001

Table 30 Pairwise Correlations of p-values ($[0, 200]$ with 10 equally sized subintervals): S stands for the standard test, C for the CU test, L for the Log test and Le for the Lewis test.

$Case$	$Subcase$	(S, C)	(S, L)	(S, Le)	(C, L)	(C, Le)	(L, Le)
Exp	$-$	0.03	0.19	0.31	0.32	0.02	0.30
E_k	$k = 2$	-0.05	0.51	0.29	-0.12	-0.03	0.24
	$k = 4$		0.21	0.07	-0.05		0.03
	$k = 6$	-0.04	0.04	0.24	-0.04		
H_2	$c^2 = 1.25$	0.05	0.23	0.36	0.25	0.10	0.37
	$c^2 = 1.5$	0.05	0.26	0.35	0.18	0.13	0.40
	$c^2 = 2$	0.04	0.19	0.21	0.04	0.11	0.36
	$c^2 = 4$		0.03		-0.07	0.08	0.24
	$c^2 = 10$				-0.03	0.06	0.30
Z	$-$	-0.06	0.47	0.46	-0.10	-0.09	0.52
LN	$-$	-0.05	0.45	0.46	-0.09	-0.08	0.56
RRI	$p = 0.1$	0.03	0.18	0.29	0.27		0.31
	$p = 0.5$		0.11	0.09	0.11	-0.07	0.25
	$p = 0.9$	-0.04	0.02		-0.04		0.14
$EARMA$	0.25		0.13	0.24	0.31	-0.04	0.28
	0.5	0.03	0.11	0.15	0.30	-0.08	0.28
	1		0.09	0.08	0.23	-0.10	0.33
	3	-0.05	0.03	-0.02	-0.07	-0.22	0.48
	5.25			-0.04	-0.13	-0.09	0.37
mH_2	$m = 2$	0.03	0.13	0.17	0.15	0.12	0.37
	$m = 5$	0.04	0.10	0.20	0.31	0.07	0.33
	$m = 10$	0.02	0.11	0.19	0.32	0.02	0.32
	$m = 20$		0.11	0.22	0.34		0.30
$RRI(H_2)$	$p = 0.1$	0.03	0.04	0.04	-0.05	0.05	0.23
	$p = 0.5$	0.05	0.08	0.12	0.08	0.11	0.21
	$p = 0.9$				0.45	0.41	0.87

Table 31 Results for all possible composite tests based on the four alternatives for the time interval ($[0, 200]$ with 10 equally sized subintervals) using the Bonferroni procedure: The test order is (standard, CU, Log, Lewis) and is indicated by 1 if the test is included in the combination and 0 otherwise. Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

		2 tests						3 tests				4 tests
$Case$	$Subcase$	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
Exp	–	9371	9360	9383	9579	9554	9553	9467	9348	9335	9315	9406
E_k	$k = 2$	235	64	3	230	3	1	2	2	3	5	3
	$k = 4$	0	0	0	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	8061	8084	7712	9063	8394	8451	8362	7891	7814	8012	8021
	$c^2 = 1.5$	6146	5927	4916	8096	6125	6074	6012	5128	5129	5579	5425
	$c^2 = 2$	3018	2565	1279	5863	2326	2218	2015	1262	1311	1785	1505
	$c^2 = 4$	152	93	7	2068	215	188	88	6	4	10	10
	$c^2 = 10$	12	5	1	1970	354	351	166	1	1	1	1
Z	–	3758	1993	717	3055	835	702	432	428	471	525	561
LN	$(1, 1)$	1051	646	249	1960	368	318	81	77	82	81	110
RRI	$p = 0.1$	8925	8906	8856	9303	9137	9151	8994	8811	8748	8790	8889
	$p = 0.5$	5079	4340	2579	6275	3541	3518	2679	2277	2272	3530	2450
	$p = 0.9$	37	2	0	100	2	2	0	0	0	0	0
$EARMA$	0.25	9083	9155	9200	9445	9410	9498	9362	9187	9117	9101	9189
	0.5	8488	8760	8728	9100	8939	9274	9003	8796	8557	8603	8691
	1	8288	8110	7635	9046	8366	8361	8336	7658	7659	8062	7859
	3	4076	3507	3266	3853	3056	3972	2653	2687	2439	2856	2276
	5.25	4757	1774	252	3247	510	491	416	262	290	1887	322
mH_2	$m = 2$	3741	3549	2653	7260	5093	5134	4126	2458	2423	2681	2644
	$m = 5$	7139	7170	7163	9325	9196	9286	8756	7113	7071	7048	7309
	$m = 10$	7849	7863	7872	9514	9443	9487	9313	7974	7940	7928	8145
	$m = 20$	8206	8228	8267	9544	9501	9524	9429	8397	8360	8339	8507
$RRI(H_2)$	$p = 0.1$	228	135	44	2456	391	370	153	20	20	33	26
	$p = 0.5$	448	407	306	3516	2059	2052	212	35	33	40	45
	$p = 0.9$	22	3	2	798	508	497	376	0	0	0	0

Table 32 Results for all possible composite tests based on the four alternatives for the time interval ($[0, 200]$ with 10 equally sized subintervals using Holm's procedure: The test order is (standard, CU, Log, Lewis) and is indicated by 1 if the test is included in the combination and 0 otherwise. Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

		2 tests						3 tests				4 tests
Case	*Subcase*	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
Exp	$-$	9362	9348	9363	9560	9549	9537	9450	9326	9313	9306	9393
E_k	$k = 2$	234	44	3	230	3	1	1	1	2	5	3
	$k = 4$	0	0	0	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	8021	8042	7645	9031	8371	8403	8277	7788	7725	7949	7982
	$c^2 = 1.5$	6095	5851	4826	8062	6080	5978	5891	4963	4991	5475	5358
	$c^2 = 2$	2968	2476	1206	5824	2289	2150	1939	1170	1228	1699	1447
	$c^2 = 4$	147	82	6	2028	206	179	78	5	3	9	8
	$c^2 = 10$	12	5	1	1923	348	333	152	1	1	1	1
Z	$-$	3745	1786	661	3049	832	651	396	380	442	507	531
LN	$(1, 1)$	1035	494	179	1941	364	284	72	57	64	71	106
RRI	$p = 0.1$	8907	8879	8816	9268	9124	9121	8953	8748	8706	8751	8866
	$p = 0.5$	4999	4236	2464	6208	3461	3431	2575	2150	2157	3419	2401
	$p = 0.9$	34	2	0	93	1	2	0	0	0	0	0
$EARMA$	0.25	9073	9144	9184	9419	9404	9481	9337	9159	9094	9080	9179
	0.5	8467	8739	8705	9058	8925	9259	8979	8764	8525	8570	8684
	1	8270	8085	7591	9016	8352	8323	8304	7621	7628	8031	7836
	3	4016	3436	3221	3781	2977	3884	2597	2634	2376	2781	2246
	5.25	4733	1709	242	3221	500	466	394	247	277	1833	308
mH_2	$m = 2$	3690	3503	2564	7212	5025	5055	3997	2355	2313	2609	2598
	$m = 5$	7117	7150	7133	9294	9173	9264	8716	7067	7027	7009	7288
	$m = 10$	7835	7843	7854	9498	9435	9468	9286	7950	7915	7909	8129
	$m = 20$	8193	8213	8244	9528	9495	9512	9408	8373	8337	8322	8489
$RRI(H_2)$	$p = 0.1$	221	123	35	2399	383	349	141	19	19	27	26
	$p = 0.5$	438	388	293	3442	2013	2000	200	34	32	37	45
	$p = 0.9$	18	2	2	776	497	492	375	0	0	0	0

Table 33 Results for all possible composite tests based on the four alternatives for the time interval ($[0, 200]$ with 10 equally sized subintervals using Simes' procedure: The test order is (standard, CU, Log, Lewis) and is indicated by 1 if the test is included in the combination and 0 otherwise. Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

		2 tests						3 tests				4 tests
$Case$	$Subcase$	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
Exp	$-$	9362	9348	9363	9560	9549	9537	9430	9294	9289	9276	9363
E_k	$k = 2$	234	44	3	230	3	1	1	1	2	4	1
	$k = 4$	0	0	0	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	8021	8042	7645	9031	8371	8403	8212	7718	7658	7889	7851
	$c^2 = 1.5$	6095	5851	4826	8062	6080	5978	5800	4876	4904	5384	5182
	$c^2 = 2$	2968	2476	1206	5824	2289	2150	1866	1123	1166	1626	1329
	$c^2 = 4$	147	82	6	2028	206	179	72	5	3	6	5
	$c^2 = 10$	12	5	1	1923	348	333	145	1	1	1	1
Z	$-$	3745	1786	661	3049	832	651	366	348	405	481	464
LN	$(1, 1)$	1035	494	179	1941	364	284	62	51	56	69	86
RRI	$p = 0.1$	8907	8879	8816	9268	9124	9121	8916	8702	8665	8706	8791
	$p = 0.5$	4999	4236	2464	6208	3461	3431	2491	2070	2084	3319	2248
	$p = 0.9$	34	2	0	93	1	2	0	0	0	0	0
$EARMA$	0.25	9073	9144	9184	9419	9404	9481	9318	9136	9067	9055	9132
	0.5	8467	8739	8705	9058	8925	9259	8940	8730	8481	8526	8623
	1	8270	8085	7591	9016	8352	8323	8262	7582	7593	7990	7766
	3	4016	3436	3221	3781	2977	3884	2537	2589	2319	2714	2138
	5.25	4733	1709	242	3221	500	466	375	235	263	1779	289
mH_2	$m = 2$	3690	3503	2564	7212	5025	5055	3907	2293	2250	2535	2474
	$m = 5$	7117	7150	7133	9294	9173	9264	8680	7028	6980	6957	7219
	$m = 10$	7835	7843	7854	9498	9435	9468	9258	7918	7892	7883	8088
	$m = 20$	8193	8213	8244	9528	9495	9512	9386	8350	8315	8303	8454
$RRI(H_2)$	$p = 0.1$	221	123	35	2399	383	349	137	17	18	26	25
	$p = 0.5$	438	388	293	3442	2013	2000	186	32	31	35	40
	$p = 0.9$	18	2	2	776	497	492	373	0	0	0	0

Table 34 Results for individual and composite tests based on the Standard KS, CU, and Lilliefors tests for the time interval ($[0, 200]$ with 10 equally sized subintervals using the Bonferroni procedure: Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

$Case$	$Subcase$	Standard	Lilliefors	CU	(Standard+CU)	(Lilliefors+CU)
Exp	$-$	9206	9493	9544	9371	9546
E_k	$k = 2$	87	1	9979	235	3
	$k = 4$	0	0	10000	0	0
	$k = 6$	0	0	10000	0	0
H_2	$c^2 = 1.25$	7687	8550	9147	8061	8667
	$c^2 = 1.5$	5546	6257	8848	6146	6823
	$c^2 = 2$	2441	2397	8214	3018	3032
	$c^2 = 4$	118	183	6790	152	262
	$c^2 = 10$	11	324	6323	12	387
Z	$-$	2274	233	9882	3758	432
LN	$(1, 1)$	328	15	9730	1051	52
RRI	$p = 0.1$	8768	9031	9311	8925	9128
	$p = 0.5$	4824	4224	7992	5079	4669
	$p = 0.9$	21	7	5890	37	13
$EARMA$	0.25	9013	9485	9314	9083	9401
	0.5	8589	9439	8923	8488	9075
	1	8084	9142	9238	8288	9113
	3	5227	6795	5869	4076	4971
	5.25	4345	6294	9154	4757	6736
mH_2	$m = 2$	3401	4023	8249	3741	4587
	$m = 5$	6747	8462	9291	7139	8681
	$m = 10$	7394	9284	9465	7849	9345
	$m = 20$	7774	9446	9459	8206	9479
$RRI(H_2)$	$p = 0.1$	191	220	6712	228	304
	$p = 0.5$	435	200	5764	448	248
	$p = 0.9$	15	314	4431	22	363

5 Experiments with Fixed Sample Size

In this section, we present the results of the experiments for the second scenario discussed in Section 2.6. That is, we suppose that we are given the arrival data in the form of n interarrival times. In Section 5.1, we first consider the case with 200 interarrival times. The results and analysis are similar to those given in Section 3. Next, we consider a larger sample size 2000 interarrival times, whose results and analysis are similar to those given in Section 4.1. Lastly, 10 equally sized subintervals are introduced in Section 5.3 as in Section 4.2. However, because the arrival data is given as 200 interarrival times, $X_1, \ldots, X_{200}$, we transform it to an arrival process by letting $t_0 = 0$ and $t_i = \sum_{j=1}^{i} X_j$ for $i = 1, ..., 200$. We then divide the interval $[0, t_{200}]$ into 10 equally sized subintervals. Again, the results and analysis are similar to those given in Section 4.2.

5.1 Experiments with $n = 200$

Table 35 Summary statistics of the arrival processes with $n = 200$ interarrival times. Associated 95% confidence intervals are also provided. All results are based on 10000 replications. $\{X_k : 1 \le k \le 200\}$ are the interarrival times and $\{t_k : 1 \le k \le 200\}$ are the arrival times such that $t_i = \sum_{j=1}^{i} X_j$.

$Case$	$Subcase$	$E[X]$	$c^2[X]$	n	t_{200}	$Min[X]$	$Max[X]$
Exp	–	1.00 ± 0.0014	1.00 ± 0.0028	200	200 ± 0.3	0.005 ± 0.0001	5.9 ± 0.03
E_k	$k = 2$	1.00 ± 0.0010	0.50 ± 0.0013	200	200 ± 0.2	0.046 ± 0.0005	4.1 ± 0.02
	$k = 4$	1.00 ± 0.0007	0.25 ± 0.0005	200	200 ± 0.1	0.153 ± 0.0009	2.9 ± 0.01
	$k = 6$	1.00 ± 0.0006	0.17 ± 0.0004	200	200 ± 0.1	0.236 ± 0.0011	2.5 ± 0.01
H_2	$c^2 = 1.25$	1.00 ± 0.0016	1.24 ± 0.0043	200	200 ± 0.3	0.004 ± 0.0001	7.2 ± 0.04
	$c^2 = 1.5$	1.00 ± 0.0017	1.48 ± 0.0057	200	200 ± 0.3	0.004 ± 0.0001	8.3 ± 0.05
	$c^2 = 2$	1.00 ± 0.0019	1.95 ± 0.0088	200	200 ± 0.4	0.004 ± 0.0001	10.2 ± 0.06
	$c^2 = 4$	1.00 ± 0.0028	3.77 ± 0.0225	200	200 ± 0.6	0.003 ± 0.0001	16.4 ± 0.11
	$c^2 = 10$	1.00 ± 0.0044	8.64 ± 0.0742	200	200 ± 0.9	0.003 ± 0.0001	29.7 ± 0.26
Z	–	1.00 ± 0.0014	0.95 ± 0.0109	200	200 ± 0.3	0.018 ± 0.0003	8.3 ± 0.10
LN	–	1.00 ± 0.0014	0.97 ± 0.0065	200	200 ± 0.3	0.075 ± 0.0005	7.4 ± 0.06
RRI	$p = 0.1$	1.00 ± 0.0015	0.99 ± 0.0030	200	200 ± 0.3	0.005 ± 0.0001	5.8 ± 0.03
	$p = 0.5$	1.00 ± 0.0024	0.98 ± 0.0045	200	200 ± 0.5	0.010 ± 0.0002	5.2 ± 0.03
	$p = 0.9$	1.00 ± 0.0058	0.88 ± 0.0084	200	200 ± 1.2	0.050 ± 0.0010	3.6 ± 0.02
$EARMA$	0.25	1.00 ± 0.0017	0.99 ± 0.0028	200	200 ± 0.3	0.005 ± 0.0001	5.8 ± 0.03
	0.5	1.00 ± 0.0019	0.99 ± 0.0030	200	200 ± 0.4	0.005 ± 0.0001	5.8 ± 0.03
	1	1.00 ± 0.0024	0.97 ± 0.0033	200	200 ± 0.5	0.005 ± 0.0001	5.6 ± 0.03
	3	1.00 ± 0.0036	0.97 ± 0.0050	200	200 ± 0.7	0.020 ± 0.0004	5.5 ± 0.03
	5.25	1.00 ± 0.0046	0.90 ± 0.0049	200	201 ± 0.9	0.005 ± 0.0001	4.9 ± 0.03
mH_2	$m = 2$	1.00 ± 0.0028	2.35 ± 0.0186	200	200 ± 0.6	0.004 ± 0.0001	13.4 ± 0.11
	$m = 5$	1.00 ± 0.0027	1.32 ± 0.0079	200	200 ± 0.5	0.004 ± 0.0001	8.1 ± 0.07
	$m = 10$	1.00 ± 0.0027	1.11 ± 0.0046	200	200 ± 0.5	0.005 ± 0.0001	6.6 ± 0.04
	$m = 20$	1.00 ± 0.0025	1.03 ± 0.0030	200	200 ± 0.5	0.005 ± 0.0001	6.1 ± 0.03
$RRI(H_2)$	$p = 0.1$	1.00 ± 0.0031	3.74 ± 0.0238	200	201 ± 0.6	0.003 ± 0.0001	16.0 ± 0.11
	$p = 0.5$	1.00 ± 0.0048	3.43 ± 0.0293	200	200 ± 1.0	0.006 ± 0.0001	13.3 ± 0.11
	$p = 0.9$	1.00 ± 0.0119	2.21 ± 0.0351	200	200 ± 2.4	0.032 ± 0.0007	6.8 ± 0.10

Table 36 Average and c^2 of transformed interarrival times ($n = 200$) with associated 95% confidence intervals. Transformations are based on the fact that $u_j = t_j/t_{200}$ ($j = 1, ..., n-1$) are distributed as uniform on $[0, 1]$ and each sample has 199 transformed values. All results are based on 10000 replications.

$Case$	$Subcase$	$E[X^{CU}]$	$c^2[X^{CU}]$	$E[X^{Log}]$	$c^2[X^{Log}]$	$E[X^{Lewis}]$	$c^2[X^{Lewis}]$
Exp	–	0.50 ± 0.0004	0.33 ± 0.0007	1.00 ± 0.0014	1.00 ± 0.0028	0.50 ± 0.0004	0.34 ± 0.0007
E_k	$k = 2$	0.50 ± 0.0003	0.33 ± 0.0005	0.99 ± 0.0010	0.50 ± 0.0013	0.62 ± 0.0003	0.16 ± 0.0003
	$k = 4$	0.50 ± 0.0002	0.33 ± 0.0003	0.99 ± 0.0007	0.25 ± 0.0006	0.73 ± 0.0003	0.08 ± 0.0002
	$k = 6$	0.50 ± 0.0002	0.33 ± 0.0003	0.99 ± 0.0006	0.17 ± 0.0004	0.77 ± 0.0002	0.05 ± 0.0001
H_2	$c^2 = 1.25$	0.50 ± 0.0004	0.34 ± 0.0007	1.02 ± 0.0018	1.47 ± 0.0187	0.48 ± 0.0004	0.36 ± 0.0007
	$c^2 = 1.5$	0.50 ± 0.0005	0.34 ± 0.0008	1.02 ± 0.0018	1.59 ± 0.0180	0.46 ± 0.0004	0.38 ± 0.0008
	$c^2 = 2$	0.50 ± 0.0006	0.34 ± 0.0010	1.01 ± 0.0019	1.91 ± 0.0085	0.42 ± 0.0005	0.40 ± 0.0009
	$c^2 = 4$	0.50 ± 0.0008	0.34 ± 0.0014	1.02 ± 0.0026	3.57 ± 0.0209	0.35 ± 0.0007	0.44 ± 0.0012
	$c^2 = 10$	0.50 ± 0.0012	0.36 ± 0.0021	1.05 ± 0.0036	7.74 ± 0.0653	0.31 ± 0.0011	0.43 ± 0.0016
Z	–	0.50 ± 0.0004	0.34 ± 0.0007	1.00 ± 0.0013	0.91 ± 0.0099	0.58 ± 0.0006	0.20 ± 0.0004
LN	–	0.50 ± 0.0004	0.33 ± 0.0007	1.00 ± 0.0013	0.96 ± 0.0058	0.56 ± 0.0005	0.18 ± 0.0004
RRI	$p = 0.1$	0.50 ± 0.0004	0.34 ± 0.0007	1.00 ± 0.0015	0.99 ± 0.0030	0.50 ± 0.0004	0.33 ± 0.0008
	$p = 0.5$	0.50 ± 0.0007	0.34 ± 0.0012	1.01 ± 0.0023	0.99 ± 0.0047	0.50 ± 0.0007	0.34 ± 0.0012
	$p = 0.9$	0.50 ± 0.0015	0.37 ± 0.0029	1.07 ± 0.0054	0.94 ± 0.0130	0.54 ± 0.0016	0.33 ± 0.0030
$EARMA$	0.25	0.50 ± 0.0005	0.34 ± 0.0008	1.00 ± 0.0016	0.99 ± 0.0028	0.50 ± 0.0004	0.33 ± 0.0007
	0.5	0.50 ± 0.0006	0.34 ± 0.0009	1.01 ± 0.0019	0.98 ± 0.0029	0.50 ± 0.0004	0.33 ± 0.0007
	1	0.50 ± 0.0004	0.34 ± 0.0008	1.02 ± 0.0017	1.16 ± 0.0182	0.52 ± 0.0004	0.32 ± 0.0007
	3	0.50 ± 0.0007	0.34 ± 0.0013	1.02 ± 0.0026	0.95 ± 0.0180	0.55 ± 0.0008	0.28 ± 0.0013
	5.25	0.50 ± 0.0012	0.36 ± 0.0022	1.04 ± 0.0040	0.80 ± 0.0039	0.52 ± 0.0008	0.32 ± 0.0012
mH_2	$m = 2$	0.50 ± 0.0008	0.34 ± 0.0013	1.02 ± 0.0025	2.22 ± 0.0170	0.42 ± 0.0007	0.38 ± 0.0008
	$m = 5$	0.50 ± 0.0007	0.34 ± 0.0013	1.01 ± 0.0024	1.28 ± 0.0070	0.47 ± 0.0005	0.36 ± 0.0007
	$m = 10$	0.50 ± 0.0004	0.34 ± 0.0007	1.00 ± 0.0014	1.02 ± 0.0029	0.50 ± 0.0004	0.34 ± 0.0007
	$m = 20$	0.50 ± 0.0004	0.34 ± 0.0007	1.00 ± 0.0014	1.00 ± 0.0027	0.50 ± 0.0004	0.34 ± 0.0007
$RRI(H_2)$	$p = 0.1$	0.50 ± 0.0009	0.35 ± 0.0015	1.03 ± 0.0027	3.54 ± 0.0220	0.35 ± 0.0008	0.44 ± 0.0014
	$p = 0.5$	0.50 ± 0.0012	0.36 ± 0.0022	1.05 ± 0.0039	3.22 ± 0.0273	0.37 ± 0.0011	0.46 ± 0.0024
	$p = 0.9$	0.50 ± 0.0022	0.43 ± 0.0050	1.14 ± 0.0075	2.18 ± 0.0399	0.44 ± 0.0022	0.49 ± 0.0065

Table 37 Performance of Alternative Tests for $n = 200$: Number of KS tests passed (denoted by $\#P$) at significance level 0.05 out of 10,000 replications and the average p-values (denoted by $E[p\text{-value}]$) with associated 95% confidence intervals.

		Standard KS		Conditional		Log		Lewis	
$Case$	$Subcase$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$
Exp	–	9487	0.50 ± 0.006	9511	0.50 ± 0.006	9478	0.50 ± 0.006	9493	0.50 ± 0.006
E_k	$k = 2$	28	0.00 ± 0.000	9985	0.78 ± 0.004	21	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 4$	0	0.00 ± 0.000	10000	0.94 ± 0.002	0	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 6$	0	0.00 ± 0.000	10000	0.98 ± 0.001	0	0.00 ± 0.000	0	0.00 ± 0.000
H_2	$c^2 = 1.25$	8843	0.42 ± 0.006	9169	0.45 ± 0.006	9015	0.43 ± 0.006	8138	0.35 ± 0.006
	$c^2 = 1.5$	7204	0.27 ± 0.005	8811	0.40 ± 0.006	7940	0.33 ± 0.006	5441	0.18 ± 0.005
	$c^2 = 2$	3603	0.09 ± 0.003	7186	0.24 ± 0.005	4447	0.15 ± 0.004	695	0.02 ± 0.001
	$c^2 = 4$	90	0.00 ± 0.000	3648	0.08 ± 0.003	1323	0.04 ± 0.002	22	0.00 ± 0.000
	$c^2 = 10$	0	0.00 ± 0.000	935	0.02 ± 0.001	1205	0.04 ± 0.003	67	0.00 ± 0.001
Z	–	1200	0.02 ± 0.001	9438	0.57 ± 0.006	1228	0.02 ± 0.001	187	0.00 ± 0.000
LN	–	98	0.00 ± 0.000	9517	0.53 ± 0.006	219	0.01 ± 0.000	24	0.00 ± 0.000
RRI	$p = 0.1$	9048	0.41 ± 0.006	9044	0.42 ± 0.006	9056	0.42 ± 0.006	9121	0.41 ± 0.005
	$p = 0.5$	4659	0.11 ± 0.003	5587	0.16 ± 0.004	5118	0.13 ± 0.003	4624	0.11 ± 0.003
	$p = 0.9$	16	0.00 ± 0.000	701	0.01 ± 0.001	83	0.00 ± 0.000	13	0.00 ± 0.000
$EARMA$	0.25	9284	0.47 ± 0.006	8564	0.36 ± 0.005	9266	0.47 ± 0.006	9498	0.50 ± 0.006
	0.5	8865	0.43 ± 0.006	7519	0.27 ± 0.005	8908	0.43 ± 0.006	9393	0.49 ± 0.006
	1	8178	0.37 ± 0.006	9238	0.48 ± 0.006	8918	0.42 ± 0.006	8115	0.34 ± 0.006
	3	5209	0.21 ± 0.005	5671	0.19 ± 0.005	4534	0.13 ± 0.004	4686	0.17 ± 0.005
	5.25	4100	0.14 ± 0.004	1598	0.03 ± 0.002	4216	0.14 ± 0.004	5680	0.21 ± 0.005
mH_2	$m = 2$	4398	0.14 ± 0.004	4355	0.11 ± 0.003	5332	0.21 ± 0.005	1546	0.04 ± 0.002
	$m = 5$	7514	0.32 ± 0.006	5400	0.17 ± 0.004	7922	0.35 ± 0.006	7228	0.29 ± 0.006
	$m = 10$	7818	0.35 ± 0.006	9446	0.48 ± 0.006	9475	0.49 ± 0.006	9457	0.50 ± 0.006
	$m = 20$	7996	0.37 ± 0.006	9494	0.50 ± 0.006	9517	0.50 ± 0.006	9489	0.50 ± 0.006
$RRI(H_2)$	$p = 0.1$	104	0.00 ± 0.000	2986	0.07 ± 0.002	1432	0.04 ± 0.002	37	0.00 ± 0.000
	$p = 0.5$	253	0.00 ± 0.001	1105	0.02 ± 0.001	1738	0.04 ± 0.002	215	0.00 ± 0.001
	$p = 0.9$	4	0.00 ± 0.000	229	0.00 ± 0.001	52	0.00 ± 0.000	5	0.00 ± 0.000

Table 38 Performance of Alternative Tests for Untransformed Interarrival Times with $n = 200$: Number of KS tests passed (denoted by $\#P$) at significance level 0.05 out of $10,000$ replications and the average p-values (denoted by $E[p\text{-value}]$) with associated 95% confidence intervals.

		KS with estimated mean		Lillifors test	
$Case$	$Subcase$	$\#P$	$E[p\text{-value}]$	$\#P$	$E[p\text{-value}]$
Exp	$-$	9945	0.65 ± 0.005	9495	0.37 ± 0.003
E_k	$k = 2$	2	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 4$	0	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 6$	0	0.00 ± 0.000	0	0.00 ± 0.000
H_2	$c^2 = 1.25$	9516	0.50 ± 0.006	8161	0.28 ± 0.004
	$c^2 = 1.5$	8030	0.29 ± 0.005	5260	0.14 ± 0.003
	$c^2 = 2$	3818	0.09 ± 0.003	1507	0.03 ± 0.002
	$c^2 = 4$	348	0.01 ± 0.001	90	0.00 ± 0.000
	$c^2 = 10$	360	0.01 ± 0.001	153	0.01 ± 0.001
Z	$-$	1042	0.02 ± 0.001	161	0.00 ± 0.000
LN	$-$	147	0.01 ± 0.000	4	0.00 ± 0.000
RRI	$p = 0.1$	9842	0.56 ± 0.005	9020	0.32 ± 0.003
	$p = 0.5$	6857	0.19 ± 0.004	3820	0.08 ± 0.002
	$p = 0.9$	50	0.00 ± 0.000	4	0.00 ± 0.000
$EARMA$	0.25	9947	0.65 ± 0.005	9521	0.37 ± 0.003
	0.5	9918	0.65 ± 0.005	9415	0.37 ± 0.003
	1	9829	0.60 ± 0.005	9073	0.34 ± 0.003
	3	8387	0.43 ± 0.007	6744	0.23 ± 0.004
	5.25	8002	0.34 ± 0.006	5891	0.18 ± 0.004
mH_2	$m = 2$	4908	0.15 ± 0.004	2628	0.07 ± 0.003
	$m = 5$	9421	0.49 ± 0.006	7946	0.27 ± 0.004
	$m = 10$	9877	0.62 ± 0.005	9183	0.35 ± 0.003
	$m = 20$	9929	0.65 ± 0.005	9487	0.37 ± 0.003
$RRI(H_2)$	$p = 0.1$	432	0.01 ± 0.001	122	0.00 ± 0.000
	$p = 0.5$	674	0.01 ± 0.001	216	0.01 ± 0.001
	$p = 0.9$	19	0.00 ± 0.000	0	0.00 ± 0.000

Table 39 Pairwise Correlations of p-values ($n = 200$): S stands for the standard test, C for the CU test, L for the Log test and Le for the Lewis test.

$Case$	$Subcase$	(S,C)	(S,L)	(S,Le)	(C,L)	(C,Le)	(L,Le)
Exp	$-$		0.30	0.49	0.32		0.45
E_k	$k = 2$	-0.05	0.49	0.28	-0.14	-0.03	0.32
	$k = 4$	-0.03	0.08	0.04	-0.03	-0.03	0.04
	$k = 6$		0.20	0.09	-0.02		0.11
H_2	$c^2 = 1.25$	0.05	0.19	0.33	0.27	0.11	0.36
	$c^2 = 1.5$	0.06	0.24	0.31	0.18	0.13	0.40
	$c^2 = 2$	0.06	0.20	0.18	0.04	0.14	0.31
	$c^2 = 4$	0.03	0.03	0.03		0.11	0.16
	$c^2 = 10$				0.22	0.41	0.22
Z	$-$	-0.06	0.57	0.53	-0.12	-0.13	0.52
LN	$-$	-0.04	0.50	0.54	-0.07	-0.09	0.53
RRI	$p = 0.1$		0.29	0.50	0.28		0.43
	$p = 0.5$		0.27	0.47	0.16		0.37
	$p = 0.9$		0.06	0.27			0.14
$EARMA$	0.25		0.23	0.41	0.32		0.38
	0.5		0.19	0.37	0.32		0.34
	1	-0.04	0.10	0.11	0.23	-0.11	0.32
	3	-0.03	0.06	0.04	-0.05	-0.19	0.44
	5.25	-0.08	0.15	0.35	0.00	-0.18	0.29
mH_2	$m = 2$	0.05	0.14	0.09	0.17	0.28	0.32
	$m = 5$		0.13	0.20	0.34	0.27	0.34
	$m = 10$		0.09	0.19	0.31	0.02	0.31
	$m = 20$	0.02	0.11	0.21	0.32		0.30
$RRI(H_2)$	$p = 0.1$	0.04	0.04		0.05	0.13	0.16
	$p = 0.5$		0.05	0.03	0.13	0.21	0.17
	$p = 0.9$			0.17			0.14

Table 40 Results for all possible composite tests based on the four alternatives for $n = 200$ using the Bonferroni procedure: The test order is (standard, CU, Log, Lewis) and is indicated by 1 if the test is included in the combination and 0 otherwise. Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

		2 tests						3 tests				4 tests
Case	*Subcase*	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
Exp	–	9529	9541	9578	9561	9522	9555	9508	9539	9510	9458	9550
E_k	$k = 2$	82	31	0	96	0	0	0	0	0	1	0
	$k = 4$	0	0	0	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	8906	8854	8336	9106	8420	8470	8239	8210	8173	8346	8349
	$c^2 = 1.5$	7527	7172	5703	8114	6148	6099	5625	5368	5345	5838	5698
	$c^2 = 2$	3904	2889	739	4372	984	973	1113	857	863	1586	991
	$c^2 = 4$	94	43	2	765	34	34	44	2	1	2	1
	$c^2 = 10$	0	0	0	387	80	85	87	0	0	0	0
Z	–	2115	1160	341	2091	371	348	326	305	316	369	396
LN	$(1, 1)$	382	184	58	711	77	64	29	22	26	23	39
RRI	$p = 0.1$	8951	9005	9120	9046	8987	9100	8925	8986	8897	8788	8966
	$p = 0.5$	3850	4008	4147	4547	3787	4238	3277	3309	2981	2771	3032
	$p = 0.9$	0	4	11	17	5	6	0	0	0	0	0
$EARMA$	0.25	8732	9221	9381	8900	8905	9419	8999	9297	8886	8816	8997
	0.5	7767	8710	9078	8003	8088	9068	8198	8854	8030	7876	8147
	1	8358	8189	7767	9017	8378	8369	8306	7787	7793	8111	7924
	3	3829	3477	3563	3590	3156	3918	2563	2768	2437	2643	2248
	5.25	898	2524	3635	1278	967	3664	963	2375	683	604	699
mH_2	$m = 2$	2964	3375	1196	3671	1511	1806	1690	1353	1150	1687	1318
	$m = 5$	5149	7017	6606	5980	5523	7152	5837	6440	5116	5245	5399
	$m = 10$	8224	8247	8252	9521	9446	9488	9296	8303	8273	8256	8444
	$m = 20$	8392	8422	8458	9556	9513	9552	9470	8586	8557	8537	8709
$RRI(H_2)$	$p = 0.1$	90	60	0	748	49	52	59	1	2	12	2
	$p = 0.5$	71	131	24	550	174	215	140	21	15	19	17
	$p = 0.9$	1	1	1	4	1	3	0	0	0	0	0

Table 41 Results for all possible composite tests based on the four alternatives for $n = 200$ using Holm's procedure: The test order is (standard, CU, Log, Lewis) and is indicated by 1 if the test is included in the combination and 0 otherwise. Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

		2 tests						3 tests				4 tests
$Case$	$Subcase$	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
Exp	$-$	9519	9528	9553	9541	9509	9519	9459	9487	9471	9439	9531
E_k	$k = 2$	81	17	0	96	0	0	0	0	0	1	0
	$k = 4$	0	0	0	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	8887	8813	8281	9070	8393	8431	8176	8135	8091	8309	8307
	$c^2 = 1.5$	7481	7103	5608	8080	6087	6026	5504	5226	5214	5750	5609
	$c^2 = 2$	3810	2812	703	4314	960	949	1046	795	804	1513	951
	$c^2 = 4$	88	39	2	726	33	34	44	2	1	2	1
	$c^2 = 10$	0	0	0	366	76	84	84	0	0	0	0
Z	$-$	2081	1003	295	2062	359	313	269	234	253	337	352
LN	$(1, 1)$	375	119	36	702	75	52	24	18	21	20	32
RRI	$p = 0.1$	8936	8980	9080	9014	8967	9069	8873	8922	8835	8747	8939
	$p = 0.5$	3733	3893	4011	4433	3679	4107	3101	3132	2799	2636	2931
	$p = 0.9$	0	3	8	15	5	6	0	0	0	0	0
$EARMA$	0.25	8712	9207	9365	8861	8892	9394	8951	9259	8845	8787	8976
	0.5	7725	8682	9051	7941	8066	9041	8147	8804	7979	7842	8119
	1	8341	8164	7721	8984	8368	8326	8274	7743	7753	8081	7909
	3	3772	3421	3515	3505	3081	3831	2489	2719	2390	2571	2214
	5.25	859	2476	3564	1223	920	3610	895	2296	629	574	670
mH_2	$m = 2$	2882	3307	1148	3607	1474	1771	1616	1284	1078	1600	1271
	$m = 5$	5097	6970	6554	5899	5454	7088	5730	6339	5006	5176	5352
	$m = 10$	8203	8228	8233	9500	9441	9477	9277	8280	8249	8227	8437
	$m = 20$	8380	8401	8436	9541	9505	9537	9453	8561	8535	8509	8701
$RRI(H_2)$	$p = 0.1$	83	54	0	701	47	51	57	1	1	11	1
	$p = 0.5$	61	125	23	530	168	204	134	21	15	18	17
	$p = 0.9$	0	0	1	2	1	3	0	0	0	0	0

Table 42 Results for all possible composite tests based on the four alternatives for $n = 200$ using Simes' procedure: The test order is (standard, CU, Log, Lewis) and is indicated by 1 if the test is included in the combination and 0 otherwise. Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

		2 tests						3 tests				4 tests
$Case$	$Subcase$	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
Exp	$-$	9519	9528	9553	9541	9509	9519	9425	9459	9450	9413	9485
E_k	$k = 2$	81	17	0	96	0	0	0	0	0	1	0
	$k = 4$	0	0	0	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	8887	8813	8281	9070	8393	8431	8114	8080	8041	8262	8199
	$c^2 = 1.5$	7481	7103	5608	8080	6087	6026	5379	5104	5101	5657	5424
	$c^2 = 2$	3810	2812	703	4314	960	949	1007	756	763	1443	869
	$c^2 = 4$	88	39	2	726	33	34	41	2	1	2	1
	$c^2 = 10$	0	0	0	366	76	84	80	0	0	0	0
Z	$-$	2081	1003	295	2062	359	313	245	220	234	320	284
LN	$(1, 1)$	375	119	36	702	75	52	21	17	18	19	24
RRI	$p = 0.1$	8936	8980	9080	9014	8967	9069	8821	8883	8790	8710	8861
	$p = 0.5$	3733	3893	4011	4433	3679	4107	2990	3017	2702	2539	2756
	$p = 0.9$	0	3	8	15	5	6	0	0	0	0	0
$EARMA$	0.25	8712	9207	9365	8861	8892	9394	8906	9224	8811	8743	8923
	0.5	7725	8682	9051	7941	8066	9041	8089	8762	7935	7784	8058
	1	8341	8164	7721	8984	8368	8326	8216	7683	7703	8046	7831
	3	3772	3421	3515	3505	3081	3831	2435	2669	2341	2523	2116
	5.25	859	2476	3564	1223	920	3610	864	2255	601	553	619
mH_2	$m = 2$	2882	3307	1148	3607	1474	1771	1570	1241	1039	1548	1181
	$m = 5$	5097	6970	6554	5899	5454	7088	5647	6262	4927	5092	5222
	$m = 10$	8203	8228	8233	9500	9441	9477	9245	8254	8220	8194	8382
	$m = 20$	8380	8401	8436	9541	9505	9537	9426	8534	8496	8476	8659
$RRI(H_2)$	$p = 0.1$	83	54	0	701	47	51	56	1	1	10	1
	$p = 0.5$	61	125	23	530	168	204	123	18	15	16	15
	$p = 0.9$	0	0	1	2	1	3	0	0	0	0	0

Table 43 Results for individual and composite tests based on the Standard KS, CU, and Lilliefors tests for $n = 200$ using the Bonferroni procedure: Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

$Case$	$Subcase$	Standard	Lilliefors	CU	(Standard+CU)	(Lilliefors+CU)
Exp	$-$	9487	9495	9511	9529	9506
E_k	$k = 2$	28	0	9985	82	0
	$k = 4$	0	0	10000	0	0
	$k = 6$	0	0	10000	0	0
H_2	$c^2 = 1.25$	8843	8161	9169	8906	8430
	$c^2 = 1.5$	7204	5260	8811	7527	5980
	$c^2 = 2$	3603	1507	7186	3904	1973
	$c^2 = 4$	90	90	3648	94	122
	$c^2 = 10$	0	153	935	0	162
Z	$-$	1200	161	9438	2115	297
LN	$(1, 1)$	98	4	9517	382	15
RRI	$p = 0.1$	9048	9020	9044	8951	8946
	$p = 0.5$	4659	3820	5587	3850	3256
	$p = 0.9$	16	4	701	0	0
$EARMA$	0.25	9284	9521	8564	8732	8904
	0.5	8865	9415	7519	7767	8093
	1	8178	9073	9238	8358	9047
	3	5209	6744	5671	3829	4851
	5.25	4100	5891	1598	898	1055
mH_2	$m = 2$	4398	2628	4355	2964	2305
	$m = 5$	7514	7946	5400	5149	5783
	$m = 10$	7818	9183	9446	8224	9299
	$m = 20$	7996	9487	9494	8392	9505
$RRI(H_2)$	$p = 0.1$	104	122	2986	90	160
	$p = 0.5$	253	216	1105	71	181
	$p = 0.9$	4	0	229	1	0

5.2 Experiments with $n = 2000$

Table 44 Summary statistics of the arrival processes with $n = 2000$ interarrival times. Associated 95% confidence intervals are also provided. All results are based on 10000 replications. $\{X_k : 1 \leq k \leq 200\}$ are the interarrival times and $\{t_k : 1 \leq k \leq 200\}$ are the arrival times such that $t_i = \sum_{j=1}^{i} X_j$.

$Case$	$Subcase$	$E[X]$	$c^2[X]$	n	t_{200}	$Min[X]$	$Max[X]$
Exp	$-$	1.00 ± 0.0004	1.00 ± 0.0009	2000	2001 ± 0.9	0.001 ± 0.0000	8.2 ± 0.03
E_k	$k = 2$	1.00 ± 0.0003	0.50 ± 0.0004	2000	2000 ± 0.6	0.014 ± 0.0001	5.5 ± 0.02
	$k = 4$	1.00 ± 0.0002	0.25 ± 0.0002	2000	1999 ± 0.4	0.080 ± 0.0005	3.7 ± 0.01
	$k = 6$	1.00 ± 0.0002	0.17 ± 0.0001	2000	2000 ± 0.4	0.149 ± 0.0006	3.0 ± 0.01
H_2	$c^2 = 1.25$	1.00 ± 0.0005	1.25 ± 0.0014	2000	2000 ± 1.0	0.000 ± 0.0000	10.6 ± 0.04
	$c^2 = 1.5$	1.00 ± 0.0005	1.50 ± 0.0019	2000	1999 ± 1.1	0.000 ± 0.0000	12.5 ± 0.05
	$c^2 = 2$	1.00 ± 0.0006	2.00 ± 0.0029	2000	2000 ± 1.3	0.000 ± 0.0000	15.6 ± 0.06
	$c^2 = 4$	1.00 ± 0.0009	3.98 ± 0.0077	2000	2000 ± 1.8	0.000 ± 0.0000	26.7 ± 0.11
	$c^2 = 10$	1.00 ± 0.0014	9.86 ± 0.0290	2000	2000 ± 2.8	0.000 ± 0.0000	53.9 ± 0.27
Z	$-$	1.00 ± 0.0004	0.99 ± 0.0042	2000	2001 ± 0.9	0.002 ± 0.0000	17.9 ± 0.11
LN	$-$	1.00 ± 0.0004	1.00 ± 0.0022	2000	2000 ± 0.9	0.042 ± 0.0002	12.9 ± 0.08
RRI	$p = 0.1$	1.00 ± 0.0005	1.00 ± 0.0010	2000	2000 ± 1.0	0.001 ± 0.0000	8.1 ± 0.03
	$p = 0.5$	1.00 ± 0.0008	1.00 ± 0.0015	2000	2001 ± 1.5	0.001 ± 0.0000	7.5 ± 0.03
	$p = 0.9$	1.00 ± 0.0019	0.98 ± 0.0035	2000	1999 ± 3.8	0.005 ± 0.0001	5.9 ± 0.02
$EARMA$	0.25	1.00 ± 0.0005	1.00 ± 0.0010	2000	2000 ± 1.1	0.000 ± 0.0000	8.1 ± 0.03
	0.5	1.00 ± 0.0006	1.00 ± 0.0010	2000	2000 ± 1.2	0.001 ± 0.0000	8.1 ± 0.03
	1	1.00 ± 0.0008	1.00 ± 0.0011	2000	2000 ± 1.5	0.000 ± 0.0000	8.0 ± 0.03
	3	1.00 ± 0.0011	1.00 ± 0.0017	2000	1999 ± 2.3	0.002 ± 0.0000	8.0 ± 0.03
	5.25	1.00 ± 0.0015	0.99 ± 0.0020	2000	2000 ± 3.0	0.001 ± 0.0000	7.5 ± 0.03
mH_2	$m = 2$	1.00 ± 0.0009	2.49 ± 0.0067	2000	2000 ± 1.7	0.000 ± 0.0000	23.6 ± 0.11
	$m = 5$	1.00 ± 0.0009	1.38 ± 0.0031	2000	2000 ± 1.7	0.000 ± 0.0000	15.1 ± 0.10
	$m = 10$	1.00 ± 0.0009	1.14 ± 0.0015	2000	2001 ± 1.7	0.000 ± 0.0000	10.5 ± 0.05
	$m = 20$	1.00 ± 0.0009	1.06 ± 0.0011	2000	1999 ± 1.7	0.000 ± 0.0000	9.1 ± 0.04
$RRI(H_2)$	$p = 0.1$	1.00 ± 0.0010	3.98 ± 0.0086	2000	2002 ± 2.0	0.000 ± 0.0000	26.2 ± 0.11
	$p = 0.5$	1.00 ± 0.0015	3.93 ± 0.0129	2000	1999 ± 3.1	0.001 ± 0.0000	23.5 ± 0.11
	$p = 0.9$	1.00 ± 0.0038	3.62 ± 0.0259	2000	2002 ± 7.6	0.003 ± 0.0001	16.5 ± 0.11

Table 45 Average and c^2 of transformed interarrival times ($n = 2000$) with associated 95% confidence intervals. Transformations are based on the fact that $u_j = t_j/t_{200}\,(j = 1, ..., n-1)$ are distributed as uniform on $[0, 1]$ and each sample has 199 transformed values. All results are based on 10000 replications.

$Case$	$Subcase$	$E[X^{CU}]$	$c^2[X^{CU}]$	$E[X^{Log}]$	$c^2[X^{Log}]$	$E[X^{Lewis}]$	$c^2[X^{Lewis}]$
Exp	–	0.50 ± 0.0001	0.33 ± 0.0002	1.00 ± 0.0004	1.00 ± 0.0009	0.50 ± 0.0001	0.33 ± 0.0002
E_k	$k = 2$	0.50 ± 0.0001	0.33 ± 0.0001	1.00 ± 0.0003	0.50 ± 0.0004	0.62 ± 0.0001	0.16 ± 0.0001
	$k = 4$	0.50 ± 0.0001	0.33 ± 0.0001	1.00 ± 0.0002	0.25 ± 0.0001	0.73 ± 0.0001	0.08 ± 0.0001
	$k = 6$	0.50 ± 0.0001	0.33 ± 0.0001	1.00 ± 0.0002	0.17 ± 0.0001	0.77 ± 0.0001	0.05 ± 0.0001
H_2	$c^2 = 1.25$	0.50 ± 0.0001	0.33 ± 0.0002	1.00 ± 0.0005	1.25 ± 0.0014	0.47 ± 0.0001	0.36 ± 0.0002
	$c^2 = 1.5$	0.50 ± 0.0002	0.33 ± 0.0003	1.00 ± 0.0005	1.49 ± 0.0019	0.45 ± 0.0001	0.38 ± 0.0002
	$c^2 = 2$	0.50 ± 0.0002	0.33 ± 0.0003	1.00 ± 0.0006	1.99 ± 0.0029	0.42 ± 0.0002	0.40 ± 0.0003
	$c^2 = 4$	0.50 ± 0.0003	0.33 ± 0.0004	1.00 ± 0.0009	3.94 ± 0.0076	0.35 ± 0.0002	0.44 ± 0.0004
	$c^2 = 10$	0.50 ± 0.0004	0.34 ± 0.0007	1.01 ± 0.0013	9.64 ± 0.0282	0.30 ± 0.0003	0.42 ± 0.0005
Z	–	0.50 ± 0.0001	0.33 ± 0.0002	1.00 ± 0.0004	0.99 ± 0.0041	0.58 ± 0.0002	0.19 ± 0.0001
LN	–	0.50 ± 0.0001	0.33 ± 0.0002	1.00 ± 0.0004	0.99 ± 0.0022	0.56 ± 0.0002	0.18 ± 0.0001
RRI	$p = 0.1$	0.50 ± 0.0001	0.33 ± 0.0002	1.00 ± 0.0005	1.00 ± 0.0010	0.50 ± 0.0001	0.33 ± 0.0002
	$p = 0.5$	0.50 ± 0.0002	0.33 ± 0.0004	1.00 ± 0.0007	1.00 ± 0.0015	0.50 ± 0.0002	0.33 ± 0.0004
	$p = 0.9$	0.50 ± 0.0005	0.34 ± 0.0009	1.02 ± 0.0018	1.00 ± 0.0046	0.50 ± 0.0005	0.33 ± 0.0009
$EARMA$	0.25	0.50 ± 0.0002	0.33 ± 0.0003	1.00 ± 0.0005	1.00 ± 0.0010	0.50 ± 0.0001	0.33 ± 0.0002
	0.5	0.50 ± 0.0002	0.33 ± 0.0003	1.00 ± 0.0006	1.00 ± 0.0010	0.50 ± 0.0001	0.33 ± 0.0002
	1	0.50 ± 0.0002	0.33 ± 0.0004	1.00 ± 0.0007	0.99 ± 0.0011	0.50 ± 0.0002	0.33 ± 0.0002
	3	0.50 ± 0.0003	0.34 ± 0.0006	1.01 ± 0.0012	0.99 ± 0.0017	0.50 ± 0.0003	0.33 ± 0.0005
	5.25	0.50 ± 0.0004	0.34 ± 0.0007	1.01 ± 0.0014	0.97 ± 0.0019	0.50 ± 0.0003	0.33 ± 0.0004
mH_2	$m = 2$	0.50 ± 0.0003	0.33 ± 0.0004	1.00 ± 0.0009	2.46 ± 0.0066	0.42 ± 0.0002	0.38 ± 0.0003
	$m = 5$	0.50 ± 0.0002	0.33 ± 0.0004	1.00 ± 0.0008	1.37 ± 0.0030	0.47 ± 0.0002	0.36 ± 0.0002
	$m = 10$	0.50 ± 0.0003	0.33 ± 0.0004	1.00 ± 0.0008	1.14 ± 0.0015	0.48 ± 0.0001	0.35 ± 0.0002
	$m = 20$	0.50 ± 0.0002	0.33 ± 0.0004	1.00 ± 0.0008	1.06 ± 0.0011	0.49 ± 0.0001	0.34 ± 0.0002
$RRI(H_2)$	$p = 0.1$	0.50 ± 0.0003	0.33 ± 0.0005	1.01 ± 0.0009	3.93 ± 0.0084	0.35 ± 0.0002	0.44 ± 0.0004
	$p = 0.5$	0.50 ± 0.0004	0.34 ± 0.0007	1.01 ± 0.0014	3.86 ± 0.0125	0.35 ± 0.0004	0.44 ± 0.0007
	$p = 0.9$	0.50 ± 0.0010	0.35 ± 0.0018	1.05 ± 0.0032	3.47 ± 0.0250	0.36 ± 0.0009	0.45 ± 0.0018

Table 46 Performance of Alternative Tests for $n = 2000$: Number of KS tests passed (denoted by $\#P$) at significance level 0.05 out of 10,000 replications and the average p-values (denoted by $E[p\text{-value}]$) with associated 95% confidence intervals.

		Standard KS		Conditional		Log		Lewis	
$Case$	$Subcase$	$\#P$	$E[p\text{-value}]$	$\#P$	$E[p\text{-value}]$	$\#P$	$E[p\text{-value}]$	$\#P$	$E[p\text{-value}]$
Exp	–	9515	0.50 ± 0.006	9481	0.50 ± 0.006	9502	0.50 ± 0.006	9495	0.50 ± 0.006
E_k	$k = 2$	0	0.00 ± 0.000	9985	0.79 ± 0.004	0	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 4$	0	0.00 ± 0.000	10000	0.95 ± 0.002	0	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 6$	0	0.00 ± 0.000	10000	0.98 ± 0.001	0	0.00 ± 0.000	0	0.00 ± 0.000
H_2	$c^2 = 1.25$	3380	0.08 ± 0.003	8957	0.40 ± 0.005	3636	0.10 ± 0.003	281	0.01 ± 0.001
	$c^2 = 1.5$	68	0.00 ± 0.000	8313	0.32 ± 0.005	216	0.00 ± 0.001	0	0.00 ± 0.000
	$c^2 = 2$	0	0.00 ± 0.000	6893	0.21 ± 0.004	0	0.00 ± 0.000	0	0.00 ± 0.000
	$c^2 = 4$	0	0.00 ± 0.000	2879	0.05 ± 0.002	0	0.00 ± 0.000	0	0.00 ± 0.000
	$c^2 = 10$	0	0.00 ± 0.000	183	0.00 ± 0.000	0	0.00 ± 0.000	0	0.00 ± 0.000
Z	–	0	0.00 ± 0.000	9502	0.52 ± 0.006	0	0.00 ± 0.000	0	0.00 ± 0.000
LN	–	0	0.00 ± 0.000	9513	0.51 ± 0.006	0	0.00 ± 0.000	0	0.00 ± 0.000
RRI	$p = 0.1$	9010	0.41 ± 0.005	9129	0.41 ± 0.005	9027	0.41 ± 0.006	9014	0.40 ± 0.005
	$p = 0.5$	4410	0.10 ± 0.003	4666	0.11 ± 0.003	4613	0.11 ± 0.003	4531	0.10 ± 0.003
	$p = 0.9$	0	0.00 ± 0.000	25	0.00 ± 0.000	6	0.00 ± 0.000	0	0.00 ± 0.000
$EARMA$	0.25	9336	0.47 ± 0.006	8326	0.33 ± 0.005	9139	0.46 ± 0.006	9429	0.49 ± 0.006
	0.5	8806	0.42 ± 0.006	7063	0.22 ± 0.004	8829	0.43 ± 0.006	9408	0.49 ± 0.006
	1	8210	0.37 ± 0.006	4722	0.12 ± 0.003	7994	0.35 ± 0.006	8901	0.43 ± 0.006
	3	5247	0.21 ± 0.005	822	0.01 ± 0.001	5396	0.20 ± 0.005	6715	0.29 ± 0.006
	5.25	4111	0.14 ± 0.005	193	0.00 ± 0.000	4185	0.14 ± 0.004	5769	0.21 ± 0.005
mH_2	$m = 2$	0	0.00 ± 0.000	3070	0.06 ± 0.002	20	0.00 ± 0.000	0	0.00 ± 0.000
	$m = 5$	3135	0.09 ± 0.003	3440	0.07 ± 0.002	3603	0.11 ± 0.004	182	0.00 ± 0.000
	$m = 10$	6428	0.25 ± 0.005	3732	0.09 ± 0.003	6646	0.26 ± 0.005	4432	0.13 ± 0.004
	$m = 20$	7364	0.31 ± 0.006	4365	0.11 ± 0.003	7709	0.34 ± 0.006	8127	0.35 ± 0.006
$RRI(H_2)$	$p = 0.1$	0	0.00 ± 0.000	1952	0.04 ± 0.001	0	0.00 ± 0.000	0	0.00 ± 0.000
	$p = 0.5$	0	0.00 ± 0.000	179	0.00 ± 0.000	0	0.00 ± 0.000	0	0.00 ± 0.000
	$p = 0.9$	0	0.00 ± 0.000	0	0.00 ± 0.000	0	0.00 ± 0.000	0	0.00 ± 0.000

Table 47 Performance of Alternative Tests for Untransformed Interarrival Times with $n = 2000$: Number of KS tests passed (denoted by $\#P$) at significance level 0.05 out of $10,000$ replications and the average p-values (denoted by $E[p\text{-value}]$) with associated 95% confidence intervals.

		KS with estimated mean		Lillifors test	
$Case$	$Subcase$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$
Exp	$-$	9943	0.66 ± 0.005	9525	0.37 ± 0.003
E_k	$k = 2$	0	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 4$	0	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 6$	0	0.00 ± 0.000	0	0.00 ± 0.000
H_2	$c^2 = 1.25$	3120	0.06 ± 0.002	824	0.02 ± 0.001
	$c^2 = 1.5$	7	0.00 ± 0.000	0	0.00 ± 0.000
	$c^2 = 2$	0	0.00 ± 0.000	0	0.00 ± 0.000
	$c^2 = 4$	0	0.00 ± 0.000	0	0.00 ± 0.000
	$c^2 = 10$	0	0.00 ± 0.000	0	0.00 ± 0.000
Z	$-$	0	0.00 ± 0.000	0	0.00 ± 0.000
LN	$-$	0	0.00 ± 0.000	0	0.00 ± 0.000
RRI	$p = 0.1$	9843	0.56 ± 0.005	8939	0.31 ± 0.004
	$p = 0.5$	6752	0.17 ± 0.003	3606	0.07 ± 0.002
	$p = 0.9$	1	0.00 ± 0.000	0	0.00 ± 0.000
$EARMA$	0.25	9925	0.65 ± 0.005	9478	0.37 ± 0.003
	0.5	9921	0.65 ± 0.005	9438	0.37 ± 0.003
	1	9835	0.60 ± 0.005	9023	0.34 ± 0.004
	3	8310	0.43 ± 0.007	6695	0.23 ± 0.004
	5.25	8046	0.35 ± 0.006	6076	0.19 ± 0.004
mH_2	$m = 2$	0	0.00 ± 0.000	0	0.00 ± 0.000
	$m = 5$	2369	0.05 ± 0.002	625	0.01 ± 0.001
	$m = 10$	8345	0.31 ± 0.005	5713	0.16 ± 0.003
	$m = 20$	9671	0.54 ± 0.006	8530	0.30 ± 0.004
$RRI(H_2)$	$p = 0.1$	0	0.00 ± 0.000	0	0.00 ± 0.000
	$p = 0.5$	0	0.00 ± 0.000	0	0.00 ± 0.000
	$p = 0.9$	0	0.00 ± 0.000	0	0.00 ± 0.000

Table 48 Pairwise Correlations of p-values ($n = 2000$): S stands for the standard test, C for the CU test, L for the Log test and Le for the Lewis test.

$Case$	$Subcase$	(S,C)	(S,L)	(S,Le)	(C,L)	(C,Le)	(L,Le)
Exp	$-$		0.34	0.50	0.29		0.49
E_k	$k = 2$						
	$k = 4$			0.07			
	$k = 6$						
H_2	$c^2 = 1.25$	0.03	0.33	0.27	-0.08	0.03	0.31
	$c^2 = 1.5$		0.14		-0.11		0.04
	$c^2 = 2$	0.02			-0.02		0.05
	$c^2 = 4$						
	$c^2 = 10$						
Z	$-$			0.08	-0.02		
LN	$-$			0.32	-0.04		0.28
RRI	$p = 0.1$		0.32	0.50	0.27		0.47
	$p = 0.5$		0.30	0.44	0.16		0.40
	$p = 0.9$		0.06	0.18			0.11
$EARMA$	0.25		0.25	0.40	0.31		0.40
	0.5		0.21	0.36	0.30		0.37
	1		0.18	0.34	0.27		0.31
	3	-0.02	0.22	0.47	0.12	-0.02	0.39
	5.25	-0.02	0.16	0.30	0.06	-0.05	0.29
mH_2	$m = 2$				-0.03		0.03
	$m = 5$	0.04	0.17	0.12		0.07	0.21
	$m = 10$	0.03	0.21	0.27	0.20	0.14	0.33
	$m = 20$		0.15	0.28	0.30	0.10	0.30
$RRI(H_2)$	$p = 0.1$					0.02	
	$p = 0.5$						
	$p = 0.9$					0.03	

Table 49 Results for all possible composite tests based on the four alternatives for $n = 2000$ using the Bonferroni procedure: The test order is (standard, CU, Log, Lewis) and is indicated by 1 if the test is included in the combination and 0 otherwise. Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

		2 tests						3 tests				4 tests
Case	*Subcase*	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
Exp	$-$	9514	9553	9570	9548	9501	9553	9523	9557	9520	9477	9561
E_k	$k = 2$	0	0	0	0	0	0	0	0	0	0	0
	$k = 4$	0	0	0	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	4303	2802	454	4362	490	480	589	555	567	1390	657
	$c^2 = 1.5$	127	29	0	266	0	0	0	0	0	0	0
	$c^2 = 2$	0	0	0	0	0	0	0	0	0	0	0
	$c^2 = 4$	0	0	0	0	0	0	0	0	0	0	0
	$c^2 = 10$	0	0	0	0	0	0	0	0	0	0	0
Z	$-$	0	0	0	0	0	0	0	0	0	0	0
LN	$(1, 1)$	0	0	0	0	0	0	0	0	0	0	0
RRI	$p = 0.1$	8966	8974	9043	9054	8967	9053	8912	8941	8878	8772	8924
	$p = 0.5$	3163	3633	3926	3792	3190	3998	2804	3051	2470	2299	2589
	$p = 0.9$	0	0	1	1	0	0	0	0	0	0	0
$EARMA$	0.25	8639	9205	9394	8673	8678	9299	8810	9267	8793	8695	8864
	0.5	7354	8676	9065	7661	7681	9095	7919	8852	7707	7598	7903
	1	5067	7572	8249	5463	5404	8161	5781	7682	5424	5335	5731
	3	790	4057	5275	1072	977	5195	1152	4083	905	879	1041
	5.25	149	2498	3610	240	197	3615	255	2314	163	162	210
mH_2	$m = 2$	0	0	0	3	0	0	0	0	0	0	0
	$m = 5$	1772	1981	181	2016	194	282	253	225	171	467	212
	$m = 10$	3422	5438	4290	4038	2950	4614	3263	4130	2755	3085	3021
	$m = 20$	4281	6784	7253	4941	4786	7566	5160	6771	4537	4505	4808
$RRI(H_2)$	$p = 0.1$	0	0	0	0	0	0	0	0	0	0	0
	$p = 0.5$	0	0	0	0	0	0	0	0	0	0	0
	$p = 0.9$	0	0	0	0	0	0	0	0	0	0	0

Table 50 Results for all possible composite tests based on the four alternatives for $n = 2000$ using Holm's procedure: The test order is (standard, CU, Log, Lewis) and is indicated by 1 if the test is included in the combination and 0 otherwise. Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

		2 tests						3 tests				4 tests
$Case$	$Subcase$	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
Exp	$-$	9503	9536	9546	9526	9494	9529	9483	9508	9484	9452	9544
E_k	$k = 2$	0	0	0	0	0	0	0	0	0	0	0
	$k = 4$	0	0	0	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	4251	2661	420	4339	479	463	549	506	523	1318	634
	$c^2 = 1.5$	122	24	0	247	0	0	0	0	0	0	0
	$c^2 = 2$	0	0	0	0	0	0	0	0	0	0	0
	$c^2 = 4$	0	0	0	0	0	0	0	0	0	0	0
	$c^2 = 10$	0	0	0	0	0	0	0	0	0	0	0
Z	$-$	0	0	0	0	0	0	0	0	0	0	0
LN	$(1, 1)$	0	0	0	0	0	0	0	0	0	0	0
RRI	$p = 0.1$	8951	8946	8996	9012	8957	9014	8842	8852	8804	8735	8886
	$p = 0.5$	3071	3484	3770	3658	3072	3846	2619	2845	2281	2154	2450
	$p = 0.9$	0	0	0	1	0	0	0	0	0	0	0
$EARMA$	0.25	8615	9184	9372	8624	8667	9272	8762	9215	8744	8667	8846
	0.5	7318	8648	9042	7589	7658	9069	7861	8810	7656	7544	7869
	1	5022	7532	8214	5387	5348	8123	5690	7607	5347	5269	5684
	3	746	4000	5208	1045	939	5141	1115	4005	859	849	1015
	5.25	135	2460	3551	231	184	3553	240	2237	153	148	197
mH_2	$m = 2$	0	0	0	3	0	0	0	0	0	0	0
	$m = 5$	1691	1917	172	1939	184	267	226	204	150	434	202
	$m = 10$	3347	5381	4201	3962	2846	4541	3117	3987	2611	2982	2940
	$m = 20$	4209	6740	7196	4869	4728	7519	5054	6675	4445	4415	4760
$RRI(H_2)$	$p = 0.1$	0	0	0	0	0	0	0	0	0	0	0
	$p = 0.5$	0	0	0	0	0	0	0	0	0	0	0
	$p = 0.9$	0	0	0	0	0	0	0	0	0	0	0

Table 51 Results for all possible composite tests based on the four alternatives for $n = 2000$ using Simes' procedure: The test order is (standard, CU, Log, Lewis) and is indicated by 1 if the test is included in the combination and 0 otherwise. Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

		2 tests						3 tests				4 tests
Case	*Subcase*	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
Exp	−	9503	9536	9546	9526	9494	9529	9452	9486	9459	9435	9493
E_k	$k = 2$	0	0	0	0	0	0	0	0	0	0	0
	$k = 4$	0	0	0	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	4251	2661	420	4339	479	463	522	483	498	1265	581
	$c^2 = 1.5$	122	24	0	247	0	0	0	0	0	0	0
	$c^2 = 2$	0	0	0	0	0	0	0	0	0	0	0
	$c^2 = 4$	0	0	0	0	0	0	0	0	0	0	0
	$c^2 = 10$	0	0	0	0	0	0	0	0	0	0	0
Z	−	0	0	0	0	0	0	0	0	0	0	0
LN	$(1, 1)$	0	0	0	0	0	0	0	0	0	0	0
RRI	$p = 0.1$	8951	8946	8996	9012	8957	9014	8788	8804	8756	8695	8798
	$p = 0.5$	3071	3484	3770	3658	3072	3846	2510	2752	2192	2063	2287
	$p = 0.9$	0	0	0	1	0	0	0	0	0	0	0
$EARMA$	0.25	8615	9184	9372	8624	8667	9272	8717	9188	8711	8631	8790
	0.5	7318	8648	9042	7589	7658	9069	7813	8779	7619	7499	7797
	1	5022	7532	8214	5387	5348	8123	5638	7555	5300	5212	5573
	3	746	4000	5208	1045	939	5141	1091	3959	833	821	975
	5.25	135	2460	3551	231	184	3553	234	2202	145	140	186
mH_2	$m = 2$	0	0	0	3	0	0	0	0	0	0	0
	$m = 5$	1691	1917	172	1939	184	267	217	194	141	408	170
	$m = 10$	3347	5381	4201	3962	2846	4541	3026	3882	2535	2906	2775
	$m = 20$	4209	6740	7196	4869	4728	7519	4993	6614	4394	4351	4634
$RRI(H_2)$	$p = 0.1$	0	0	0	0	0	0	0	0	0	0	0
	$p = 0.5$	0	0	0	0	0	0	0	0	0	0	0
	$p = 0.9$	0	0	0	0	0	0	0	0	0	0	0

Table 52 Results for individual and composite tests based on the Standard KS, CU, and Lilliefors tests for $n = 2000$ using the Bonferroni procedure: Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

$Case$	$Subcase$	Standard	Lilliefors	CU	(Standard+CU)	(Lilliefors+CU)
Exp	$-$	9515	9525	9481	9514	9524
E_k	$k = 2$	0	0	9985	0	0
	$k = 4$	0	0	10000	0	0
	$k = 6$	0	0	10000	0	0
H_2	$c^2 = 1.25$	3380	824	8957	4303	1350
	$c^2 = 1.5$	68	0	8313	127	1
	$c^2 = 2$	0	0	6893	0	0
	$c^2 = 4$	0	0	2879	0	0
	$c^2 = 10$	0	0	183	0	0
Z	$-$	0	0	9502	0	0
LN	$(1, 1)$	0	0	9513	0	0
RRI	$p = 0.1$	9010	8939	9129	8966	8933
	$p = 0.5$	4410	3606	4666	3163	2731
	$p = 0.9$	0	0	25	0	0
$EARMA$	0.25	9336	9478	8326	8639	8723
	0.5	8806	9438	7063	7354	7695
	1	8210	9023	4722	5067	5484
	3	5247	6695	822	790	997
	5.25	4111	6076	193	149	219
mH_2	$m = 2$	0	0	3070	0	0
	$m = 5$	3135	625	3440	1772	576
	$m = 10$	6428	5713	3732	3422	3504
	$m = 20$	7364	8530	4365	4281	4896
$RRI(H_2)$	$p = 0.1$	0	0	1952	0	0
	$p = 0.5$	0	0	179	0	0
	$p = 0.9$	0	0	0	0	0

5.3 Experiments with $n = 200$ with 10 Subintervals

Table 53 Summary statistics of the arrival processes with $n = 200$ interarrival times and 10 equally sized subintervals. Associated 95% confidence intervals are also presented. All results are based on 10000 replications.

$Case$	$Subcase$	$E[X]$	$c^2[X]$	n	t_{200}	$Min[X]$	$Max[X]$
Exp	−	0.96 ± 0.0014	0.99 ± 0.0027	199	190 ± 0.3	0.005 ± 0.0001	5.6 ± 0.02
E_k	$k = 2$	0.97 ± 0.0010	0.51 ± 0.0013	199	192 ± 0.2	0.036 ± 0.0005	4.0 ± 0.02
	$k = 4$	0.97 ± 0.0007	0.26 ± 0.0006	199	193 ± 0.1	0.078 ± 0.0011	2.9 ± 0.01
	$k = 6$	0.97 ± 0.0006	0.18 ± 0.0004	199	194 ± 0.1	0.093 ± 0.0014	2.5 ± 0.01
H_2	$c^2 = 1.25$	0.95 ± 0.0015	1.20 ± 0.0040	199	189 ± 0.3	0.004 ± 0.0001	6.7 ± 0.03
	$c^2 = 1.5$	0.94 ± 0.0016	1.41 ± 0.0052	199	188 ± 0.3	0.004 ± 0.0001	7.5 ± 0.04
	$c^2 = 2$	0.93 ± 0.0018	1.78 ± 0.0076	199	185 ± 0.4	0.004 ± 0.0001	8.9 ± 0.05
	$c^2 = 4$	0.89 ± 0.0024	2.92 ± 0.0152	199	177 ± 0.5	0.003 ± 0.0001	12.1 ± 0.07
	$c^2 = 10$	0.78 ± 0.0030	4.30 ± 0.0322	199	156 ± 0.6	0.003 ± 0.0001	14.6 ± 0.10
Z	−	0.96 ± 0.0012	0.81 ± 0.0066	199	190 ± 0.2	0.016 ± 0.0003	6.7 ± 0.07
LN	−	0.95 ± 0.0013	0.91 ± 0.0045	199	190 ± 0.3	0.053 ± 0.0006	6.6 ± 0.04
RRI	$p = 0.1$	0.95 ± 0.0015	0.99 ± 0.0029	199	190 ± 0.3	0.005 ± 0.0001	5.5 ± 0.02
	$p = 0.5$	0.96 ± 0.0023	0.98 ± 0.0042	199	190 ± 0.5	0.009 ± 0.0002	5.1 ± 0.02
	$p = 0.9$	0.96 ± 0.0056	0.88 ± 0.0081	199	190 ± 1.1	0.035 ± 0.0007	3.6 ± 0.02
$EARMA$	0.25	0.96 ± 0.0016	0.99 ± 0.0027	199	190 ± 0.3	0.005 ± 0.0001	5.5 ± 0.02
	0.5	0.96 ± 0.0019	0.98 ± 0.0028	199	190 ± 0.4	0.005 ± 0.0001	5.5 ± 0.02
	1	0.96 ± 0.0023	0.97 ± 0.0031	199	190 ± 0.5	0.005 ± 0.0001	5.4 ± 0.03
	3	0.96 ± 0.0035	0.97 ± 0.0047	199	191 ± 0.7	0.017 ± 0.0003	5.3 ± 0.03
	5.25	0.96 ± 0.0044	0.90 ± 0.0047	199	191 ± 0.9	0.005 ± 0.0001	4.8 ± 0.03
mH_2	$m = 2$	0.92 ± 0.0025	1.88 ± 0.0116	199	183 ± 0.5	0.004 ± 0.0001	10.2 ± 0.07
	$m = 5$	0.95 ± 0.0026	1.24 ± 0.0055	199	188 ± 0.5	0.004 ± 0.0001	7.1 ± 0.05
	$m = 10$	0.95 ± 0.0025	1.08 ± 0.0036	199	190 ± 0.5	0.005 ± 0.0001	6.1 ± 0.04
	$m = 20$	0.95 ± 0.0024	1.02 ± 0.0030	199	190 ± 0.5	0.005 ± 0.0001	5.8 ± 0.03
$RRI(H_2)$	$p = 0.1$	0.89 ± 0.0027	2.92 ± 0.0160	199	177 ± 0.5	0.003 ± 0.0001	12.1 ± 0.07
	$p = 0.5$	0.89 ± 0.0041	2.78 ± 0.0205	199	177 ± 0.8	0.006 ± 0.0001	11.1 ± 0.08
	$p = 0.9$	0.92 ± 0.0108	2.00 ± 0.0280	199	183 ± 2.1	0.024 ± 0.0005	6.5 ± 0.09

Table 54 Average and c^2 of transformed interarrival times ($n = 200$ with 10 equally sized subintervals) with associated 95% confidence intervals. All results are based on 10000 replications.

$Case$	$Subcase$	$E[X^{CU}]$	$c^2[X^{CU}]$	$E[X^{Log}]$	$c^2[X^{Log}]$	$E[X^{Lewis}]$	$c^2[X^{Lewis}]$
Exp	–	0.50 ± 0.0004	0.33 ± 0.0007	1.02 ± 0.0017	1.31 ± 0.0189	0.50 ± 0.0004	0.34 ± 0.0007
E_k	$k = 2$	0.50 ± 0.0003	0.33 ± 0.0005	1.01 ± 0.0014	0.86 ± 0.0196	0.62 ± 0.0003	0.17 ± 0.0004
	$k = 4$	0.50 ± 0.0002	0.33 ± 0.0004	1.01 ± 0.0013	0.62 ± 0.0197	0.71 ± 0.0003	0.10 ± 0.0002
	$k = 6$	0.50 ± 0.0002	0.33 ± 0.0003	1.01 ± 0.0012	0.54 ± 0.0197	0.75 ± 0.0002	0.07 ± 0.0002
H_2	$c^2 = 1.25$	0.50 ± 0.0004	0.34 ± 0.0007	1.02 ± 0.0018	1.47 ± 0.0187	0.48 ± 0.0004	0.36 ± 0.0007
	$c^2 = 1.5$	0.50 ± 0.0005	0.34 ± 0.0008	1.02 ± 0.0018	1.59 ± 0.0180	0.46 ± 0.0004	0.38 ± 0.0008
	$c^2 = 2$	0.50 ± 0.0005	0.34 ± 0.0009	1.02 ± 0.0019	1.85 ± 0.0182	0.43 ± 0.0005	0.40 ± 0.0009
	$c^2 = 4$	0.50 ± 0.0006	0.34 ± 0.0011	1.02 ± 0.0022	2.47 ± 0.0189	0.39 ± 0.0007	0.45 ± 0.0011
	$c^2 = 10$	0.50 ± 0.0006	0.34 ± 0.0012	1.02 ± 0.0022	3.02 ± 0.0262	0.39 ± 0.0010	0.46 ± 0.0014
Z	–	0.50 ± 0.0003	0.33 ± 0.0006	1.01 ± 0.0016	1.03 ± 0.0194	0.58 ± 0.0004	0.21 ± 0.0005
LN	–	0.50 ± 0.0004	0.33 ± 0.0006	1.01 ± 0.0016	1.14 ± 0.0195	0.56 ± 0.0004	0.19 ± 0.0004
RRI	$p = 0.1$	0.50 ± 0.0004	0.34 ± 0.0007	1.02 ± 0.0018	1.31 ± 0.0188	0.50 ± 0.0004	0.33 ± 0.0008
	$p = 0.5$	0.50 ± 0.0005	0.34 ± 0.0009	1.02 ± 0.0020	1.28 ± 0.0186	0.54 ± 0.0006	0.32 ± 0.0011
	$p = 0.9$	0.50 ± 0.0008	0.34 ± 0.0014	1.02 ± 0.0028	1.19 ± 0.0242	0.67 ± 0.0014	0.23 ± 0.0019
$EARMA$	0.25	0.50 ± 0.0004	0.34 ± 0.0007	1.02 ± 0.0017	1.28 ± 0.0183	0.51 ± 0.0004	0.33 ± 0.0007
	0.5	0.50 ± 0.0005	0.34 ± 0.0008	1.01 ± 0.0018	1.23 ± 0.0180	0.51 ± 0.0004	0.33 ± 0.0007
	1	0.50 ± 0.0004	0.34 ± 0.0008	1.02 ± 0.0017	1.16 ± 0.0182	0.52 ± 0.0004	0.32 ± 0.0007
	3	0.50 ± 0.0007	0.34 ± 0.0013	1.02 ± 0.0026	0.95 ± 0.0180	0.55 ± 0.0008	0.28 ± 0.0013
	5.25	0.50 ± 0.0005	0.34 ± 0.0008	1.01 ± 0.0017	0.80 ± 0.0161	0.59 ± 0.0005	0.27 ± 0.0007
mH_2	$m = 2$	0.50 ± 0.0005	0.34 ± 0.0009	1.00 ± 0.0017	1.45 ± 0.0068	0.45 ± 0.0005	0.38 ± 0.0009
	$m = 5$	0.50 ± 0.0004	0.34 ± 0.0007	1.00 ± 0.0015	1.08 ± 0.0033	0.49 ± 0.0004	0.35 ± 0.0007
	$m = 10$	0.50 ± 0.0004	0.34 ± 0.0007	1.00 ± 0.0014	1.02 ± 0.0029	0.50 ± 0.0004	0.34 ± 0.0007
	$m = 20$	0.50 ± 0.0004	0.34 ± 0.0007	1.00 ± 0.0014	1.00 ± 0.0027	0.50 ± 0.0004	0.34 ± 0.0007
$RRI(H_2)$	$p = 0.1$	0.50 ± 0.0006	0.34 ± 0.0011	1.02 ± 0.0022	2.47 ± 0.0196	0.40 ± 0.0007	0.45 ± 0.0012
	$p = 0.5$	0.50 ± 0.0007	0.34 ± 0.0012	1.02 ± 0.0024	2.24 ± 0.0219	0.45 ± 0.0009	0.42 ± 0.0016
	$p = 0.9$	0.50 ± 0.0009	0.35 ± 0.0018	1.03 ± 0.0032	1.85 ± 0.0382	0.62 ± 0.0018	0.29 ± 0.0027

Table 55 Performance of Alternative Tests for $n = 200$ with 10 equally sized subintervals: Number of KS tests passed (denoted by $\#P$) at significance level 0.05 out of $10,000$ replications and the average p-values (denoted by $E[p\text{-value}]$) with associated 95% confidence intervals.

		Standard KS		Conditional		Log		Lewis	
$Case$	$Subcase$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$
Exp	–	9159	0.46 ± 0.006	9486	0.50 ± 0.006	9511	0.50 ± 0.006	9502	0.50 ± 0.006
E_k	$k = 2$	131	0.00 ± 0.000	9966	0.73 ± 0.005	75	0.00 ± 0.000	1	0.00 ± 0.000
	$k = 4$	0	0.00 ± 0.000	10000	0.89 ± 0.003	0	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 6$	0	0.00 ± 0.000	10000	0.95 ± 0.002	0	0.00 ± 0.000	0	0.00 ± 0.000
H_2	$c^2 = 1.25$	7819	0.32 ± 0.006	9169	0.45 ± 0.006	9015	0.43 ± 0.006	8138	0.35 ± 0.006
	$c^2 = 1.5$	5631	0.18 ± 0.005	8811	0.40 ± 0.006	7940	0.33 ± 0.006	5441	0.18 ± 0.005
	$c^2 = 2$	2331	0.05 ± 0.002	8256	0.34 ± 0.006	5676	0.19 ± 0.005	1947	0.05 ± 0.003
	$c^2 = 4$	41	0.00 ± 0.000	6850	0.25 ± 0.005	2485	0.07 ± 0.003	336	0.01 ± 0.001
	$c^2 = 10$	0	0.00 ± 0.000	6565	0.25 ± 0.005	2824	0.09 ± 0.004	835	0.03 ± 0.002
Z	–	2240	0.04 ± 0.001	9843	0.65 ± 0.005	2072	0.04 ± 0.002	437	0.01 ± 0.001
LN	–	311	0.01 ± 0.000	9690	0.58 ± 0.006	888	0.02 ± 0.001	112	0.00 ± 0.000
RRI	$p = 0.1$	8741	0.38 ± 0.006	9318	0.47 ± 0.006	9207	0.44 ± 0.006	9104	0.42 ± 0.006
	$p = 0.5$	4725	0.11 ± 0.003	8033	0.33 ± 0.006	5948	0.17 ± 0.004	3177	0.07 ± 0.002
	$p = 0.9$	27	0.00 ± 0.000	5680	0.22 ± 0.005	72	0.00 ± 0.000	1	0.00 ± 0.000
$EARMA$	0.25	8992	0.43 ± 0.006	9312	0.48 ± 0.006	9437	0.49 ± 0.006	9473	0.50 ± 0.006
	0.5	8596	0.40 ± 0.006	8916	0.43 ± 0.006	9265	0.47 ± 0.006	9269	0.47 ± 0.006
	1	7939	0.35 ± 0.006	9238	0.48 ± 0.006	8918	0.42 ± 0.006	8115	0.34 ± 0.006
	3	5111	0.20 ± 0.005	5671	0.19 ± 0.005	4534	0.13 ± 0.004	4686	0.17 ± 0.005
	5.25	4192	0.14 ± 0.005	9098	0.48 ± 0.006	2484	0.05 ± 0.002	338	0.01 ± 0.001
mH_2	$m = 2$	3215	0.10 ± 0.004	8271	0.35 ± 0.006	7268	0.30 ± 0.006	4628	0.16 ± 0.005
	$m = 5$	6661	0.27 ± 0.006	9213	0.46 ± 0.006	9326	0.47 ± 0.006	9136	0.45 ± 0.006
	$m = 10$	7275	0.31 ± 0.006	9446	0.48 ± 0.006	9475	0.49 ± 0.006	9457	0.50 ± 0.006
	$m = 20$	7614	0.33 ± 0.006	9494	0.50 ± 0.006	9517	0.50 ± 0.006	9489	0.50 ± 0.006
$RRI(H_2)$	$p = 0.1$	55	0.00 ± 0.000	6728	0.24 ± 0.005	2826	0.08 ± 0.003	529	0.01 ± 0.001
	$p = 0.5$	180	0.00 ± 0.000	5846	0.19 ± 0.005	4221	0.12 ± 0.004	2252	0.05 ± 0.002
	$p = 0.9$	3	0.00 ± 0.000	4229	0.15 ± 0.005	163	0.00 ± 0.000	7	0.00 ± 0.000

Table 56 Performance of Alternative Tests for Untransformed Interarrival Times with $n = 200$ and 10 equally sized subintervals: Number of KS tests passed (denoted by $\#P$) at significance level 0.05 out of $10,000$ replications and the average p-values (denoted by $E[p\text{-value}]$) with associated 95% confidence intervals.

		KS with estimated mean		Lillifors test	
$Case$	$Subcase$	$\#P$	$E[p$-value$]$	$\#P$	$E[p$-value$]$
Exp	$-$	9951	0.65 ± 0.005	9488	0.37 ± 0.003
E_k	$k = 2$	13	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 4$	0	0.00 ± 0.000	0	0.00 ± 0.000
	$k = 6$	0	0.00 ± 0.000	0	0.00 ± 0.000
H_2	$c^2 = 1.25$	9644	0.53 ± 0.006	8476	0.30 ± 0.004
	$c^2 = 1.5$	8588	0.35 ± 0.006	6171	0.18 ± 0.004
	$c^2 = 2$	5313	0.14 ± 0.004	2575	0.06 ± 0.002
	$c^2 = 4$	1261	0.03 ± 0.002	428	0.01 ± 0.001
	$c^2 = 10$	1807	0.05 ± 0.003	908	0.02 ± 0.002
Z	$-$	1165	0.02 ± 0.001	186	0.00 ± 0.000
LN	$-$	185	0.01 ± 0.000	9	0.00 ± 0.000
RRI	$p = 0.1$	9851	0.57 ± 0.005	9062	0.32 ± 0.003
	$p = 0.5$	7175	0.20 ± 0.004	4192	0.09 ± 0.002
	$p = 0.9$	78	0.00 ± 0.000	4	0.00 ± 0.000
$EARMA$	0.25	9958	0.66 ± 0.005	9507	0.37 ± 0.003
	0.5	9936	0.65 ± 0.005	9454	0.37 ± 0.003
	1	9859	0.61 ± 0.005	9170	0.35 ± 0.003
	3	8454	0.44 ± 0.007	6845	0.24 ± 0.004
	5.25	8266	0.37 ± 0.006	6212	0.19 ± 0.004
mH_2	$m = 2$	6710	0.23 ± 0.005	4133	0.11 ± 0.003
	$m = 5$	9609	0.53 ± 0.006	8453	0.30 ± 0.004
	$m = 10$	9904	0.63 ± 0.005	9312	0.36 ± 0.003
	$m = 20$	9938	0.65 ± 0.005	9463	0.37 ± 0.003
$RRI(H_2)$	$p = 0.1$	1267	0.03 ± 0.002	442	0.01 ± 0.001
	$p = 0.5$	1285	0.03 ± 0.002	465	0.01 ± 0.001
	$p = 0.9$	38	0.00 ± 0.000	2	0.00 ± 0.000

Table 57 Pairwise Correlations of p-values ($n = 200$ with 10 equally sized subintervals): S stands for the standard test, C for the CU test, L for the Log test and Le for the Lewis test.

$Case$	$Subcase$	(S,C)	(S,L)	(S,Le)	(C,L)	(C,Le)	(L,Le)
Exp	–	0.03	0.19	0.32	0.32		0.28
E_k	$k=2$	−0.04	0.44	0.26	−0.13		0.35
	$k=4$	−0.03	0.06	0.08	−0.05		0.03
	$k=6$	−0.06	0.92	0.51	−0.06	−0.04	0.56
H_2	$c^2=1.25$	0.04	0.20	0.33	0.27	0.11	0.36
	$c^2=1.5$	0.05	0.25	0.29	0.18	0.13	0.40
	$c^2=2$	0.05	0.15	0.13	0.06	0.11	0.38
	$c^2=4$					0.07	0.32
	$c^2=10$				0.12	0.17	0.46
Z	–	−0.05	0.48	0.45	−0.10	−0.08	0.50
LN	–	0.00	0.40	0.43	−0.08	−0.05	0.50
RRI	$p=0.1$	0.02	0.18	0.29	0.28	−0.02	0.29
	$p=0.5$	0.00	0.10	0.04	0.13	−0.07	0.25
	$p=0.9$	−0.02			−0.04		0.02
$EARMA$	0.25	0.02	0.13	0.23	0.32	−0.04	0.28
	0.5		0.09	0.15	0.30	−0.06	0.28
	1	−0.03	0.08	0.05	0.23	−0.11	0.32
	3	−0.03	0.03		−0.05	−0.19	0.44
	5.25	−0.02		−0.04	−0.12	−0.09	0.37
mH_2	$m=2$		0.07	0.06	0.17	0.15	0.42
	$m=5$	0.02	0.10	0.17	0.32	0.06	0.31
	$m=10$		0.09	0.18	0.31	0.02	0.31
	$m=20$	0.03	0.12	0.21	0.32		0.30
$RRI(H_2)$	$p=0.1$	0.02	0.04	0.03	0.03	0.11	0.35
	$p=0.5$		0.02	0.04	0.11	0.12	0.23
	$p=0.9$			0.02	−0.05		0.05

Table 58 Results for all possible composite tests based on the four alternatives for $n = 200$ (with 10 equally sized subintervals) using the Bonferroni procedure: The test order is (standard, CU, Log, Lewis) and is indicated by 1 if the test is included in the combination and 0 otherwise. Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

		2 tests						3 tests				4 tests
$Case$	$Subcase$	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
Exp	$-$	9310	9341	9357	9566	9507	9546	9463	9360	9309	9286	9376
E_k	$k = 2$	352	103	2	226	2	2	0	0	1	1	0
	$k = 4$	0	0	0	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	8213	8209	7807	9106	8420	8470	8373	7934	7897	8060	8091
	$c^2 = 1.5$	6245	6088	4925	8114	6148	6099	5985	5084	5067	5595	5432
	$c^2 = 2$	2920	2450	1179	5976	2500	2448	2276	1258	1288	1756	1464
	$c^2 = 4$	73	38	2	2483	441	430	304	4	4	13	5
	$c^2 = 10$	0	0	0	2759	1005	986	697	0	0	0	0
Z	$-$	3615	1977	614	3079	726	616	381	372	392	456	461
LN	$(1, 1)$	955	494	147	1708	254	198	44	43	45	44	62
RRI	$p = 0.1$	8908	8897	8847	9278	9133	9150	9002	8777	8752	8750	8856
	$p = 0.5$	5033	4214	2528	6206	3554	3499	2690	2214	2272	3380	2382
	$p = 0.9$	33	1	0	64	1	0	0	0	0	0	0
$EARMA$	0.25	9073	9139	9192	9416	9399	9494	9354	9187	9113	9074	9200
	0.5	8473	8710	8710	9085	8969	9254	8977	8703	8490	8528	8635
	1	8210	8020	7575	9017	8378	8369	8346	7646	7663	8005	7794
	3	3842	3342	3310	3590	3156	3918	2596	2632	2338	2589	2135
	5.25	4635	1665	229	3133	494	455	381	216	239	1673	250
mH_2	$m = 2$	3571	3346	2314	7322	5065	5064	4187	2253	2231	2478	2481
	$m = 5$	7035	7114	7062	9306	9098	9206	8692	6989	6899	6925	7155
	$m = 10$	7746	7775	7782	9521	9446	9488	9352	7921	7899	7873	8086
	$m = 20$	8020	8049	8085	9556	9513	9552	9476	8259	8229	8204	8397
$RRI(H_2)$	$p = 0.1$	72	49	8	2791	659	642	376	7	7	9	10
	$p = 0.5$	205	188	115	3780	2273	2228	463	31	28	33	35
	$p = 0.9$	10	1	1	119	4	2	0	0	0	0	0

Table 59 Results for all possible composite tests based on the four alternatives for $n = 200$ (with 10 equally sized subintervals) using Holm's procedure: The test order is (standard, CU, Log, Lewis) and is indicated by 1 if the test is included in the combination and 0 otherwise. Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

		2 tests						3 tests				4 tests
$Case$	$Subcase$	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
Exp	$-$	9303	9324	9329	9538	9504	9533	9442	9330	9284	9263	9365
E_k	$k = 2$	351	71	2	224	2	2	0	0	1	1	0
	$k = 4$	0	0	0	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	8181	8156	7724	9070	8393	8431	8316	7843	7802	8004	8052
	$c^2 = 1.5$	6192	6001	4806	8080	6087	6026	5865	4946	4936	5499	5357
	$c^2 = 2$	2870	2372	1109	5940	2463	2388	2172	1159	1183	1667	1412
	$c^2 = 4$	69	33	2	2434	430	413	283	2	2	11	5
	$c^2 = 10$	0	0	0	2726	985	965	668	0	0	0	0
Z	$-$	3605	1791	560	3072	716	579	345	321	355	432	436
LN	$(1, 1)$	935	375	115	1692	247	176	41	38	41	43	57
RRI	$p = 0.1$	8882	8871	8814	9259	9125	9116	8961	8717	8708	8718	8836
	$p = 0.5$	4972	4106	2419	6155	3481	3399	2585	2087	2152	3264	2328
	$p = 0.9$	31	1	0	59	1	0	0	0	0	0	0
$EARMA$	0.25	9051	9120	9176	9398	9391	9471	9329	9163	9089	9056	9184
	0.5	8452	8695	8693	9057	8958	9221	8949	8671	8476	8509	8628
	1	8188	7991	7538	8984	8368	8326	8317	7604	7619	7973	7775
	3	3783	3276	3274	3505	3081	3831	2523	2574	2284	2514	2112
	5.25	4606	1617	217	3101	486	434	361	202	230	1628	243
mH_2	$m = 2$	3527	3289	2246	7284	5017	5002	4080	2162	2141	2407	2427
	$m = 5$	7005	7085	7020	9281	9087	9174	8659	6943	6859	6890	7130
	$m = 10$	7725	7757	7761	9500	9441	9477	9322	7889	7861	7852	8077
	$m = 20$	8006	8036	8057	9541	9505	9537	9451	8224	8200	8179	8387
$RRI(H_2)$	$p = 0.1$	71	45	7	2728	642	628	365	5	5	7	10
	$p = 0.5$	190	180	106	3699	2216	2155	446	30	26	32	33
	$p = 0.9$	9	0	0	108	4	2	0	0	0	0	0

Table 60 Results for all possible composite tests based on the four alternatives for $n = 200$ (with 10 equally sized subintervals) using Simes' procedure: The test order is (standard, CU, Log, Lewis) and is indicated by 1 if the test is included in the combination and 0 otherwise. Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

		2 tests						3 tests				4 tests
$Case$	$Subcase$	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
Exp	$-$	9303	9324	9329	9538	9504	9533	9409	9294	9252	9235	9327
E_k	$k = 2$	351	71	2	224	2	2	0	0	1	1	0
	$k = 4$	0	0	0	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	8181	8156	7724	9070	8393	8431	8254	7779	7740	7954	7934
	$c^2 = 1.5$	6192	6001	4806	8080	6087	6026	5744	4826	4820	5408	5194
	$c^2 = 2$	2870	2372	1109	5940	2463	2388	2086	1118	1136	1598	1286
	$c^2 = 4$	69	33	2	2434	430	413	269	2	2	9	3
	$c^2 = 10$	0	0	0	2726	985	965	646	0	0	0	0
Z	$-$	3605	1791	560	3072	716	579	313	295	330	414	382
LN	$(1, 1)$	935	375	115	1692	247	176	38	33	35	40	48
RRI	$p = 0.1$	8882	8871	8814	9259	9125	9116	8921	8685	8671	8680	8757
	$p = 0.5$	4972	4106	2419	6155	3481	3399	2497	1994	2062	3159	2174
	$p = 0.9$	31	1	0	59	1	0	0	0	0	0	0
$EARMA$	0.25	9051	9120	9176	9398	9391	9471	9303	9135	9058	9031	9139
	0.5	8452	8695	8693	9057	8958	9221	8905	8631	8435	8473	8582
	1	8188	7991	7538	8984	8368	8326	8265	7543	7564	7928	7710
	3	3783	3276	3274	3505	3081	3831	2463	2525	2229	2460	2010
	5.25	4606	1617	217	3101	486	434	349	188	215	1591	219
mH_2	$m = 2$	3527	3289	2246	7284	5017	5002	4025	2100	2078	2349	2292
	$m = 5$	7005	7085	7020	9281	9087	9174	8624	6891	6815	6850	7066
	$m = 10$	7725	7757	7761	9500	9441	9477	9294	7860	7829	7816	8017
	$m = 20$	8006	8036	8057	9541	9505	9537	9421	8195	8175	8147	8338
$RRI(H_2)$	$p = 0.1$	71	45	7	2728	642	628	350	4	4	6	8
	$p = 0.5$	190	180	106	3699	2216	2155	427	29	26	30	30
	$p = 0.9$	9	0	0	108	4	2	0	0	0	0	0

Table 61 Results for individual and composite tests based on the Standard KS, CU, and Lilliefors tests for $n = 200$ (with 10 equally sized subintervals) using the Bonferroni procedure: Number of tests passed at significance level 0.05 out of $10,000$ replications are shown.

$Case$	$Subcase$	Standard	Lilliefors	CU	(Standard+CU)	(Lilliefors+CU)
Exp	$-$	9159	9488	9486	9310	9506
E_k	$k = 2$	131	0	9966	352	2
	$k = 4$	0	0	10000	0	0
	$k = 6$	0	0	10000	0	0
H_2	$c^2 = 1.25$	7819	8476	9169	8213	8637
	$c^2 = 1.5$	5631	6171	8811	6245	6687
	$c^2 = 2$	2331	2575	8256	2920	3160
	$c^2 = 4$	41	428	6850	73	536
	$c^2 = 10$	0	908	6565	0	1049
Z	$-$	2240	186	9843	3615	341
LN	$(1, 1)$	311	9	9690	955	29
RRI	$p = 0.1$	8741	9062	9318	8908	9117
	$p = 0.5$	4725	4192	8033	5033	4607
	$p = 0.9$	27	4	5680	33	10
$EARMA$	0.25	8992	9507	9312	9073	9401
	0.5	8596	9454	8916	8473	9089
	1	7939	9170	9238	8210	9124
	3	5111	6845	5671	3842	4921
	5.25	4192	6212	9098	4635	6665
mH_2	$m = 2$	3215	4133	8271	3571	4582
	$m = 5$	6661	8453	9213	7035	8635
	$m = 10$	7275	9312	9446	7746	9367
	$m = 20$	7614	9463	9494	8020	9489
$RRI(H_2)$	$p = 0.1$	55	442	6728	72	598
	$p = 0.5$	180	465	5846	205	564
	$p = 0.9$	3	2	4229	10	3

6 Extending Section 5.3: The Standard KS Test with Estimated Mean

In Tables 4, 8, and 10 of the main paper, we see that the simulation results of the invalid standard KS test with estimated mean is not consistent with the fixed significance level of $\alpha = 0.05$ because the null hypothesis is not rejected in over 99% of the cases. That is, we see that using the estimated mean in this setting tends to make rejection less likely.

To obtain a valid test of this form, we conduct additional simulation experiments with increased α until the null hypothesis is not rejected in only 95% of the cases in Tables 62 - 64, extending the results in Tables 4, 8, and 10 of the main paper, respectively. We observe that a realized significance level of $\alpha = 0.05$ is achieved if we increase the nominal significance level to $\alpha = 0.18$; then the results are very close to the results for the Lilliefors (1969) test in Tables 4, 8, and 10 of the main paper. Since these performance results do not differ greatly, one might consider the Lilliefors (1969) test as a realizable alternative to the standard KS test. However, this is evidently inferior to the Lewis KS test in the cases with non-exponential interarrival times.

Table 62 (Extending Table 4 of the main paper) Performance of the standard KS test with estimated mean for untransformed interarrival times from fixed interval $[0, t]$ for $t = 200$: Number of K-S tests passed (denoted by $\#P$) out of 10000 replications at varying significance levels.

$Case$	$Subcase$	significance level							
		0.05	0.10	0.15	0.16	0.17	0.18	0.19	0.2
Exp	$-$	9942	9820	9645	9604	9554	9509	9461	9418
E_k	$k = 2$	8	0	0	0	0	0	0	0
	$k = 4$	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	9564	9091	8573	8472	8383	8286	8171	8068
	$c^2 = 1.5$	8063	6888	5946	5777	5620	5454	5277	5147
	$c^2 = 2$	3752	2461	1745	1632	1530	1443	1360	1284
	$c^2 = 4$	157	60	34	32	25	23	21	21
	$c^2 = 10$	76	43	28	27	27	26	25	24
Z	$-$	1230	537	297	274	245	218	201	184
LN	$(1, 1)$	281	63	24	16	13	10	7	6
RRI	$p = 0.1$	9845	9571	9227	9141	9070	8997	8933	8849
	$p = 0.5$	6873	5497	4388	4191	4010	3860	3664	3509
	$p = 0.9$	44	8	3	3	2	2	1	1
$EARMA$	0.25	9942	9811	9628	9593	9541	9485	9433	9385
	0.5	9925	9758	9591	9545	9494	9444	9386	9343
	1	9845	9587	9313	9250	9182	9131	9063	8993
	3	8341	7639	7056	6932	6827	6738	6640	6554
	5.25	8015	7105	6389	6264	6158	6046	5924	5808
mH_2	$m = 2$	4949	3638	2839	2710	2600	2484	2382	2287
	$m = 5$	9414	8858	8324	8200	8099	7995	7892	7798
	$m = 10$	9874	9652	9378	9320	9254	9203	9147	9092
	$m = 20$	9935	9793	9584	9540	9498	9442	9395	9348
$RRI(H_2)$	$p = 0.1$	202	88	56	50	46	42	34	33
	$p = 0.5$	311	184	120	111	100	97	91	83
	$p = 0.9$	36	14	4	4	3	2	2	1

Table 63 (Extending Table 8 of the main paper) Performance of the standard KS test with estimated mean for untransformed interarrival times on $[0, 200]$ with 10 equally sized subintervals of length 20: Number of K-S tests passed (denoted by $\#P$) out of 10000 replications at varying significance levels.

$Case$	$Subcase$	significance level							
		0.05	0.10	0.15	0.16	0.17	0.18	0.19	0.2
Exp	$-$	9949	9818	9646	9603	9559	9517	9461	9421
E_k	$k = 2$	20	5	1	1	1	1	1	1
	$k = 4$	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	9681	9274	8841	8749	8663	8589	8498	8405
	$c^2 = 1.5$	8601	7633	6785	6623	6471	6321	6173	6037
	$c^2 = 2$	5290	3763	2865	2717	2597	2456	2339	2224
	$c^2 = 4$	908	439	255	226	200	190	176	165
	$c^2 = 10$	971	571	394	377	355	331	309	289
Z	$-$	1276	580	329	300	264	246	218	195
LN	$(1, 1)$	297	92	25	18	15	15	14	11
RRI	$p = 0.1$	9847	9587	9281	9216	9145	9060	8987	8911
	$p = 0.5$	7235	5885	4847	4639	4455	4286	4130	3983
	$p = 0.9$	69	22	12	9	9	7	7	7
$EARMA$	0.25	9949	9826	9634	9599	9545	9502	9456	9411
	0.5	9930	9794	9597	9553	9510	9456	9414	9365
	1	9878	9638	9366	9299	9232	9175	9111	9049
	3	8432	7719	7133	7041	6942	6849	6749	6638
	5.25	8324	7460	6729	6602	6488	6356	6227	6117
mH_2	$m = 2$	6883	5523	4574	4417	4244	4085	3934	3797
	$m = 5$	9645	9217	8762	8680	8595	8506	8404	8318
	$m = 10$	9905	9704	9473	9416	9352	9305	9251	9189
	$m = 20$	9940	9788	9617	9573	9524	9474	9418	9379
$RRI(H_2)$	$p = 0.1$	873	462	285	256	236	227	213	201
	$p = 0.5$	715	408	266	241	224	205	195	188
	$p = 0.9$	503	413	347	339	329	321	315	309

Table 64 (Extending Table 10 of the main paper) Performance of the standard KS test with estimated mean for untransformed interarrival times with $n = 200$: Number of K-S tests passed (denoted by $\#P$) out of 10000 replications at varying significance levels.

$Case$	$Subcase$	significance level							
		0.05	0.10	0.15	0.16	0.17	0.18	0.19	0.2
Exp	–	9945	9815	9636	9597	9554	9516	9468	9410
E_k	$k = 2$	2	1	0	0	0	0	0	0
	$k = 4$	0	0	0	0	0	0	0	0
	$k = 6$	0	0	0	0	0	0	0	0
H_2	$c^2 = 1.25$	9516	9016	8510	8425	8324	8213	8112	8012
	$c^2 = 1.5$	8030	6810	5821	5640	5483	5326	5193	5059
	$c^2 = 2$	3818	2577	1836	1727	1629	1540	1464	1393
	$c^2 = 4$	348	182	117	111	101	95	85	80
	$c^2 = 10$	360	241	181	171	163	154	149	146
Z	–	1042	444	233	206	182	168	148	131
LN	$(1, 1)$	147	35	9	6	6	4	3	2
RRI	$p = 0.1$	9842	9585	9275	9196	9127	9056	8977	8895
	$p = 0.5$	6857	5416	4360	4186	4037	3876	3741	3592
	$p = 0.9$	50	11	4	4	4	4	4	2
$EARMA$	0.25	9947	9818	9663	9614	9573	9537	9482	9438
	0.5	9918	9778	9582	9545	9487	9440	9387	9347
	1	9829	9561	9297	9241	9187	9117	9045	8966
	3	8387	7672	7091	6987	6890	6800	6689	6578
	5.25	8002	7035	6324	6183	6062	5945	5823	5691
mH_2	$m = 2$	4908	3718	2987	2885	2776	2666	2569	2478
	$m = 5$	9421	8859	8306	8195	8095	7983	7893	7784
	$m = 10$	9877	9660	9391	9334	9272	9206	9146	9083
	$m = 20$	9929	9820	9637	9599	9551	9510	9452	9404
$RRI(H_2)$	$p = 0.1$	432	235	166	148	139	127	117	109
	$p = 0.5$	674	401	274	258	238	219	209	192
	$p = 0.9$	19	3	2	2	2	0	0	0

7 Extending Section 7: CLT View

In Section 7 of the main paper, we study the asymptotics of the KS tests and show that even though the CU KS test has so little power for non-exponential interarrival times, it is the best way to detect dependent exponential interarrival times with ample data. Figures 105-112 are supplements to Figures 4-6 in the main paper: we show histograms of $\sqrt{t}\hat{D}_t$ based on 100 replications for $t = 2 \times 10^5$ for exponential, E_2, H_2 with $c^2 = 2$, and EARMA cases. Furthermore, Figures 113-144 show plots of $\sqrt{t}|F_t(x) - F(x)|$ for $x = 0.2$, 0.4, 0.6, and 0.8 as a function of t up to $t = 2 \times 10^5$, with t expressed in log scale. For each KS test, the 95% confidence interval lines are also shown. They are again for exponential, E_2, H_2 with $c^2 = 2$, and EARMA cases, supplementing Figure 7 in the main paper.

Figure 105 Exp - Histogram of $\sqrt{t}D_t$ with $t = 2 \times 10^5$ from 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (from left to right).

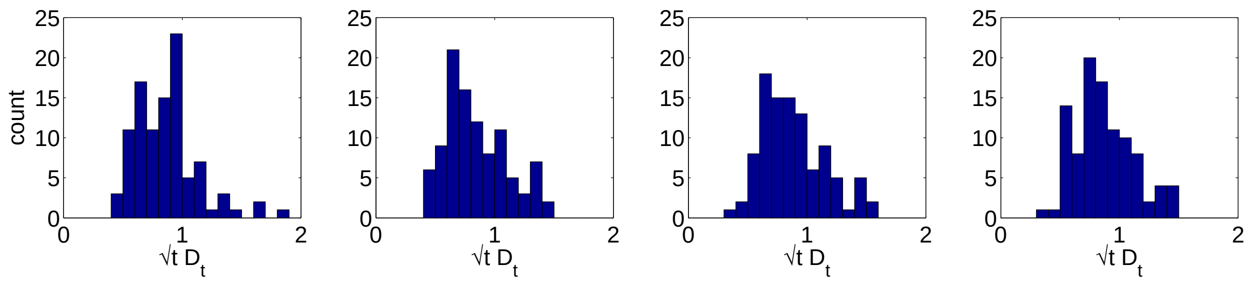


Figure 106 E_2 - Histogram of $\sqrt{t}D_t$ with $t = 2 \times 10^5$ from 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (from left to right).

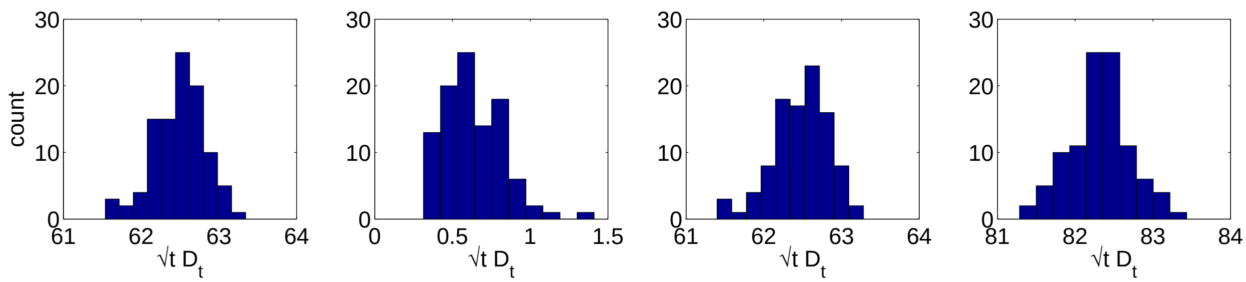


Figure 107 H_2 ($c^2 = 2$) - Histogram of $\sqrt{t}D_t$ with $t = 2 \times 10^5$ from 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (from left to right).

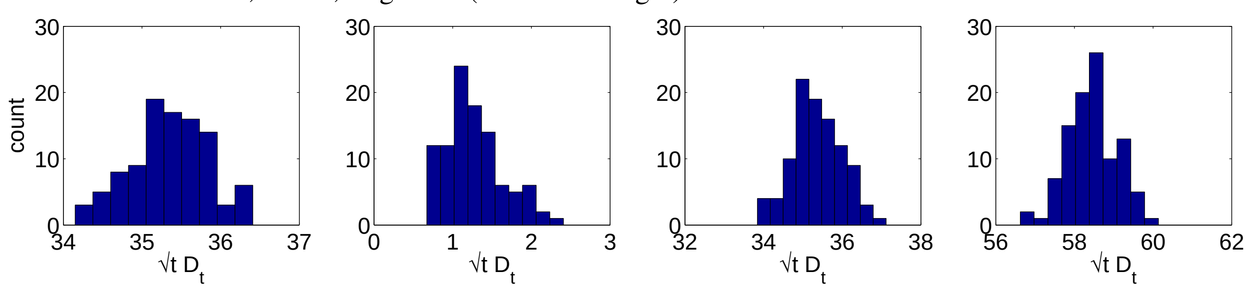


Figure 108 $EARMA$ (0.25) - Histogram of $\sqrt{t}D_t$ with $t = 2 \times 10^5$ from 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (from left to right).

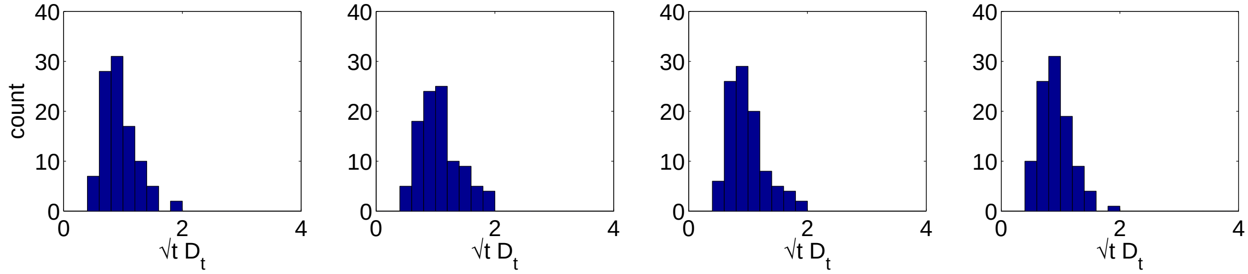


Figure 109 $EARMA$ (0.5) - Histogram of $\sqrt{t}D_t$ with $t = 2 \times 10^5$ from 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (from left to right).

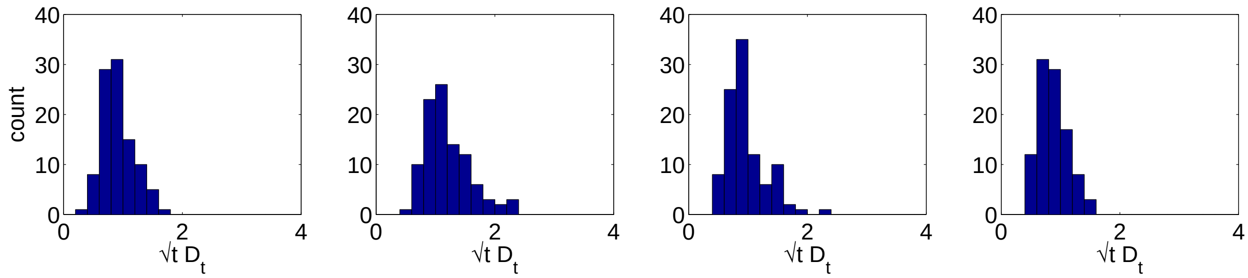


Figure 110 $EARMA$ (1) - Histogram of $\sqrt{t}D_t$ with $t = 2 \times 10^5$ from 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (from left to right).

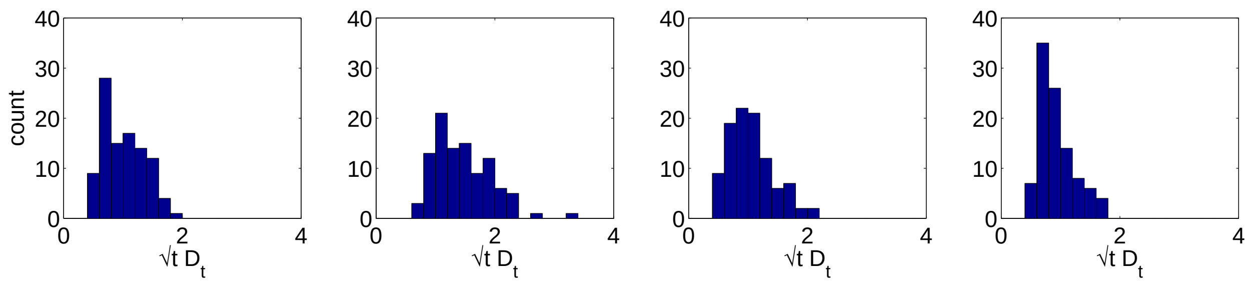


Figure 111 $EARMA$ (3) - Histogram of $\sqrt{t}D_t$ with $t = 2 \times 10^5$ from 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (from left to right).

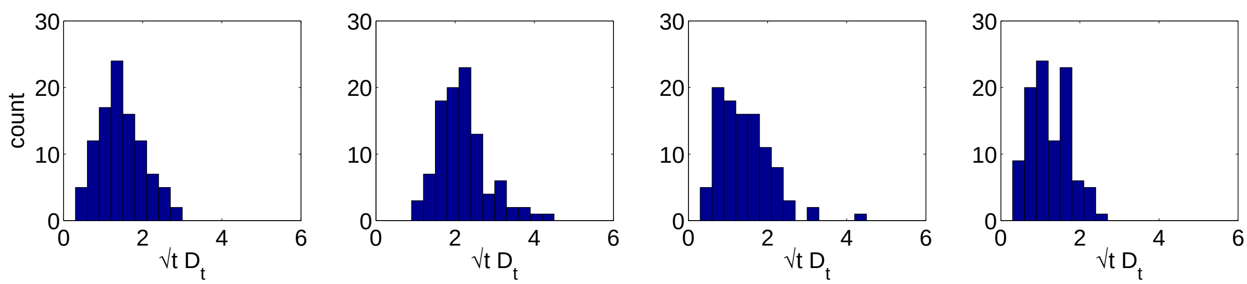


Figure 112 $EARMA$ (5.25) - Histogram of $\sqrt{t}D_t$ with $t = 2 \times 10^5$ from 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (from left to right).

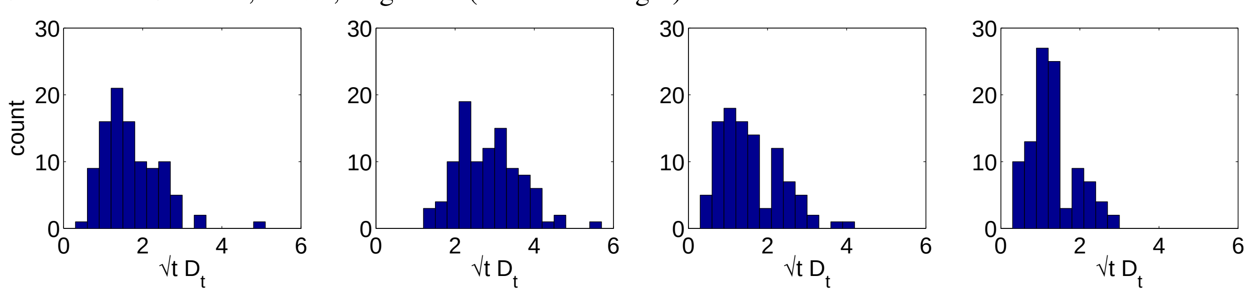


Figure 113 Exp - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.2$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

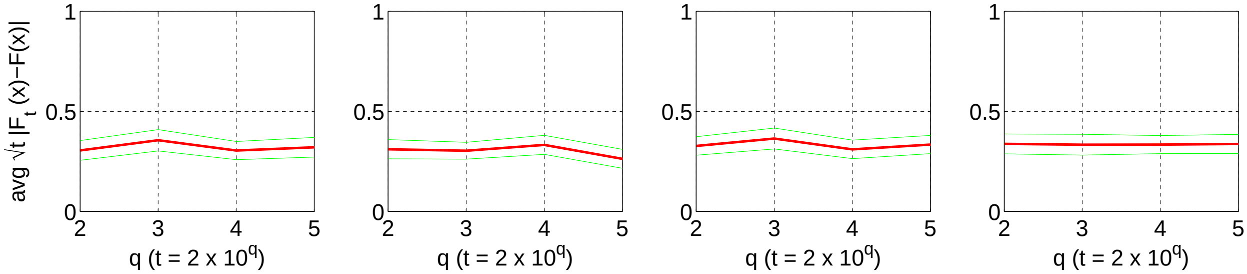


Figure 114 Exp - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.4$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

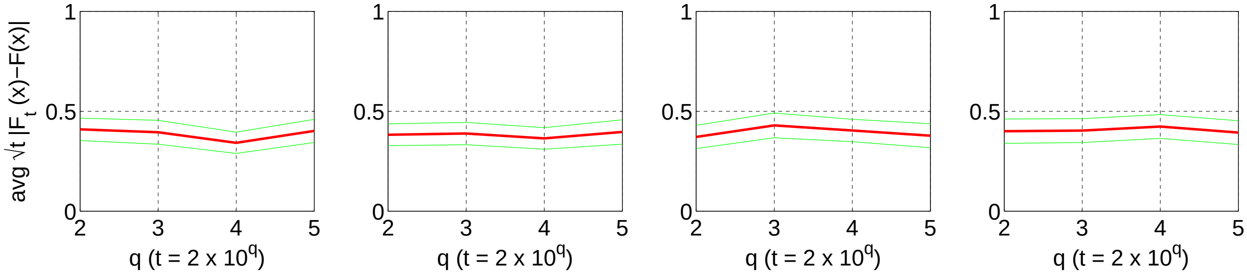


Figure 115 Exp - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.6$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

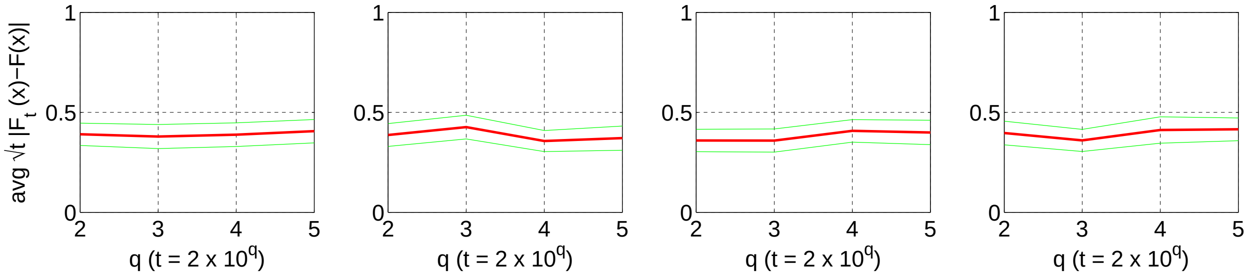


Figure 116 Exp - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.8$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

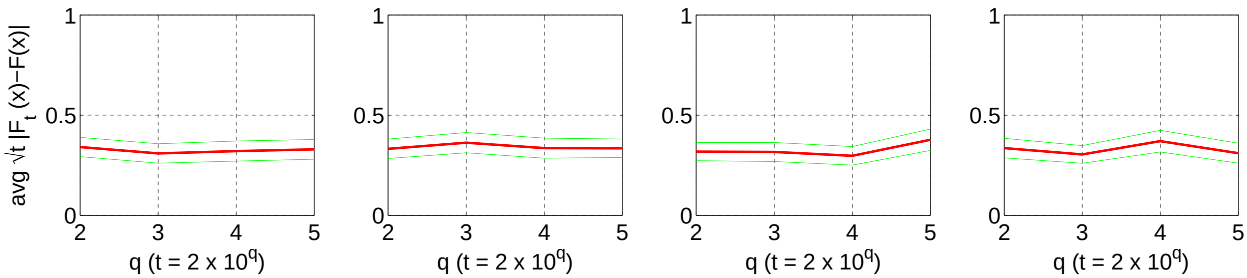


Figure 117 E_2 - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.2$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

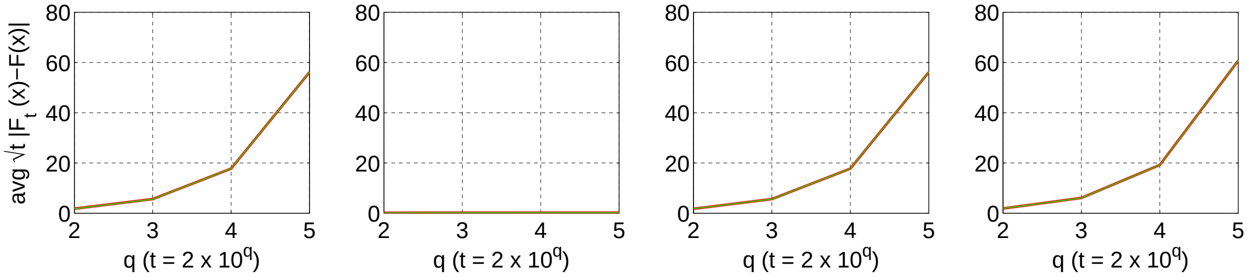


Figure 118 E_2 - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.4$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

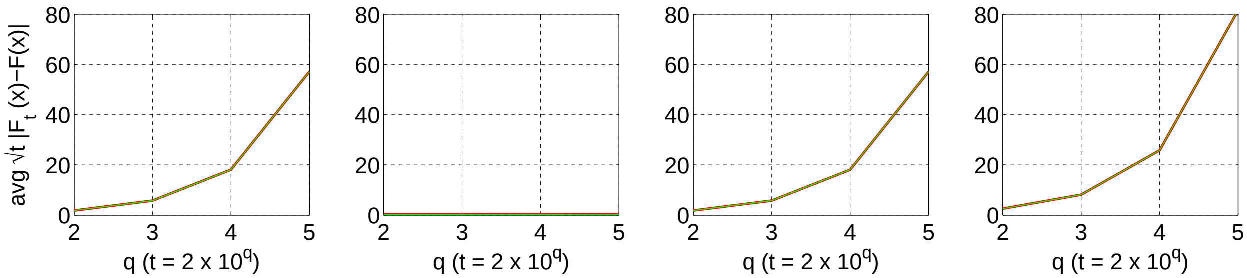


Figure 119 E_2 - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.6$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

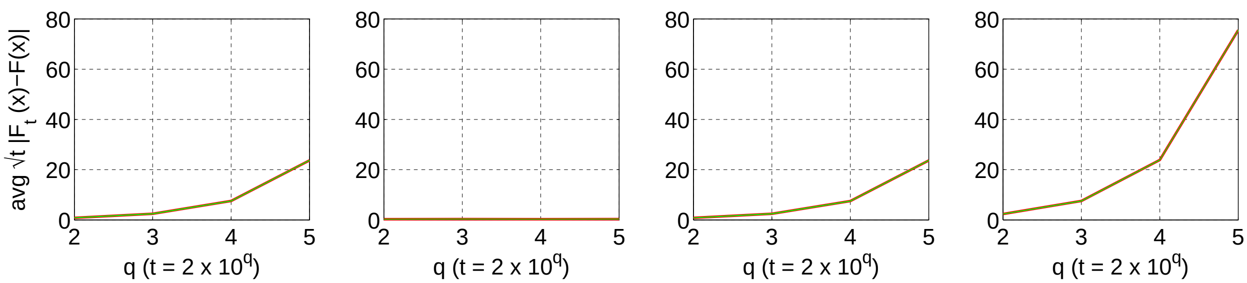


Figure 120 E_2 - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.8$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

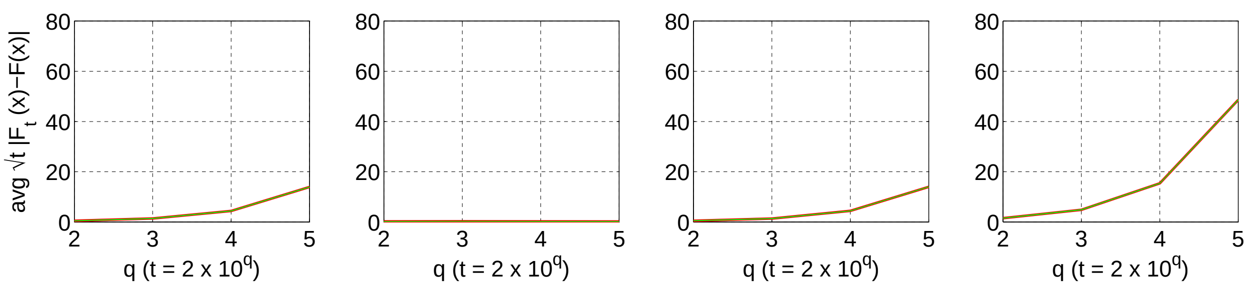


Figure 121 H_2 ($c^2 = 2$) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.2$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

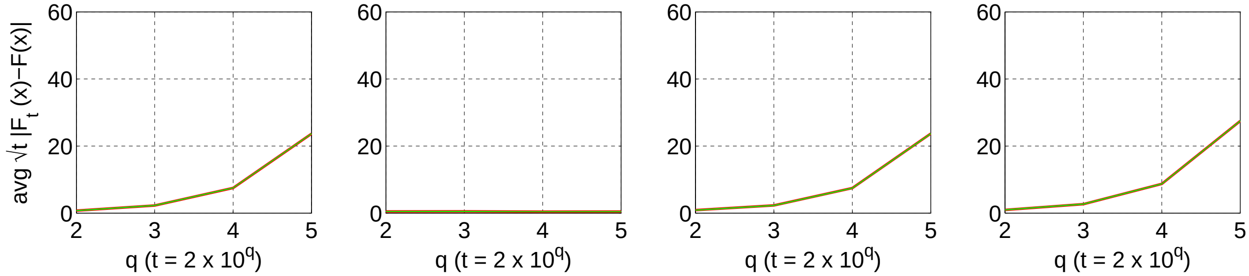


Figure 122 H_2 ($c^2 = 2$) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.4$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

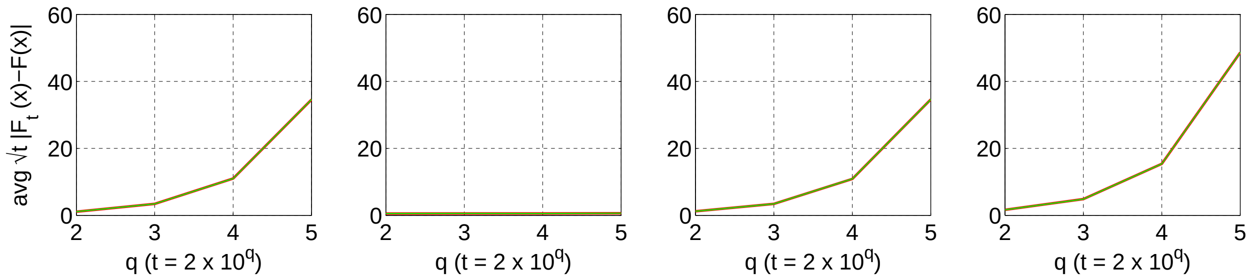


Figure 123 H_2 ($c^2 = 2$) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.6$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

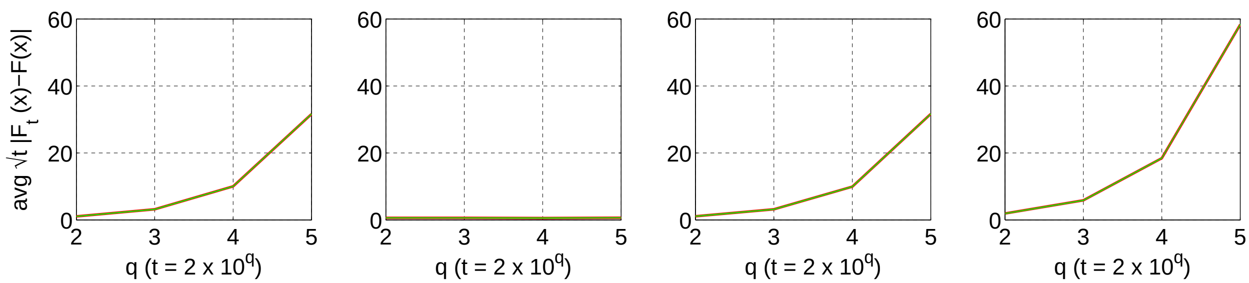


Figure 124 H_2 ($c^2 = 2$) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.8$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

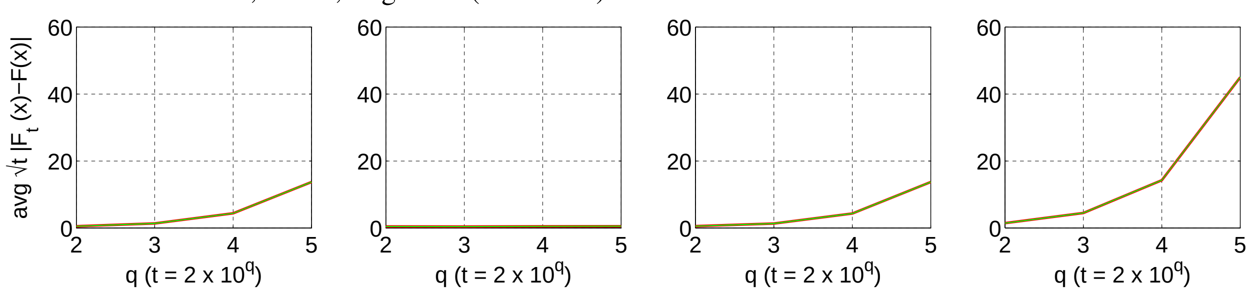


Figure 125 $EARMA$ (0.25) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.2$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

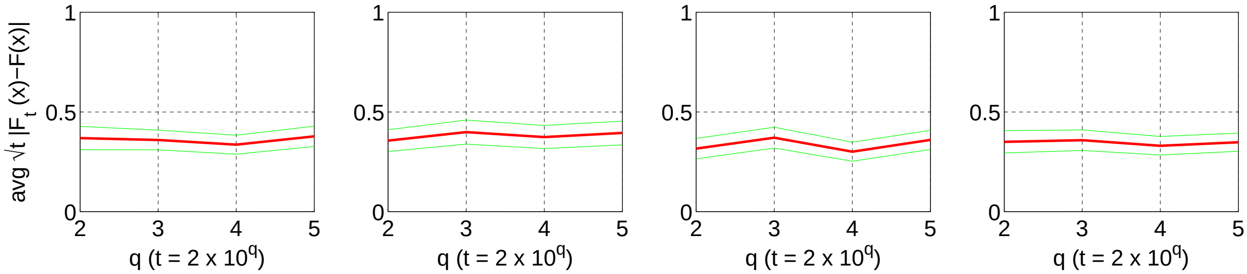


Figure 126 $EARMA$ (0.25) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.4$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

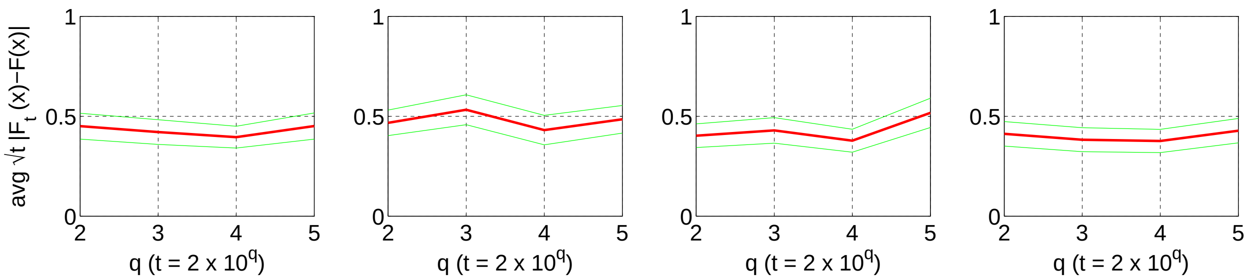


Figure 127 $EARMA$ (0.25) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.6$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

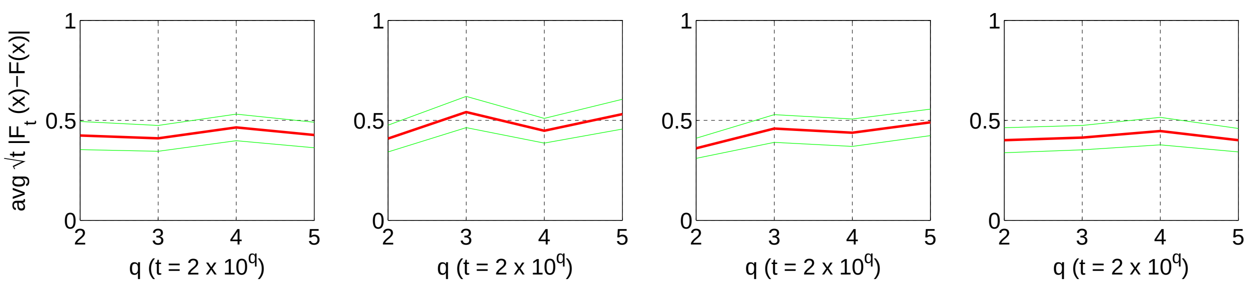


Figure 128 $EARMA$ (0.25) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.8$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

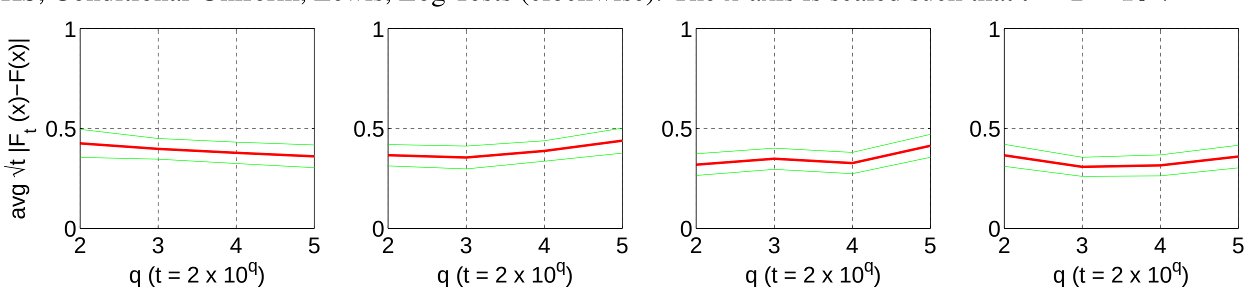


Figure 129 $EARMA$ (0.5) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.2$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

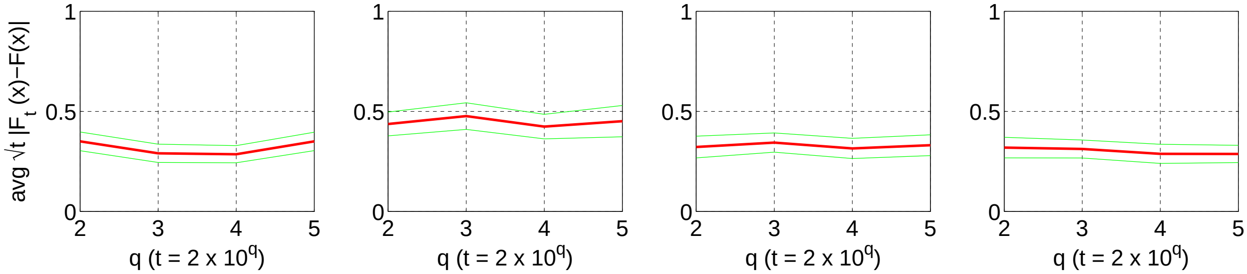


Figure 130 $EARMA$ (0.5) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.4$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

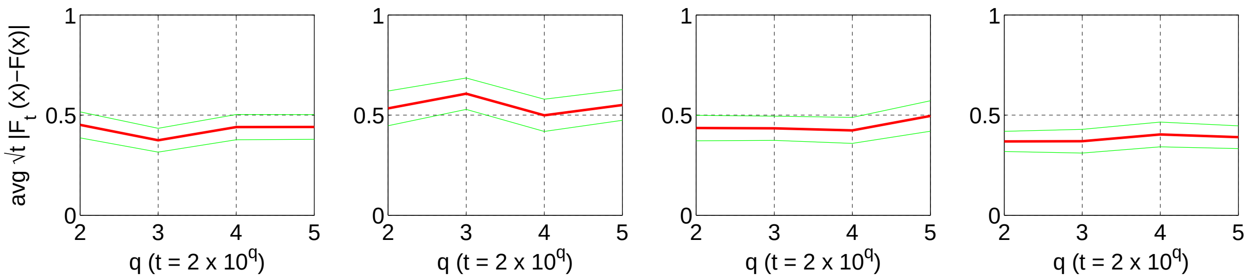


Figure 131 $EARMA$ (0.5) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.6$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

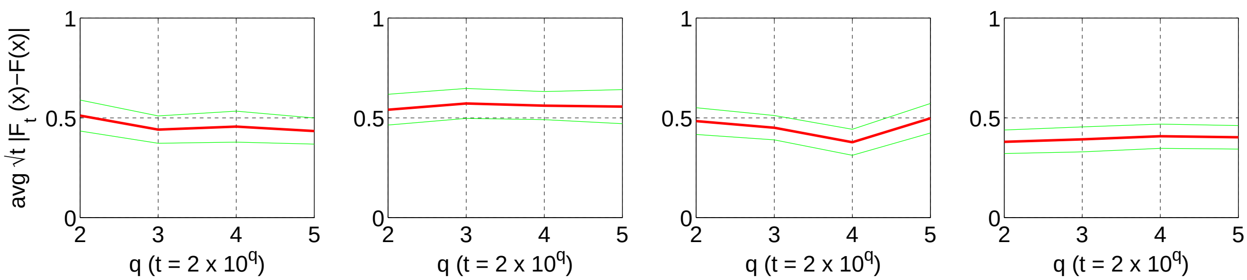


Figure 132 $EARMA$ (0.5) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.8$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

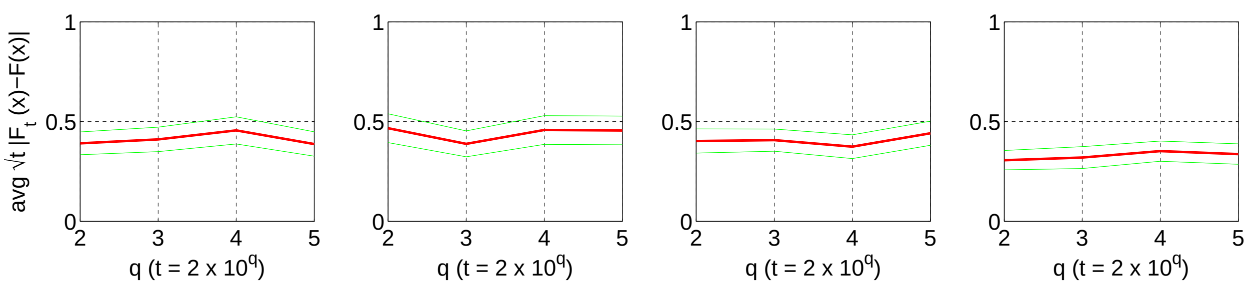


Figure 133 $EARMA$ (1) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.2$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

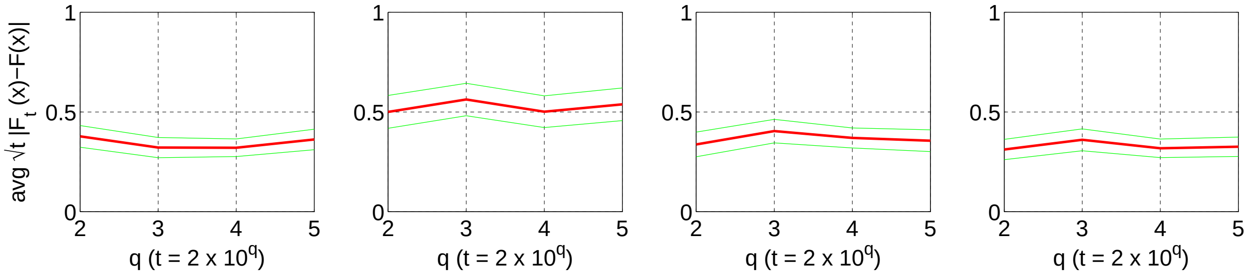


Figure 134 $EARMA$ (1) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.4$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

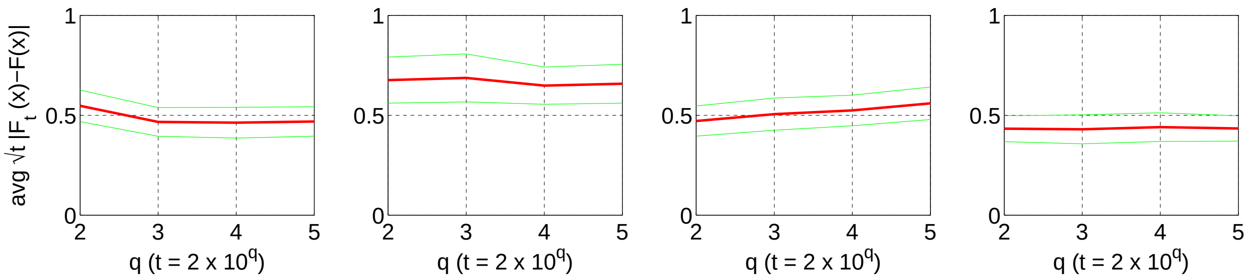


Figure 135 $EARMA$ (1) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.6$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

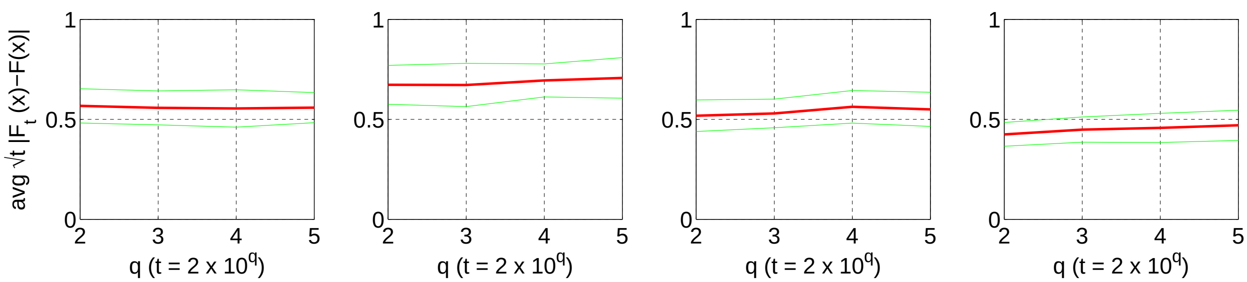


Figure 136 $EARMA$ (1) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.8$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

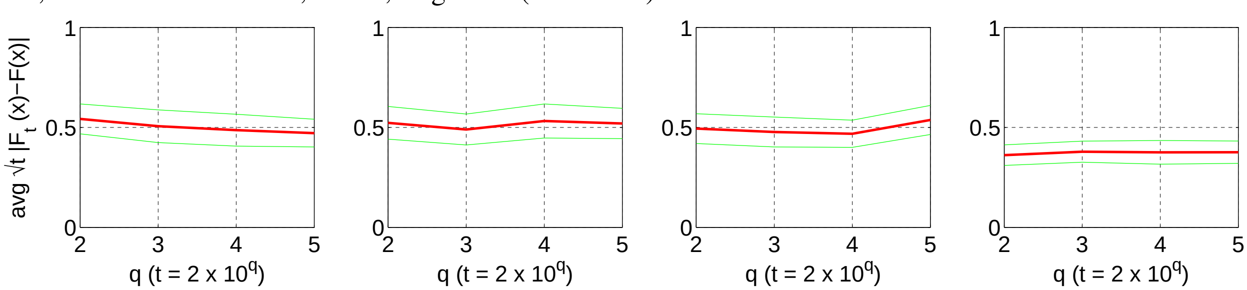


Figure 137 $EARMA$ (3) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.2$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

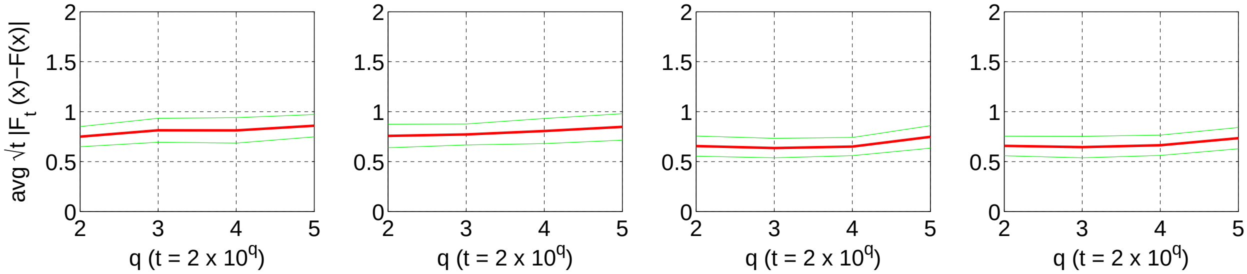


Figure 138 $EARMA$ (3) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.4$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

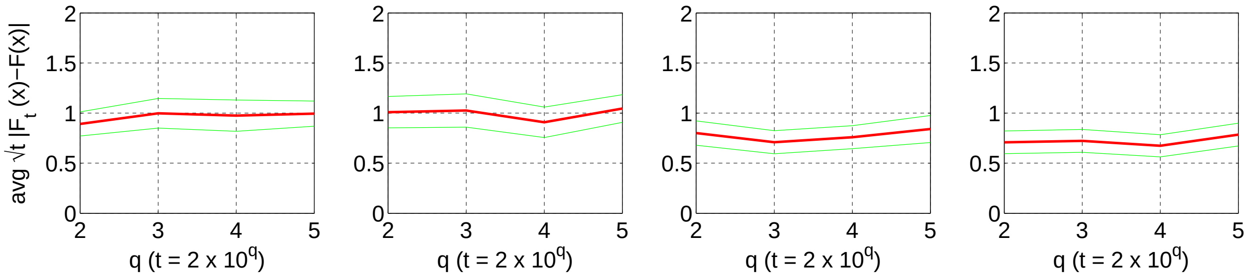


Figure 139 $EARMA$ (3) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.6$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

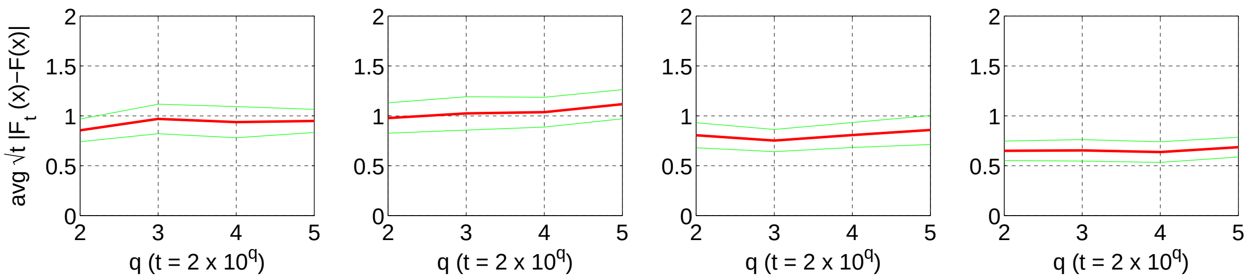


Figure 140 $EARMA$ (3) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.8$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

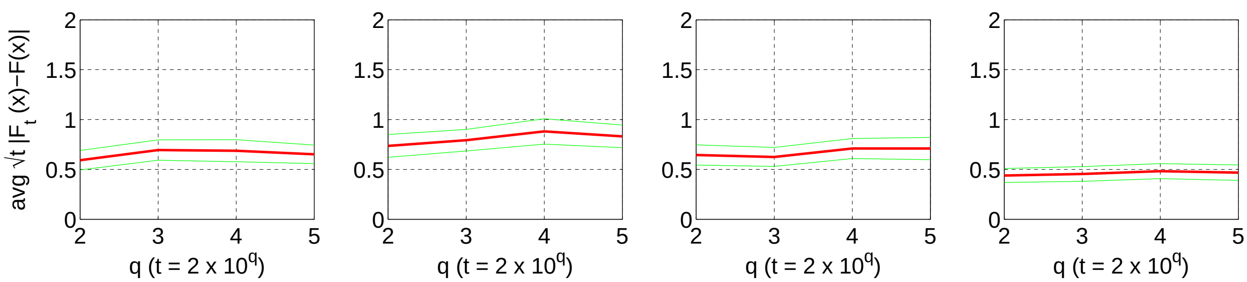


Figure 141 $EARMA$ (5.25) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.2$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

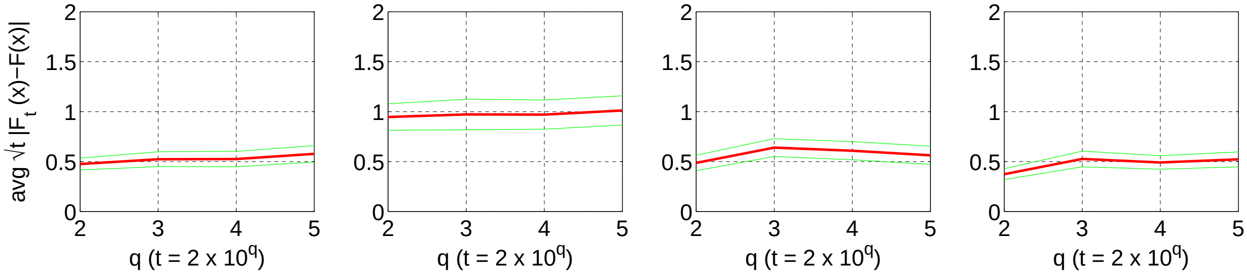


Figure 142 $EARMA$ (5.25) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.4$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

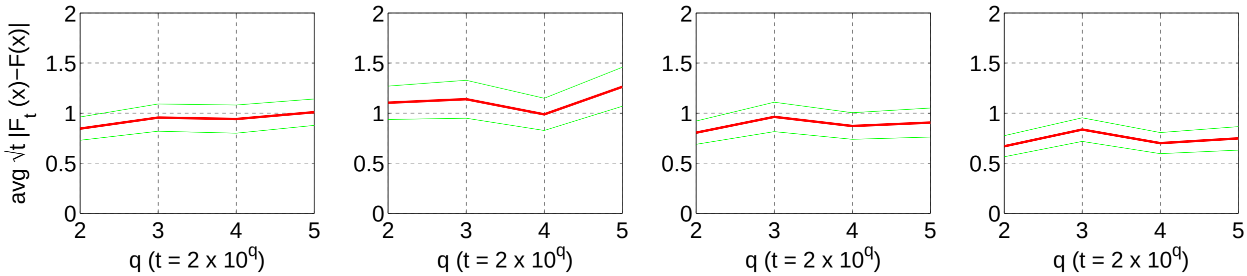


Figure 143 $EARMA$ (5.25) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.6$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.

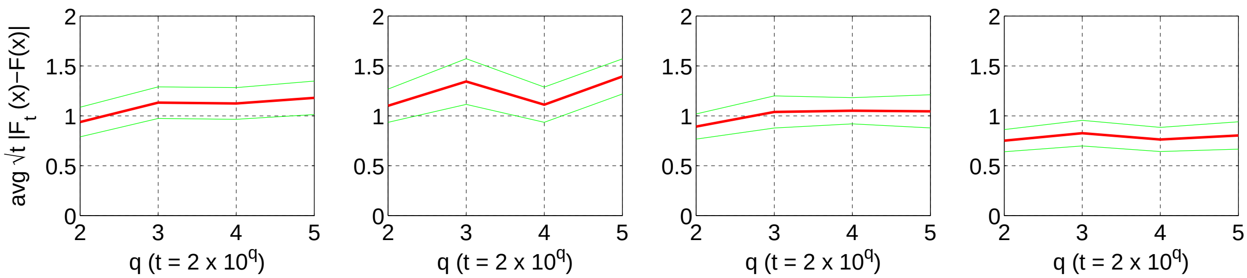
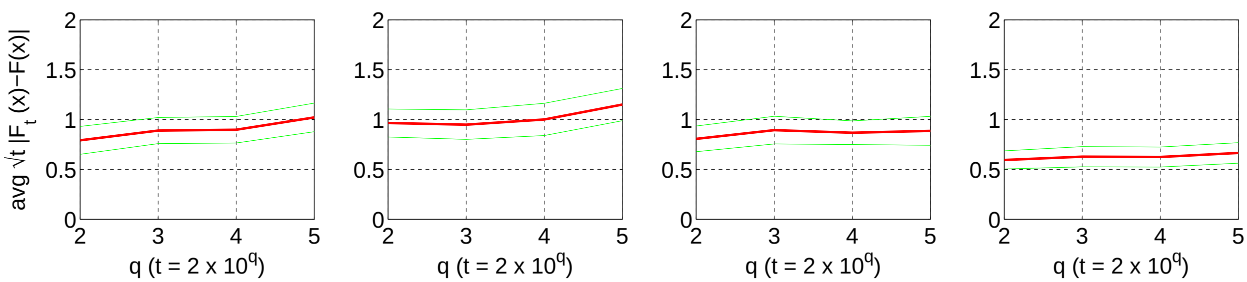


Figure 144 $EARMA$ (5.25) - Average $\sqrt{t}|F_t(x) - F(x)|$ with $F(x) = 0.8$ over 100 replications: Standard KS, Conditional-Uniform, Lewis, Log Tests (clockwise). The x-axis is scaled such that $t = 2 \times 10^q$.



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