As a post-doctoral fellow at Columbia University under Professor Michael Weinstein, the applicant will pursue further studies of linear and nonlinear dispersive equations. Such studies will include stability theory of solitons, describing the spectrum of linearized Hamiltonians, scattering thresholds for nonlinear equations, well-posedness results for various nonlinearities, parametrix constructions and well-posedness results for general classes of pseudodifferential operators, and billiard problems. As a main focus, the applicant will pursue results for the nonlinear Schrödinger equation, but will also study nonlinear wave equations and relativity. The methods required for the above projects will include, but are not limited to, Fourier analysis, microlocal/semiclassical analysis, phase space transforms, functional analysis, Sobolev estimates, numerical analysis, variational methods and spectral theory. The equations of interest have broad applications to the study of Bose-Einstein condensates, fiber optic communications, water waves and other physical applications.

Primarily, the applicant wishes to build on work done in the doctoral dissertation studying a focusing nonlinear Schrödinger equation with a saturated nonlinearity in three dimensions of the form:

$$\begin{cases} iu_t + \Delta u + \beta(|u|^2)u = 0 \\ u(0,x) = u_0(x), \end{cases} \tag{1}$$

where β is given certain structure. Soliton solutions of such an equation are solutions of the form $e^{i\lambda^2 t}Q_\lambda(x)$, where Q_λ is the unique, radial, decreasing solution to

$$-\Delta Q_\lambda + \lambda^2 Q_\lambda - \beta(|Q_\lambda|^2)Q_\lambda = 0.$$

Saturated nonlinearities are those that behave like supercritical nonlinearities for small values and subcritical nonlinearities for large values. Numerical experiments show that these nonlinearities have a soliton of minimal L^2 mass. By recent results due to Comech-Pelinovsky, it is known that such a soliton is unstable under general perturbations. Using the methods of Bourgain-Wang and Krieger-Schlag, a stable class of perturbations is constructed for this minimal mass soliton, say Q. The key methods involved are a detailed look at the spectral theory of the Hamiltonian linearized about Q, as well as development of a general distorted Fourier basis theory in order to push through the existence argument. From the knowledge that such stable perturbations exist, the applicant will further investigate the dynamics of minimal mass soliton interaction with perturbations as well as other solitons.

In addition to the above, the applicant will pursue results towards scattering and disperion. Weinstein proved that for the L^2 critical NLS equation in any dimension n, $\beta(|u|^2) = |u|^{\frac{4}{n}}$, there is a minimal mass, $M = \|Q_1\|_{L^2}$, for the initial data below which only linear dispersion occurs. In other words, if $\|u_0\|_{L^2} < M$, given the solution u in 1 above, there exists some $\phi \in L^2$ such that

$$\|u(t) - e^{it\Delta}\phi\|_{L^2} \to 0,$$

as $t \to \infty$. This is proved by relating the ideal constant of an inequality to the mass of the soliton, which is constant due to the scaling in the critical problem. Both analytically describing the shape of the soliton curve in general situations and proving dispersion below the minimal mass become incredibly difficult without scaling. The applicant will continue to work in this direction in order to anaytically prove scattering below this minimal L^2 mass. Scattering results that do not depend upon scaling are very rare and would go a long way towards understanding soliton resolution.