

# **A Staffing Algorithm for Call Centers with Skill-Based Routing**

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**Joint work with:**

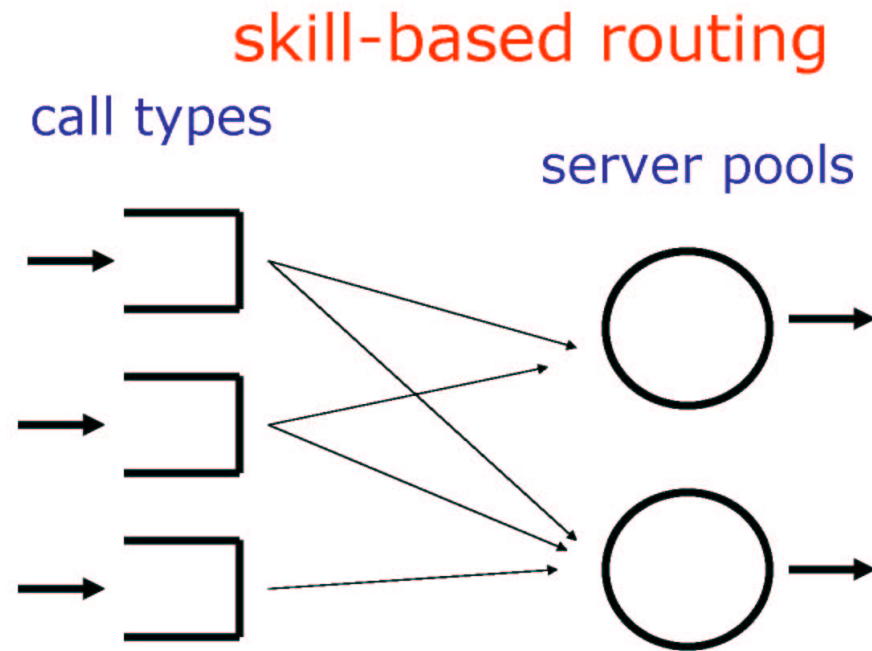
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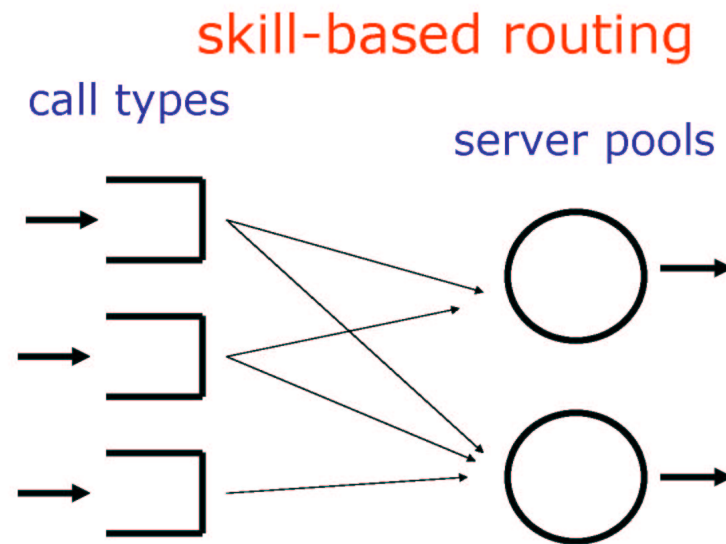
**Thesis:** Performance Modelling and Design  
of Call Centers with Skill-Based Routing

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# Multiple Types of Calls and Agents



# Multiple Types of Calls and Agents



Special case: The service-time distribution does not depend on the call type or the agent.

## First Contribution:

### Routing and Provisioning Algorithm

Minimize the Required Staff and Telephone Lines  
While Meeting the Service level Agreement (SLA)

$$P(\text{Delay} \leq 30 \text{ seconds}) \geq 0.80$$

$$P(\text{Blocking}) \leq 0.005$$

(service level may depend on call type)

## Second Contribution:

### Demonstrate Resource-Pooling Phenomenon

A small amount of cross training (multiple skills) produces almost the same performance as if all agents had all skills (as in the single-type case).

### Simulation Experiments

# Precedents

” A little bit of flexibility goes a long way.”

## Joining One of Many Queues

- Azar, Broder, Karlin and Upfal (1994)
- Vvedenskaya, Dobrushin and Karpelovich (1996)
- Turner (1996, 1998)
- Mitzenmacher (1996) and
- Mitzenmacher and Vöcking (1999)

## Flexible Manufacturing: Chaining

- Jordan and Graves (1995)
- Aksin and Karaesman (2002)
- Hopp and Van Oyen (2003)
- Jordan, Inman and Blumenfeld (2003)
- Gurumurthi and Benjaafar (2004)

# Outline

1. **SBR Call-Center Model**
2. **Resource-Pooling Experiment**
3. **Provisioning Algorithm**
4. **Simulation to Show Performance**



## $M_n/M_n/C/K/NPrPr$ **SBR Call Center**

1.  $C$  agents,  $C + K$  telephone trunklines, and  $n$  call types.
2. *Non-preemptive Priorities (NPrPr)* - Calls are processed in priority order. Calls are worked to completion once they are handed to an agent.
3. *Longest-Idle-Agent Routing (LIAR) Policy* - Calls are forwarded to the agent who has been waiting the longest since his last job completion and has the highest skill to handle the request.

## Agent-Skill Matrix - $C \times n$

4. *Agent-Skill Profile* - Predefined in an agent-skill matrix  $A \equiv (a_{ij})$  as

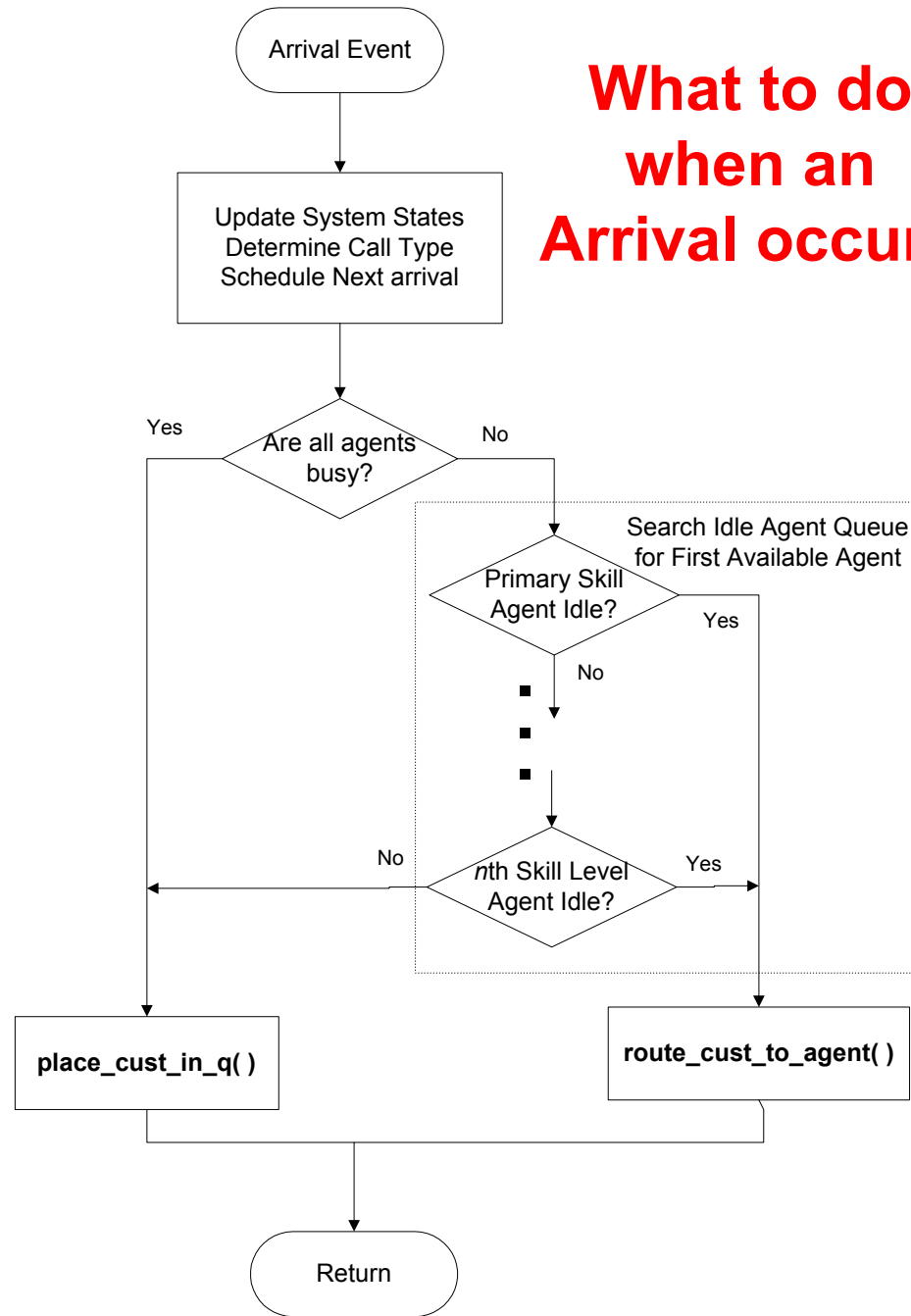
$$a_{ij} = \begin{cases} k & \text{when agent } i \text{ supports call type } k \\ & \text{at priority level } j \text{ (primary, secondary, etc),} \\ 0 & \text{otherwise.} \end{cases}$$

where  $i = 1, \dots, C$ ,  $1 \leq k \leq n$ , and  $1 \leq j \leq n$ .

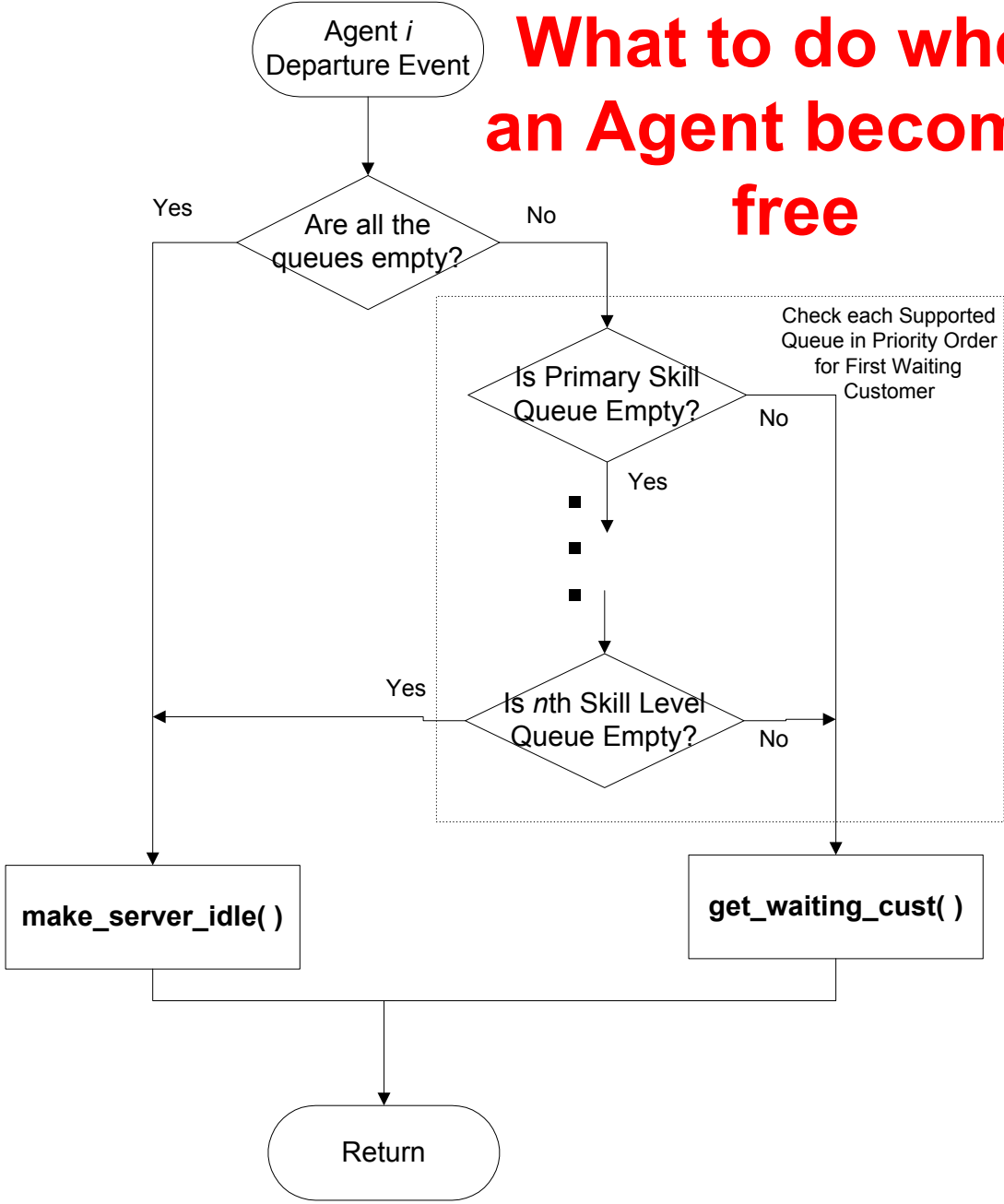
### Examples:

$$\mathbf{A}_{5 \times 1} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{A}_{3 \times 2}^{(1)} = \begin{pmatrix} 1 & 0 \\ 2 & 0 \\ 2 & 0 \end{pmatrix}, \quad \mathbf{A}_{4 \times 2} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 2 & 1 \\ 2 & 1 \end{pmatrix}, \quad \mathbf{A}_{6 \times 4} = \begin{pmatrix} 3 & 4 & 1 & 0 \\ 1 & 4 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 3 & 1 & 2 & 4 \\ 1 & 0 & 4 & 0 \end{pmatrix}$$

# What to do when an Arrival occurs



# What to do when an Agent becomes free



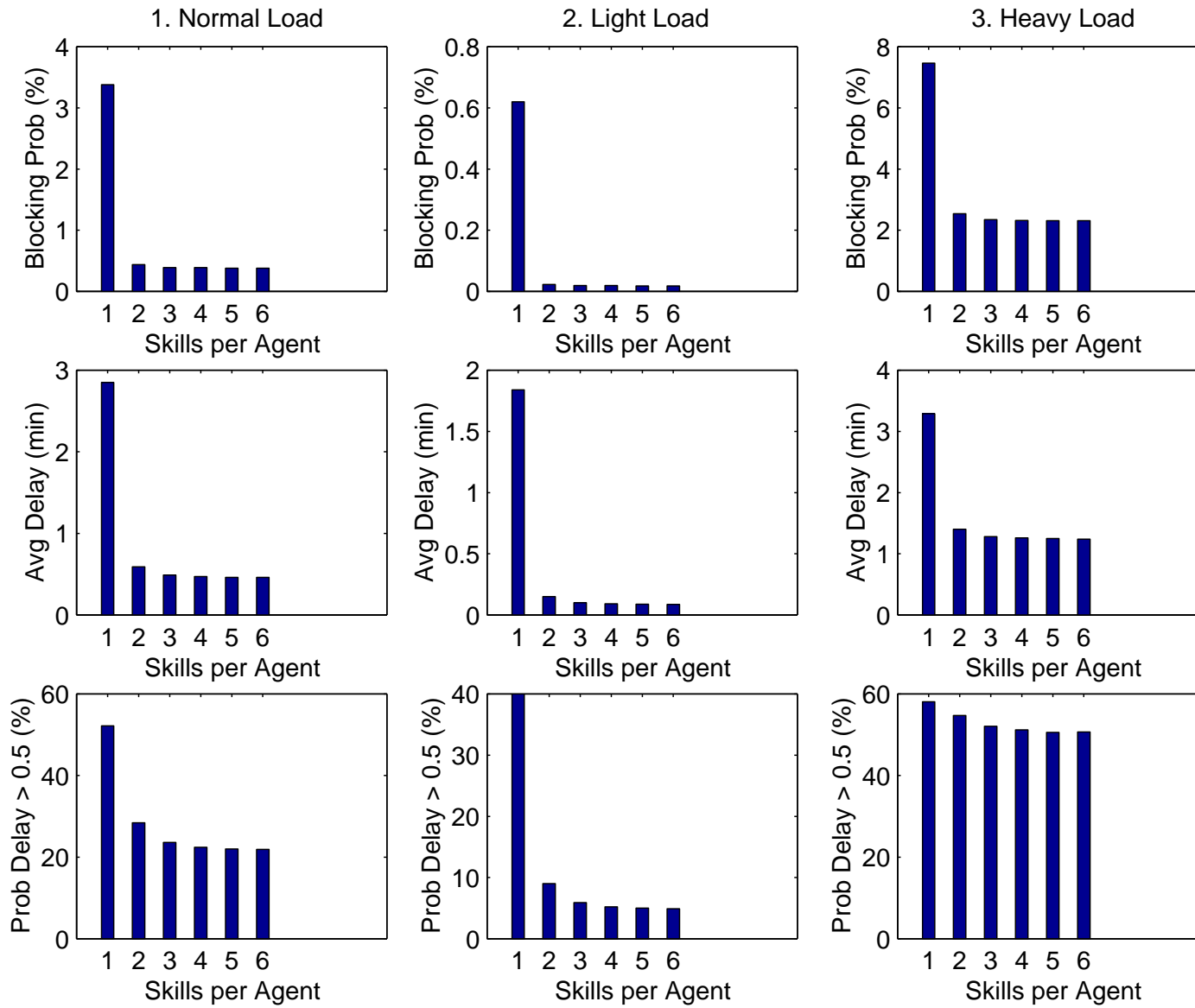
# Resource-Pooling Experiment

# Model Assumptions

1. **Arrival Process** -  $n$  types of calls arrive at the call center according to  $n$  mutually independent Poisson processes with rate  $\lambda_i$ ,  $1 \leq i \leq n$ .  
[ $n = 6$ ,  $\lambda_i = 1.40$  for all  $i$ ]
2. **Service Time Process** - Call holding (service) times are mutually independent exponential random variables with mean  $1/\mu_i$  which are independent of the arrival process,  $1 \leq i \leq n$ .  
[ $1/\mu_i = 1/\mu = 10$  minutes for all  $i$ ]
3. **Offered Loads** -  $\alpha_i = \lambda_i/\mu_i$   
[ $\alpha_i = 14$  for all  $i$ , so the total offered load is  $\alpha = 84$ ]
4. **Agents and Telephone Lines**  
[ $C = 90$  and  $K = 30$  ( $C + K = 120$ )]

Agents are given  $k$  skills,  $1 \leq k \leq 6$

Three Loads: Normal (84), Light (77.4), Heavy (90)





# Provisioning Algorithm

Find **C**, **K** and **A**

So that each agent has **at most 2 skills**  
and all performance constraints are met.

How do we know it works?

The optimal values of **C** and **K**  
are almost the same as for **M/M/C/K**  
which occurs with a single call type.

## Balanced Example

M/M/C/K: C = 90 and K = 19

SBR: C = 91 and K = 20

## SBR Balanced Provisioning Example

- Call volume is  $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = 1.375$ ,
- Service times are  $1/\mu_1 = \dots = 1/\mu_6 = 10$  mins
- Agents Skill Profile: Agents have 2 skills each.
- Service level targets
  1. Blocking service level target is 0.5%.
  2. 80% of the calls are answered within  $\tau = 0.5$  minute.
- Square-root safety method for distributing agents into work groups is used.
- It is known that the total number of agents required is between 90 (best-case) and 106 (worse-case). Similarly, the the telephone trunkline capacity is between 111 and 156.

## Unbalanced Example

**M/M/C/K: C = 90 and K = 19**

**SBR: C = 91 and K = 21**

## SBR Unbalanced Provisioning Example

- Call volume is  $\lambda_1 = \lambda_2 = 0.425$ ,  $\lambda_3 = 1.05$ ,  $\lambda_4 = 1.375$ ,  $\lambda_5 = 1.925$ , and  $\lambda_6 = 3.05$  calls/min.
- Service times are  $1/\mu_1 = \dots = 1/\mu_6 = 10$  mins
- Agents Skill Profile: Agents have 2 skills each.
- Service level targets
  1. Blocking service level target is 0.5%.
  2. 80% of the calls are answered within  $\tau = 0.5$  minute.
- Square-root safety method for distributing agents into work groups is used.
- It is known that the total number of agents required is between 90 (best-case) and 106 (worse-case). Similarly, the the telephone trunkline capacity is between 111 and 156.

## Unbalanced SBR Provisioning Example Summary

	Best Case	Actual Perf.	Worst Case
$(C, C + K)$	(90, 109)	(91, 111)	(106, 156)
Workgroup 1 $C_1$		7	7
Workgroup 2 $C_2$		7	7
Workgroup 3 $C_3$		13	14
Workgroup 4 $C_4$		15	18
Workgroup 5 $C_5$		21	24
Workgroup 6 $C_6$		28	36

## SBR Provisioning

- Solves the problem of determining the minimum number of agents  $C$  and the minimum number of telephone trunklines  $C + K$  needed to meet service level targets.
- Exploits resource pooling results.
- Exploits  $M/M/C/K$  results to determine initial estimate for  $(C, K)$ .
- Uses fair agent skill assignment scheme to construct agent skill matrix satisfying general agent skill profile.
- Simulation runs are performed to make improvements on the initial assignment using a heuristic search algorithm.



# Determining Primary Skills

$$C_k = \alpha_k + x\sqrt{\alpha_k}$$

$$x = \frac{(C - \alpha)}{\sum_{i=1}^n \sqrt{\alpha_i}}$$

**and round**

# Determining Secondary Skills

$$C_{i,k} = \frac{C_i C_k}{C - C_i}$$

**and round**

**STOP**

<b>Initial SBR Provisioning Algorithm</b>				
	<b>Number of Iterations (Agents)</b>			
<b>Performance Measure</b>	<b>1 (90)</b>	<b>2 (91)</b>	<b>3 (92)</b>	<b>4 (93)</b>
1. Blocking (%)	0.53	0.42	0.36	0.30
4. $\mathcal{P}(\text{Delay} \leq 0.5   \text{entry})$	81.3	83.9	86.5	88.8
5. $\mathcal{P}(\text{Delay}_1 \leq 0.5   \text{entry})$	68.3	75.5	78.4	80.5
5. $\mathcal{P}(\text{Delay}_2 \leq 0.5   \text{entry})$	65.2	74.9	77.8	80.3
5. $\mathcal{P}(\text{Delay}_3 \leq 0.5   \text{entry})$	79.7	81.8	84.7	88.0
5. $\mathcal{P}(\text{Delay}_4 \leq 0.5   \text{entry})$	82.0	83.6	86.5	88.8
5. $\mathcal{P}(\text{Delay}_5 \leq 0.5   \text{entry})$	83.4	86.2	87.8	89.8
5. $\mathcal{P}(\text{Delay}_6 \leq 0.5   \text{entry})$	84.4	85.8	88.7	90.9

<b>Refined SBR Provisioning Algorithm</b>						
	<b>Number of Iterations (Agents)</b>					
<b>Performance Measure</b>	<b>4 (93)</b>	<b>5 (92)</b>	<b>6 (92)</b>	<b>7 (91)</b>	<b>8 (91)</b>	<b>9 (90)</b>
1. Blocking (%)	0.30	0.35	0.36	0.43	<b>0.44</b>	<b>0.54</b>
4. $\mathcal{P}(\text{Delay} \leq 0.5   \text{entry})$	88.8	86.5	86.2	83.4	82.9	<b>79.8</b>
5. $\mathcal{P}(\text{Delay}_1 \leq 0.5   \text{entry})$	80.5	<b>78.0</b>	81.6	<b>78.6</b>	<b>82.6</b>	80.0
5. $\mathcal{P}(\text{Delay}_2 \leq 0.5   \text{entry})$	80.3	<b>77.6</b>	81.4	<b>78.6</b>	<b>81.9</b>	79.7
5. $\mathcal{P}(\text{Delay}_3 \leq 0.5   \text{entry})$	88.0	86.1	85.8	83.6	<b>83.4</b>	<b>78.6</b>
5. $\mathcal{P}(\text{Delay}_4 \leq 0.5   \text{entry})$	88.8	87.2	<b>87.0</b>	83.2	<b>82.6</b>	<b>80.5</b>
5. $\mathcal{P}(\text{Delay}_5 \leq 0.5   \text{entry})$	<b>89.8</b>	<b>87.7</b>	86.7	<b>84.6</b>	<b>83.1</b>	<b>79.4</b>
5. $\mathcal{P}(\text{Delay}_6 \leq 0.5   \text{entry})$	<b>90.9</b>	<b>88.0</b>	<b>86.9</b>	<b>84.1</b>	<b>82.9</b>	<b>80.3</b>