NOV 23, 2004

APAM HOMEWORK #7

PROBLEM #3.1

$p_0 \leftarrow \!\!-\!\!- O \rightsquigarrow \gamma$

CONSERVATION OF MOMENTUM

$$p_0 = E_\gamma / c$$
$$E_0 = p_0^2 / 2m_0 = E_\gamma^2 / 2m_0c^2$$

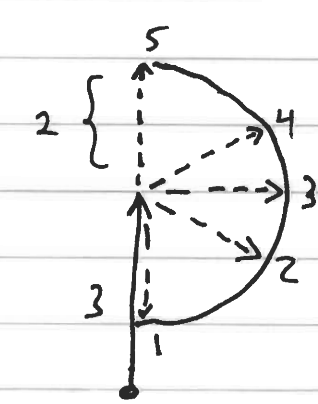
$$\text{TOTAL ENERGY OF EXCITED STATE} = E_\gamma + \frac{E_\gamma^2}{2m_0c^2} = E_\gamma\left(1 + \frac{E_\gamma}{2m_0c^2}\right)$$

$$m_0c^2 = 16u \times 931.5\,MeV = 14904\,MeV$$
$$E_\gamma / 2m_0c^2 = 2\times10^{-4} \quad \therefore \quad E^* = 6.129\,MeV$$

PROBLEM #3.2

WHEN ANGULAR MOMENTUM IS ADDED TOGETHER, THE RESULT DEPENDS UPON THE ORIENTATION OF THE ANGULAR MOMENTUM VECTORS. IN QUANTUM MECHANICS, THESE POSSIBLE COMBINATIONS ARE QUANTIZED.

FOR EXAMPLE $I_1 = 3$ AND $I_2 = 2$ COMBINE IN SEVERAL POSSIBLE WAYS



$I_1 = 3$ AND $I_2 = 2$ COMBINE TO PRODUCE A NEW ANGULAR MOMENTUM STATE WITH

$$|I_1 - I_2| \le I_{NEW} \le |I_1 + I_2|$$
$$1 \le I_{NEW} \le 5$$

#3.2)

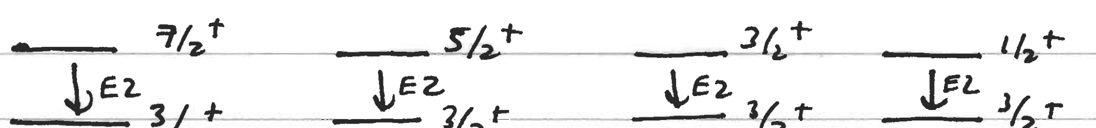
	COUPLES TO	POSSIBLE TRANSITIONS
i) $3^- \rightarrow 2^+$	$1 \leq L \leq 5$	E1, M2, E3, M4, E5
ii) $5/2^+ \rightarrow 9/2^+$	$2 \leq L \leq 7$	E2, M3, E4, M5, E6, M7
iii) $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$	$L = 1$ (0 NOT ALLOWED)	E1
iv) $\frac{3}{2}^+ \rightarrow \frac{7}{2}^+$	$2 \leq L \leq 5$	E2, M3, E4, M5

THE LOWEST-ORDER MULTIPOLE IS ALWAYS MOST LIKELY. (NOTE: THE EMITTED γ-RAYS HAVE SAME ENERGY NO MATTER THE MULTIPOLARITY.)

PROBLEM #3.3 FOR A QUADRUPOLE TRANSITION $(L=2)$, THE INITIAL AND FINAL STATES MUST SATISFY:

$$|I_1 - I_2| \leq 2 \leq |I_1 + I_2|$$

THE POSSIBLE STATES ARE:

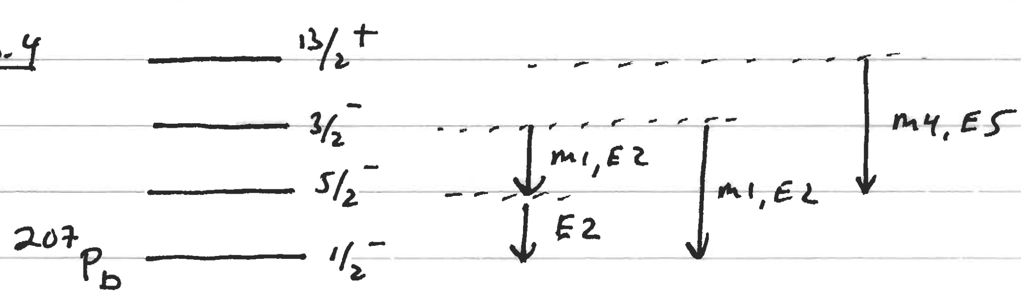


THESE ALLOW OTHER TRANSITIONS

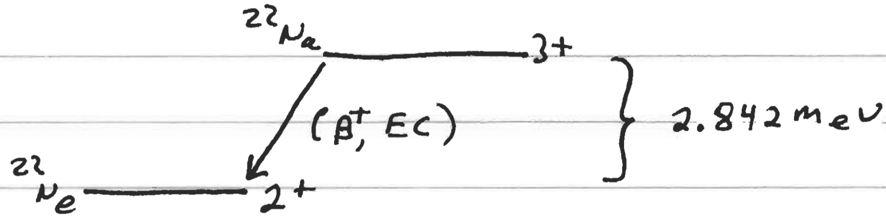
M3	M1	M1	M1
E4	M3	M3	
M5	E4		

IF THERE IS NO M1 RADIATION DETECTED, THEN THE INITIAL STATE CANNOT BE $5/2^+$, $3/2^+$, OR $1/2^+$. IT MUST BE $7/2^+$.

PROBLEM #3.4



PROBLEM #3.5



a) $^{22}Na \rightarrow {}^{22}Ne + e^+ + \nu$

$^{22}Na + e^- \rightarrow {}^{22}Ne + \nu$

b) $Q_{\beta^+} = M_A(^{22}Na) - M_A(^{22}Ne) - 2m_e$

$= 1.82 \text{ meV}$

c) $Q_{EC} = M_A(^{22}Na) - M_A(^{22}Ne) = Q_{\beta^+} + 2m_e$

$= 2.842 \text{ meV}$

WHERE WE NOTE $M_A(^{22}Na) = M_N(^{22}Na) + 11\, m_e$

$M_A(^{22}Ne) = M_N(^{22}Ne) + 10\, m_e$

(THE DIFFERENCE BETWEEN NUCLEAR MASS AND ATOMIC MASS)

PROBLEM #3.6

$n \rightarrow p + e^- + \nu$

$Q = m_n - m_p - m_e = 0.782 \text{ meV}$

THIS IS A SPECIAL "SUPERALLOWED" TRANSITION.

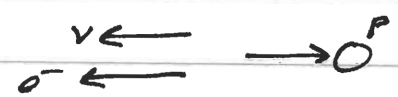
$\log_{10} f t_{1/2} \doteq 3.5 = \log_{10} f + \log_{10} t_{1/2}$

BUT $\log_{10} f \sim 0.2$ (Fig. 3.8). SO $\log_{10} t_{1/2} \doteq 3.3$

$t_{1/2} \sim 10^{3.3} \sim 33$ MINUTES (ABOUT 2× TOO SLOW)

PROBLEM #3.7

MAXIMUM (RECOIL) ENERGY OF PROTON WHEN ELECTRON (AND NEUTRINO) ARE OPPOSITELY DIRECTED.



CONSERVATION OF MOMENTUM $\quad P_p = P_e$

ENERGY (MAX) OF PROTON $\quad E_p = P_p^2/2m_p = P_e^2/2m_p$

SINCE REST MASS OF ELECTRON IS LESS THAN 0.782 MeV, THE ELECTRON IS RELATIVISTIC.

$$E_{TOT}^2 = P_e^2 + m_e^2 \qquad E_{TOT} = E_{KIN} + m_e$$

SO

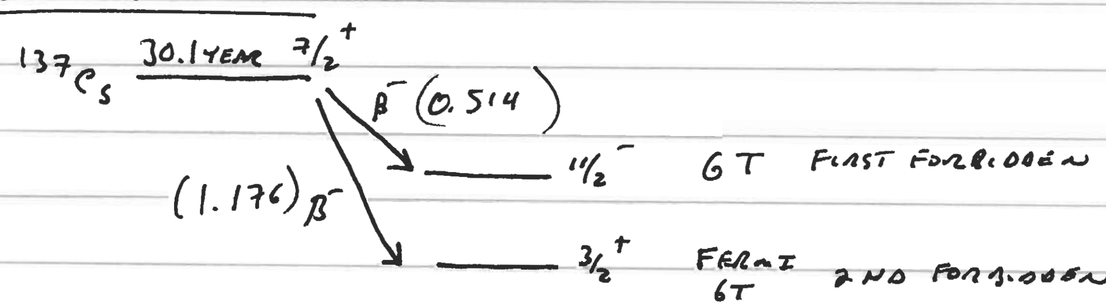
$$P_e^2 = (E_{KIN} + m_e)^2 - m_e^2 = E_{KIN}^2 + 2E_{KIN}m_e$$

ASSUME $E_{KIN} \approx 0.782\,MeV$ (SINCE ELECTRONS ARE SO LIGHT)

THEN

$$E_p = \frac{E_{KIN}^2 + 2E_{KIN}m_e}{2m_p} = 7.5\times10^{-4}\ MeV$$

PROBLEM #3.8



FROM FIG 3.8 $\quad$ 0.51 MeV $\Rightarrow \log_{10} f = 0.5$

1.176 MeV $\Rightarrow \log_{10} f \sim 1.8$

PROBLEM 3.8 (CONTINUED)

$$\log_{10} t_{1/2} = \underbrace{\log_{10} f t_{1/2}}_{} - \log_{10} f$$

USUALLY ~ 7.5 1st ~ 12 2nd

$$\frac{dN_{cs}}{dt} = -\lambda_1 N_{cs} - \lambda_2 N_{cs} = -\lambda_{TOT} N_{cs} \qquad (\lambda_1 + \lambda_2 = \lambda_{TOT})$$

BUT $\frac{\lambda_1}{\lambda_{TOT}} \simeq 94.4\%$ $\qquad \frac{\lambda_2}{\lambda_{TOT}} = 0.056$ $\qquad$ WITH $t_{1/2} = \frac{\ln(2)}{\lambda}$

$$\lambda_{TOT} = \frac{\ln(2)}{30.1\ \text{YEAR}} \sim \frac{\ln(2)}{9.5 \times 10^8} \sim 7.32 \times 10^{-10}\ \text{sec}^{-1}$$

$\lambda_1 = 6.9 \times 10^{-10}$ $\qquad \lambda_2 = 4.1 \times 10^{-11}$

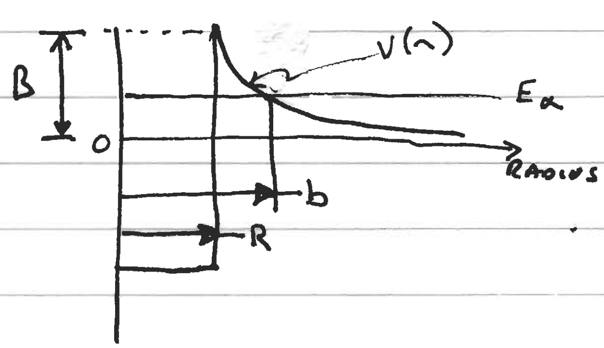
$t_{1/2} \sim 10^9$ $\qquad t_{1/2} \sim 1.7 \times 10^{10}$

$$\log_{10} f t_{1/2} = \log_{10} t_{1/2} + \log_{10} f$$

$= 9 + 0.5$ $(0.514\ \text{MeV}) \sim 9.5$ $\qquad$ CLOSE to TABLE 3.3

$= 10.23 + 1.8$ $(1.176\ \text{MeV}) \sim 12.03$

PROBLEM #3.9



For $r > R$, $V(r) = \frac{2 Z e^2}{4\pi \epsilon_0 r}$

$V(b) = 4.268 = \frac{2 Z\, 1.44}{b}$ $\qquad (Z = 90)$ Th

$B = V(R) = \frac{2 Z\, 1.44}{1.4 A^{1/3}}$ $\qquad (A = 235)$

$b = \frac{2 Z\, 1.44}{4.268} = 60.7\ \text{fm}$ $\qquad B = 30\ \text{MeV}$

PROBLEM # 7.10

$$^{209}Bi \rightarrow \alpha + {}^{205}Tl + 3.13\,MeV$$

given with $\tau_{1/2} = 2.1$ min for similar decay...

$$^{211}Bi \rightarrow \alpha + {}^{207}Tl + 6.749\,MeV$$

$$\frac{\tau^{1/2}(^{209}Bi)}{\tau^{1/2}(^{211}Bi)} \approx e^{-2[G_{211} - G_{209}]} \qquad (Z = 81)$$

$$G(Z, B, Q) \approx \frac{2}{137} Z \sqrt{\frac{2 \cdot 3727}{Q}} \left[\cos^{-1}\sqrt{\frac{Q}{B}} - \sqrt{\left(\frac{Q}{B}\right)\left(1 - \frac{Q}{B}\right)} \right]$$

$$B^{211} = \frac{2\, Z\, 1.44}{1.2(A_1^{1/3} + 4^{1/3})} = 25.91\,MeV \qquad A_1 = 207$$

$$B^{209} = \frac{2\, Z\, 1.44}{1.2(A_1^{1/3} + 4^{1/3})} = 25.976\,MeV \qquad A_1 = 205$$

$$G^{211} = 23.4 \qquad G^{209} = 51.4 \qquad \text{Therefore}$$

$$\frac{\tau^{1/2}(^{209}Bi)}{\tau^{1/2}(^{211}Bi)} \approx e^{-2[23.4 - 51.4]} \approx e^{+56} \approx 2 \times 10^{24}$$

$$\tau^{1/2}(^{209}Bi) \sim 2 \times 10^{24}\ 126\ \text{sec} = 2.5 \times 10^{26}\ \text{sec} = 8 \times 10^{18}\ \text{year}$$

Problem #3.11

$$^{238}U \rightarrow \alpha + {}^{234}Th \quad (\simeq \text{STABLE}) \qquad Z(Th) = 90$$

$$^{238}Pu \rightarrow \alpha + {}^{234}U \quad (88 \text{ YEARS}) \qquad Z(U) = 92$$

$$Q = 5\ \text{MeV} \qquad B = \frac{2Z\,1.44}{1.2(A_1^{1/3} + 4^{1/3})} = \begin{cases} 27.9\ \text{MeV} & {}^{238}U \\ 28.5\ \text{MeV} & {}^{238}Pu \end{cases}$$

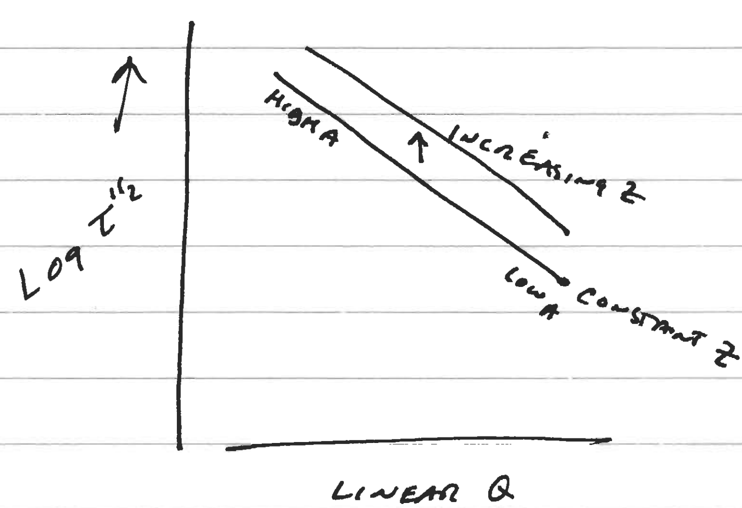
$$G(^{238}U) = 38$$

$$G(^{234}Pu) = 39.3 \qquad \text{AT } Q = 5\ \text{MeV}$$

$$\frac{\tau_{1/2}(^{238}U)}{\tau^{1/2}(^{238}Pu)} \sim e^{-2[G_{Pu} - G_U]} \sim e^{-2(39.3 - 38)}$$

$$\sim 0.07$$

THEREFORE, AS Z INCREASES SO DOES $\tau_{1/2}$ FOR A GIVEN α DECAY ENERGY.



See Figure 3.9