9/21/04

HOMEWORK #1 SOLUTIONS
AP E4010 INTRO NUC SCIENCE

1.1 AVERAGE DENSITY $= A m_u / (\frac{4}{3}\pi R^3)$

$$R = 1.2 \times A^{1/3} \text{ fm}$$

$\therefore$ AVG DEN $= 2.29 \times 10^{17}$ kg/m^3

NOTE: DENSITY OF WATER ≈ 1 g/cc
$= 1 \times 10^3$ kg/m^3

NUCLEUS IS 200,000,000,000,000 TIMES DENSITY OF H_2O

1.2 WAVELENGTH $= \lambda \sim 2\pi \hbar / p$

PHOTON: $p = E/c$
PARTICLE: $p = \sqrt{2 m_0 E}$ (NON-RELATIVISTIC)
$= \gamma m_0 c \left(\frac{v}{c}\right)$ (RELATIVISTIC)

RECALL: $E^2 = m_0^2 c^2 + p^2 c^2$ TOTAL ENERGY
TWO COMMON LIMITS
KINETIC ENERGY $= p^2/2m_0$ $\ll m_0 c^2$
$= pc$ $\gg m_0 c^2$

FOR OUR NUCLEAR PHYSICS, ONLY ELECTRONS WILL BE RELATIVISTIC: $m_e c^2 \approx 0.511$ MEV ELECTRONS
$m_p c^2 \approx 938$ MEV PROTONS

(WE'LL WORK UP TO A FEW 10'S MEV.)

EINSTEIN DEFINED $\gamma \equiv 1/\left(1 - \frac{v^2}{c^2}\right)^{1/2}$ $\left(\beta \equiv \frac{v}{c}\right)$

$$E_{K.E.} = m_0 c^2 (\gamma - 1)$$

$$\therefore \quad \gamma = 1 + E_{K.E.}/m_0 c^2$$

$$\frac{v}{c} = \sqrt{1 - 1/\gamma^2}$$

PROBLEM 1.2 (CONTINUED)

THUS, WAVELENGTHS

PHOTON: $\lambda = 2\pi \hbar/p = 2\pi \dfrac{\hbar c}{E} = 2\pi \dfrac{197.3\ fm}{E(MeV)}$

N.R. PARTICLE: $\lambda = 2\pi \hbar/p = 2\pi \dfrac{\hbar c}{\sqrt{2 m_0 c^2 E}} = 2\pi \dfrac{197.3\ fm}{\sqrt{2 \times 931.5\ m_u E(MeV)}}$

$$= \frac{2\pi\ fm}{0.2187\sqrt{m_u E(MeV)}}$$

REL. PARTICLE: $\lambda = 2\pi\ \hbar c / \gamma m_0 c^2 \left(\frac{v}{c}\right)$

$$= 2\pi \frac{197.3\ fm}{0.511\ \gamma \left(\frac{v}{c}\right)} \quad \text{(ELECTRONS)}$$

	ENERGY	PHOTON	ELECTRON	NEUTRONS	100 m_u
(NUC)	1 MeV	1.2 pm	0.87 pm	28.6 fm	2.87 fm
(IONIZATION)	10^{-5} MeV	0.12 μm	388 pm	9.0 pm	0.91 pm
(ROOM TEMP)	$\frac{1}{4}10^{-7}$ MeV	49.6 μm	7.7 nm	181 pm	18.2 pm

WHERE $m = 10^{-3}$ $\mu = 10^{-6}$ $n = 10^{-9}$ $p = 10^{-12}$ $f = 10^{-15}$

NOTE: BOHR RADIUS OF ATOM ~ 53 pm
NUCLEAR RADIUS ~ FEW fm

FINALLY,

RATIO OF 10 MeV ^{13}C AND PHOTON ....

$$\frac{mv(^{13}C)}{p(\gamma)} = \frac{M_c V_c}{(E/c)} = \frac{\sqrt{2 M_c E}}{(E/c)} = \sqrt{2\frac{M_c c^2}{E}}$$

$$= \sqrt{2\,\frac{931.5 \times 13}{E(MeV)}} = 47.3 \gg 1$$

1.3 UNCERTAINTY PRINCIPLE IMPLIES $p \approx \hbar/x$
BUT $E = pc$ FOR RELATIVISTIC PARTICLES.
THUS
$$E = \frac{\hbar c}{x} = \frac{197.3 \text{ MeV}}{10} \approx 19.7 \text{ MeV}$$

1.4 (a) $Q = \alpha + {}^{9}Be - {}^{13}C$
$= (2603 + 12182 - 3355) \times 10^{-6}\ 931.5 \text{ MeV}$
$= 10.65 \text{ MeV}$

(b) MOMENTUM CONSERVATION IMPLIES

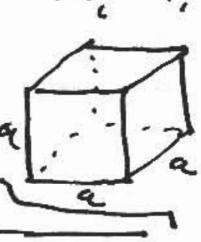
$$m_\alpha V_\alpha = m_c V_c \Rightarrow V_c = \frac{m_\alpha}{m_c} V_\alpha$$
$$E_c = \tfrac{1}{2} m_c \left(\frac{m_\alpha}{m_c}\right)^2 V_\alpha^2 = \frac{m_\alpha}{m_c} E_\alpha = 1.539 \text{ Me}$$

ENERGY CONSERVATION GIVES THE ENERGY OF THE PHOTON...

$$\text{PHOTON ENERGY} = Q + 5 \text{ MeV} - 1.539 \text{ MeV}$$
$$= 14.1 \text{ MeV}$$

1.5 (a) $$E = \left(\frac{h^2}{8ma^2}\right)\left(n_x^2 + n_y^2 + n_z^2\right) \quad n_i = 1, 2, 3, 4 \cdots$$

GROUND STATE: $n_x = n_y = n_z = 1$



$$\therefore \quad a = \sqrt{\frac{3h^2}{8mE}} = \pi(\hbar c)\sqrt{\frac{3}{2mc^2 E(\text{MeV})}}$$

$\hbar c = 197.3$ MeV fm $\quad mc^2 = 939.6$ MeV (NEUTRON

$a = 7.83$ fm (FOR $E = 10$)

(b) $E_0 \propto (1+1+1)$
$E_1 \propto (1+1+2^2) \Rightarrow \frac{E_1}{E_0} = \frac{6}{3} = 2$
3 STATES EACH $\begin{cases} E_2 \propto (1+2^2+2^2) \Rightarrow E_2/E_0 = 9/3 = 3 \\ E_3 \propto (1+1+3^2) \Rightarrow E_3/E_0 = 3.666 \end{cases}$

$E_4 \propto (2^2 + 2^2 + 2^2)$

$\frac{E_4}{E_0} = 4$

1.6 $^{234}U \rightarrow \alpha + {}^{230}Th$ (a)
$\rightarrow d + {}^{232}Pa$ (b)
$\rightarrow t + {}^{231}Pa$ (c)
$\rightarrow {}^{3}He + {}^{231}Th$ (d)

(a) $(40946 - 2603 - 33127)\,10^{-6}\,931.5 = 4.859$ MeV
(b) $(40946 - 14102 - 38582)\,10^{-6}\,931.5 = -10.93$ MeV
(c) $(40946 - 16049 - 35879)\,10^{-6}\,931.5 = -10.23$ MeV
(d) $(40946 - 16029 - 36297)\,10^{-6}\,931.5 = -10.60$ MeV

1.7 $^{212}Po \rightarrow {}^{208}Pb + \alpha(8.784)$

MOMENTUM CONSERVATION $\Rightarrow$ $E_{Pb} = \dfrac{m_\alpha}{m_{Pb}} E_\alpha = 0.169$

MASS EXCESS FOR ^{212}PO $\Rightarrow \left(2603 - 23364 + \dfrac{0.169 + 8.784}{10^{-6}\,931.5}\right)$

$= -11149.5$

1.8 $^{36}_{17}Cl \rightarrow {}^{36}_{16}S + e^+ + \nu$ (1.142)
$\rightarrow {}^{36}_{18}Ar + e^- + \bar{\nu}$ (0.709)

EXCESS:

$M_{Cl} = M_S + \dfrac{1.142}{931.5} = 35.9683$ (−31693)

$M_{Ar} = M_{Cl} - \dfrac{0.709}{931.5} = 35.9675$ (−32454)

1.9 Eq 1.17 $\Rightarrow$ $N_B(t) = \dfrac{\lambda_A}{\lambda_B - \lambda_A} N_A(0)\left[e^{-\lambda_A t} - e^{-\lambda_B t}\right]$

MAXIMUM N_B WHEN $dN_B/dt = 0$, OR

$\lambda_A e^{-\lambda_A t^*} - \lambda_B e^{-\lambda_B t^*} = 0$

$e^{(\lambda_B - \lambda_A)t^*} = \dfrac{\lambda_B}{\lambda_A}$

$(\lambda_B - \lambda_A)\, t^* = \ln(\lambda_B/\lambda_A)$

$t^* = \dfrac{0.693}{\lambda_A}$ (IF $\lambda_B = 2\lambda_A$)

PROBLEM 1.9 (CONTINUED)

THE MAXIMUM AMOUNT OF $N_B(t = t^*)$ IS

$$N_B(\text{max}) = N_A(0)\left[e^{-0.693} - e^{-2\times 0.693}\right]$$

$$= 0.25\ N_A(0)$$

1.10 RECALL: 1 Bq (BECQUEREL) = 1 DISINTEGRATION/SEC

$$1\ \mu Ci = 3.7\times 10^4\ Bq$$

Eq. 1.27 GIVES $\frac{dN}{dt} = P - \lambda N$

SOLUTION: $N(t) = K_1 e^{-\lambda t} + K_2$ (K_1, K_2 = CONSTANTS)

as $t \to \infty$ $N(\infty) = P/\lambda = K_2$

at $t \to 0$ $N(0) = 0 = K_1 + \frac{P}{\lambda} \Rightarrow K_1 = -\frac{P}{\lambda}$

$\therefore$ $N(t) = \frac{P}{\lambda}\left(1 - e^{-\lambda t}\right)$ QED

$$\lambda = \ln(2)/\tau_{1/2} = \ln(2)/(3\times 10^5 \times 3.1536\times 10^7)$$

$$= 7.326\times 10^{-14}\ \text{SEC}^{-1}$$

$100\mu Ci \Rightarrow 3.7\times 10^6 = P\left(1 - e^{-\lambda t}\right)$ (6)

($NiCl_2$)

$$P = \text{PRODUCTION} = 10^{14}\ 43\times 10^{-24}\left(\frac{2\ \ 0.758\ \ 1g}{129.6\ \ 1.66\times 10^{-24}g}\right)$$

$$= 3\times 10^{13}\ \text{EVENTS/SEC}$$

SOLVE FOR t^*:

$$e^{-\lambda t^*} = 1 - \frac{3.7\times 10^6}{3\times 10^{13}}$$

$$-\lambda t^* = -1.22\times 10^{-7}$$

$$t^* = 1.667\times 10^6\ \text{SEC}$$

$$= 0.05287\ \text{YEARS}$$

$$= 19.29\ \text{DAYS}$$

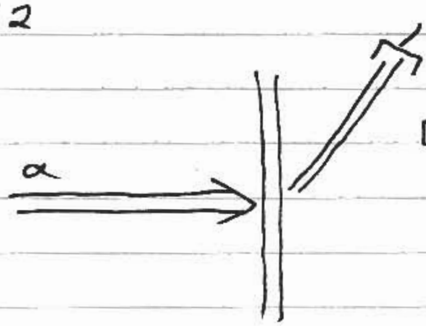
1.11

$$N(t) = 4.1344 \times 10^{26}\left(1 - e^{-\lambda t}\right) \quad ; \; \lambda = 7.3 \times 10^{-14}\ \text{sec}^{-1}$$

$$@\ t = 24 \times 60 \times 60 \Rightarrow 0.037\%$$

(ALSO $P = \phi \sigma N$. WITH $t \ll \lambda$, FRACTION $= \phi \sigma t$)

1.12



$$\text{DET RATE} = I\ (nt)\left(\frac{\sigma}{4\pi}\right) d\Omega$$

DET RATE: 15 PER SEC

$d\Omega$: 2×10^{-3}

$$nt:\ \frac{10^{-3}}{48\ 1.66 \times 10^{-27}} = 1.25 \times 10^{22}\ \text{PARTICLES PER cm}^2$$

$$I:\ \frac{10^{-8}}{2 \times 1.6 \times 10^{-19}} = 3 \times 10^{10}\ \text{PER SEC}$$

$$\therefore\ \sigma = \frac{15}{3 \times 10^{10}\ \ 1.25 \times 10^{27}\ \ \frac{2 \times 10^{-3}}{4\pi}} = 2.4 \times 10^{-28}\ \text{m}^2$$

$$= 0.00024\ \text{barn}\ \ (0.24\ \text{mb})$$

PROBLEM #2

$$\lambda = \frac{1240\ \text{fm}}{E[\text{MeV}]}$$

E	10 eV	100 keV	1 MeV
λ	0.124 nm	0.0124 nm	1240 fm

PROBLEM #3

ISOTOPES: ^{3}He, ^{4}He AND ^{12}C, ^{14}C

ISOBARS: ^{3}H, ^{3}He AND ^{12}C, ^{12}N

ISOTONES: ^{3}H, ^{4}He AND $^{99}_{43}Tc$, $^{100}_{44}Ru$

ISOMERS: ^{99}Tc, ^{99m}Tc

PROBLEM #4

TO SOLVE THIS PROBLEM, FIRST COMPUTE THE ATOMIC NUMBER DENSITY $\equiv \rho / m_A$

	m_A	$\lambda_{MFP} = \frac{1}{n\sigma}$
LH	0.0695	23.9 cm
AIR	0.000083	19,923
AL	0.090	18.4
Cu	0.141	11.8
Pb	0.0547	30.3 cm
	NUCLEI/cm^3	

PROBLEM #5

RUTHERFORD SCATTERING DIFF. CROSS SECTION IS

$$\frac{d\sigma}{d\Omega} = \frac{0.001295\,(Z_1 Z_2)^2}{E^2(\text{MeV})\,\sin\left(\frac{\theta}{2}\right)^4}\ \text{BARN}$$

$$\text{DET RATE} = I\ (nt)\ \frac{d\sigma}{d\Omega}\Big|_{45^\circ}\ d\Omega = 296\ \frac{\text{PROTONS}}{\text{SEC}}$$

$I = 10^8$

$nt = \frac{0.05}{107.9\ 1.66\times10^{-24}}$

$\frac{d\sigma}{d\Omega}\Big|_{45^\circ} = 133.4\text{ b}$, $Z_1 = 1$, $Z_2 = 47$

$d\Omega = \frac{10\text{ cm}^2}{4\pi(100\text{ cm})^2} = 7.9\times10^{-5}$