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The Generation of Tripoles from Unstable Axisymmetric Isolated Vortex Structures.

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Abstract. – Tripolar coherent vortices are shown to emerge from the unstable evolution of perturbed two-dimensional axisymmetric flows. They are obtained from the nonlinear equilibration of barotropically linearly unstable normal modes, as well as from more general initial perturbations. This instability is proposed as an important mechanism for the generation of both dipolar and tripolar coherent vortex structures.

In recent years, strong isolated coherent vortex structures have been shown to play an important role in the dynamics of two-dimensional (2D), as well as quasi-geostrophic (QG) flows. Numerous examples have been observed and studied in both laboratory experiments and geophysical situations [1, 2]. Strong vortices spontaneously appear from stochastic initial conditions in high-resolution numerical simulations of 2D [3, 4] and stratified QG [5] turbulence. They also arise from mixed barotropic-baroclinic instabilities of strong jets, such as western boundary currents in the oceans.

Many recent studies have focused on the generation and interactions (both barotropic [6, 7] and baroclinic [8]) of isolated vortices. Until not long ago, two types of vortices have received the greatest attention: the monopole and the dipole. The next structure in complexity, commonly referred to as a tripole, has three vorticity maxima, of alternate signs and rotates as a whole with constant angular velocity. Originally mentioned by Leith [9], the first tripole was observed in a very high resolution simulation of plane turbulent flows by Legras, Benzi and Santangelo [10]. Polvani and Carton [11] have investigated the steady configurations and the stability of the tripolar V-states (piecewise constant vorticity patches), while Carton [12] has shown that tripoles appear in long-time evolutions of barotropically unstable Karman streets. Moreover, tripoles have also been generated in laboratory experiments with a rotating tank by Van Heijst and Kloosterziel [13].

One mechanism for the generation of tripoles was uncovered by Larichev and Reznik [14]: they obtained a tripole via a nonsymmetric collision of two modons. Reference [12] and some preliminary numerical experiments led us to investigate an alternative, and, we believe, more important mechanism: the generation of tripoles from the instability of axisymmetric monopoles. In this letter, we wish to report on results obtained through high-resolution numerical simulation of the long-time evolution of perturbed, linearly unstable axisymmetric vortices. The results presented here are not fully comprehensive, but are representative of the main qualitative behaviors.

The general framework of the above studies was that of homogeneous, incompressible two-dimensional nearly inviscid flows, described by the Euler equation. We start by describing the linear problem of the barotropic instability and show how the eigenmodes which have the largest growth rate can be obtained. Examples where linearly unstable eigenmodes achieve a finite amplitude equilibration into a tripolar configuration are then presented and analyzed in terms of fundamental physical processes.

For greater generality, we consider the equivalent-barotropic equations:

$$[\partial_t + J(\psi, \cdot)]\, q = 0 \qquad \text{and} \qquad q = [\nabla^2 \psi - \gamma^2 \psi]\,, \tag{1}$$

where q is the potential vorticity and ψ the streamfunction, γ is the ratio of a typical length scale for the vortex to the deformation radius (a measure of the relative strength of gravity *vs.* rotation). For $\gamma = 0$, the two-dimensional Euler equations are recovered.

We first study the linear stability of an axisymmetric vorticity distribution, whose streamfunction $\bar{\psi}(r)$ is an exact solution of (1). We consider normal mode perturbations proportional to $\exp[il(\theta - ct)]$, where l is the azimuthal wave number, and write the total streamfunction $\psi(r, \theta)$ as follows:

$$\psi(r, \theta) = \bar{\psi}(r) + \psi'(r) \exp[il(\theta - ct)]\,, \tag{2}$$

where

$$A = \frac{|\psi'|}{|\bar{\psi}|} \ll 1\,.$$

After substitution of (2) into (1) the linearized equation for ψ^{prime} has to be solved numerically for the phase speed and the growth rate of the perturbation. In general, mode $l = 1$ is found to be barotropically stable [15], and mode $l = 2$ is the most unstable wave number [16]. Another important property of continuous two-dimensional flows is the existence of critical layers, which make the numerical calculation of the normal modes arduous.

The next step is consider the finite-amplitude evolution of the most unstable normal mode for a given γ, l, and $\bar{\psi}$. To study this we initialize a spectral model with the superposition of the mean profile and this eigenvector. The initial amplitude A of the perturbation is usually a few percent of the mean flow, and the horizontal resolution is $(128)^2$. Due to the enstrophy cascade, we employ the standard approach of adding a nonphysical hyperviscosity term on the right-hand side of (1), proportional to $\nabla^4 q$. This procedure does not alter the outcome of the computation, however, for the enstrophy is conserved in all cases to a few parts in a thousand.

From a large number of numerical experiments, we have found that the formation of tripoles from unstable axisymmetric vortices requires both a certain type of unperturbed radial vorticity profile (not too remote from the shape of the final structure, and moreover only slightly barotropically unstable), and a sufficiently small amplitude of the normal mode perturbation. The mean vortex must exhibit two regions of opposite-signed vorticity from the center to the periphery. An example of such profiles is given by the following family of vortices, indexed by a «steepness parameter» α:

$$\bar{q} = \left(1 - \frac{\alpha}{2} r^{\alpha}\right) \exp\left[-r^{\alpha}\right], \tag{3}$$

which corresponds to a shielded vorticity monopole (having no net circulation—the «shielding» is created by the opposite vorticity in the external region). For $\alpha = 3$, and $A = 0.05$ at $t = 0$, the most unstable ($l = 2$) normal mode is seen to equilibrate readily into a tripole (see fig. 1). In fig. 2 we present the time evolution of the amplitude of the $l = 2$ mode, which shows a rapid equilibration after the period of exponential growth.

Whether the nonlinear evolution of the perturbed state of (3) yields a tripole depends on the value of α, which controls the steepness of the vorticity gradients, and the initial amplitude of the disturbance. In fig. 3, we present the regime diagram for this family of

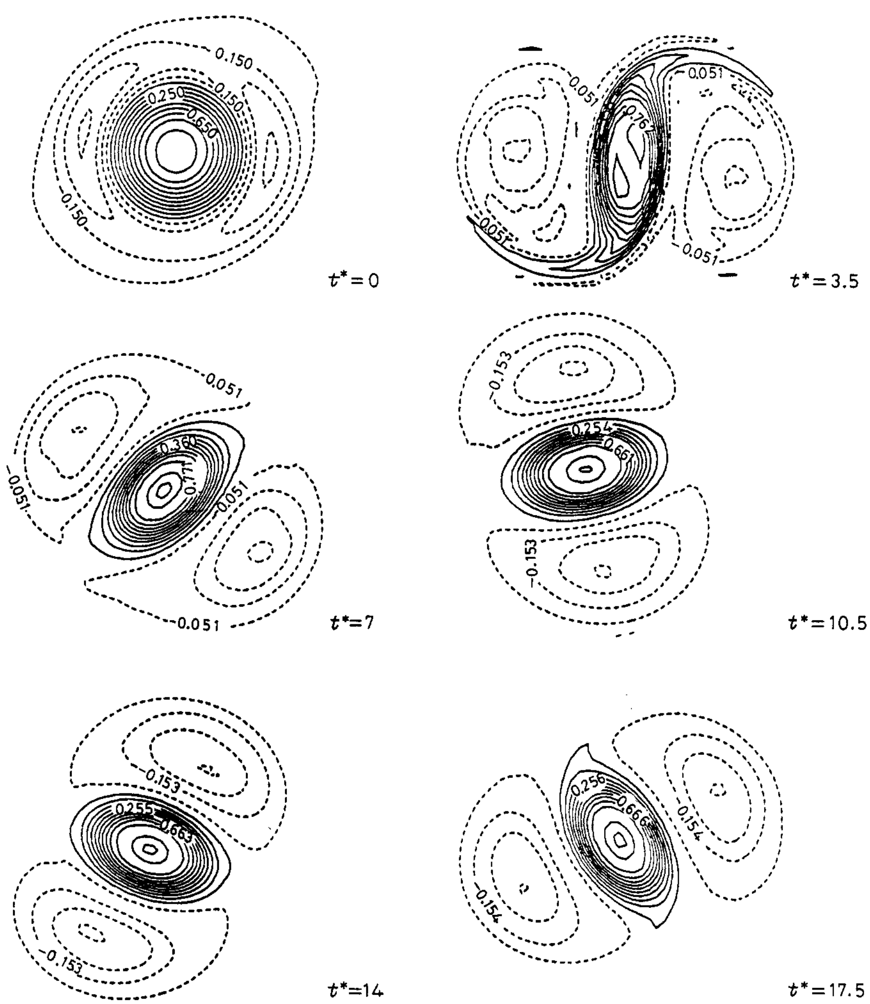

Fig. 1. – The emergence of a 2D tripole from a perturbed mean profile given by (3) with $\alpha = 3$. The initial amplitude of the disturbance is 0.05. Frames are shown at $t = 0$, 1.8τ, 3.6τ, etc., where τ is the eddy turnover time (time advances to the right and downwards).

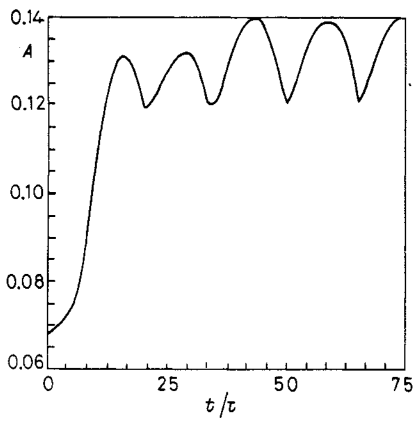

Fig. 2.

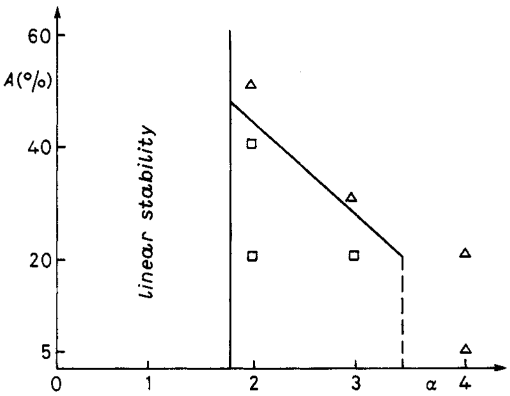

Fig. 3.

Fig. 2. – The amplitude of the $l = 2$ mode *vs.* time, as an initially perturbed two-dimensional vortex of mean profile given by (3) equilibrates into a tripole. For this run $\alpha = 3$ and $A = 0.05$ at $t = 0$. τ is the eddy turnover time.

Fig. 3. – The regime diagram for finite amplitude evolution of the most unstable normal mode disturbances on a 2D vortex, with mean profile (3). α represents the steepness of the shielded monopole and A is the perturbation amplitude at $t = 0$. Experiments were performed for every integer value of α shown here, and with a 0.1 increment in A, from $A = 0$ to $A = 0.6$.

mean profiles. Unless α and the perturbation amplitude A are moderately small, corresponding to smoothly varying mean profiles, the unstable vortex will split into two dipoles.

Several other experiments have been conducted with either $l \neq 2$ and/or with mean profiles other than a shielded monopole (we note that unshielded monopoles are uninteresting, for they are stable by the Rayleigh criterion); they were found to lead to a catastrophic growth of the perturbation, resulting in the breaking of the original vortex. We propose the following interpretation: to yield a tripole, profiles which are initially too different from a shielded monopole have to undergo a drastic topological change incompatible with the weak instability (*i.e.* small growth rate) and low perturbation amplitudes required for equilibration (such is the case of unshielded annuli, for instance). Higher-wave-number disturbances need much larger values of α at the onset of instability than those observed for $l = 2$. Since the initial conditions of the nonlinear simulations are slightly distinct from a pure normal mode (this is chiefly due to discretization errors, even at high resolution (256^2)), the spurious mode-2 component initially present grows to breaking, preventing a hypothetical fragile stabilization for high wave numbers from occurring.

In the equivalent-barotropic case ($\gamma \neq 0$), tripole generation seems to be a more common phenomenon, and is observed for a number of different families of mean profiles (shielded monopoles, shielded annuli—which differ from the former by a vanishing vorticity at the origin). The phenomenon is very similar to the one we have just illustrated: an initially unstable perturbation grows exponentially at early times, but eventually saturates. We show this explicitly in fig. 4, where the amplitude of the most unstable $l = 2$ mode is plotted *vs.* time, for a mean profile $\bar{\Psi}(r) = \exp[-r^4]$, with $\gamma = 3.5$ and $A = 0.05$ at $t = 0$. In this case, the resulting tripole exhibits larger amplitude oscillations during the nonlinear regime than the pure two-dimensional example.

Other examples of oscillating tripoles emerging from unstable axisymmetric vortices have been observed in a two-layer quasi-geostrophic model, for both shielded and

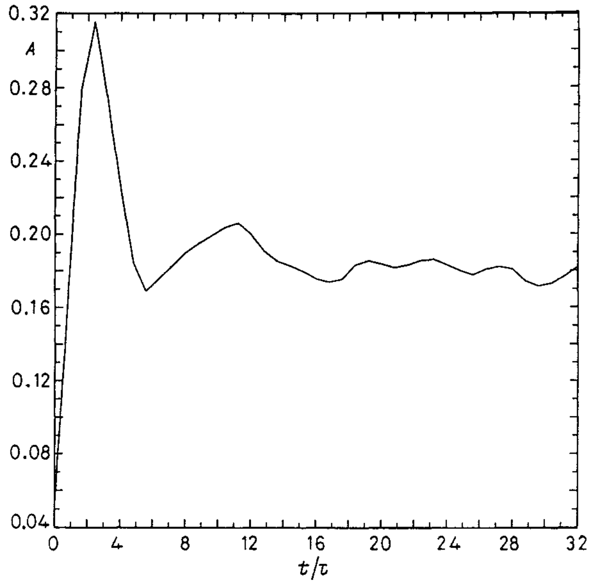

Fig. 4. – The amplitude of the nonaxisymmetric part of the potential vorticity *vs.* time, as an initially perturbed equivalent-barotropic $\psi = \exp[-r^4]$ vortex equilibrates into a tripole, for $\gamma = 3.5$ and $A = 0.05$ at $t = 0$.

unshielded vortices ([17, 18]). In these examples also, the nonlinear evolution of the tripoles is strongly oscillatory, and is complicated by the presence of baroclinic instability. The periodic variation of the phase shift between the two layers plays a critical role in the balance of the energy transfers during the finite-amplitude regime. (Incidentally, tripoles have also appeared from the same instability in nongeostrophic regimes [19].)

A modal analysis was carried out for each of the runs we have performed, and showed that, once the tripolar state is reached, most of the amplitude of the nonaxisymmetric component of the flow is concentrated in a few small wave numbers. At that time, the correction to the mean flow tends to reduce, though not completely, the linear instability. Mode $l = 2$ oscillates regularly in amplitude, though its radial structure is very different from the original eigenmode. Mode 4 is correlated in time with mode 2, but has a much smaller intensity. Higher modes are virtually negligible. This is confirmed by a box diagram of the energy exchanges [12], which shows dominant conversions between these few low modes.

Normal mode perturbations are not the only means of generating tripoles from perturbed axisymmetric initial conditions. Substitution of

$$r = [x^2 + ((1 + \varepsilon)\, y)^2]^{1/2}$$

with $\varepsilon = 0.05$ into a profile similar to (3) leads to tripoles, provided the steepness of the vorticity distribution is not too great. Runs with higher viscosity, which tends to smooth the vorticity gradients, yield tripolar states from initially steeper profiles. In general, beyond some maximum steepness, the splitting into two dipoles appears inevitable.

The barotropic (or baroclinic) instability of axisymmetric vortices is therefore an effective means of generating tripoles, although the perturbation energy involved must remain small compared to the energy of the axisymmetric flow. The variety of the long-time behaviors nevertheless indicates a trend towards more vacillation when a greater number of degrees of freedom is present. From our results, we conclude that, in all cases studied, only a few azimuthal modes of perturbation play a significant dynamical role in the nonlinear equilibration process. A Landau equation describing the finite-amplitude evolution of the perturbation amplitude A has been investigated, but the presence of a critical layer for the marginal eigenmode precludes the possibility of a simple and rigorous derivation of such an equation. A low-order Galerkin projection model, retaining only the first few harmonics, is now being developed to describe semi-analytically the nonlinear stabilization regime, and to account for its disappearance for stronger instability (*i.e.* for large α) [18].

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