AP4010 Introduction to Nuclear Science

NOTE: There will be no class on Tuesday, November 2 because of Election Day. Our next class will be November 9.
QUIZ 2: Our second open-book, open-notes quiz will be in-class on November 9. The primary subject of the quiz is nuclear structure and introductory quantum mechanics.

Question 1
Using Fig. 2.9 from our textbook (p. 48, Lilley), justify the nuclear spin and parity (+ for even, and − for odd) for the following nuclei:

\[
\begin{align*}
\frac{3}{2}\text{He}\left(\frac{1}{2}^+\right) & \quad \frac{4}{2}\text{He}(0+) \quad \frac{27}{13}\text{Al}(\frac{5}{2}+) \\
\frac{28}{14}\text{Si}(0+) & \quad \frac{38}{18}\text{Ar}(0+) \quad \frac{41}{19}\text{K}(\frac{3}{2}+) \\
\frac{63}{29}\text{Cu}(\frac{3}{2}^-) & \quad \frac{65}{29}\text{Cu}(\frac{3}{2}^-) \quad \frac{64}{30}\text{Zn}(0+) 
\end{align*}
\]

Question 2
What is the minimum photon energy required to dissociate \( ^2\text{H} \) (deuterium)?

\[
\gamma + ^2\text{H} \rightarrow n + p
\]
Assume the binding energy to be 2.224589 MeV (and don’t forget to conserve total energy and momentum).
Question 3

Fig. 1.5 (p. 13) from the textbook is attached showing a curve of lowest mass isobars, sometimes called the Segré chart, (i.e. the most stable combination of neutrons, \( N \), and protons, \( Z \), for a given atomic number, \( A \equiv Z + N \).)

For each combination of \( N \) and \( Z \), the nucleus mass is approximately given by a so-called “semi-empirical mass formula”, or SEMF. The SEMF is

\[
M(Z, N) = Zm_H + Nm_n - B(Z, N)/c^2
\]

\[
B(Z, N) = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} \pm \Delta
\]

where \( a_v = 15.56 \text{ MeV} \), \( a_s = 17.23 \text{ MeV} \), \( a_c = 0.7 \text{ MeV} \), and \( a_a = 23.28 \text{ MeV} \) and \( \Delta \sim 12/A^{1/2} \text{ MeV} \). Also, \( m_H c^2 = 938.8 \text{ MeV} \) and \( m_n c^2 = 939.6 \text{ MeV} \).

Part a

For each value of \( A \), there is a minimum \( M \) corresponding to the most stable isobar. If the ratio of \( N \) to \( Z \) is not equal to this minimum, a radioactive decay can occur. What type of radioactive decay reduces \( M \) without changing \( A \)?

Part b

Use the semi-empirical mass formula (below) to derive an analytical expression for the most stable ratio of \( N/Z \) as a function of \( A \). This formula is: \( N/Z = 0.98 + 0.015A^{2/3} \).

[Hint: To find the minimum mass as \( Z \) is changed keeping \( A \) constant, you need first express the SEMF in terms of \( Z \) and \( A \) only. Also, explain why or why not \( \Delta \) can be ignored in your estimated formula. After finding the lowest energy for each \( A \), substitute \( Z \to A/(1 + N/Z) \) and solve for the quantity \((N/Z)\).]
Question 4

Heavy nuclei (i.e. large $A \sim 150$) become unstable to alpha decay even though they may be stable to beta decay.

Estimate, for large values of $A$, the maximum nucleus size that remains stable to alpha decay processes.

Hint: To form your estimate, you should estimate the change of mass (or the change of energy) upon release of an $\alpha$ particle:

$$\Delta_\alpha M = M(Z, N) - M(Z - 2, N - 2) - m_\alpha$$

$$= 2(m_H + m_n) - m_\alpha - \frac{1}{c^2}[B(Z, N) - B(Z - 2, N - 2)]$$

$$\approx 2(m_H + m_n) - m_\alpha - 4 \frac{dB(A/2.4, 0.58A)}{dA}$$

In the above formula, $N/Z \approx 1.4$ was assumed. This approximation (see from Question 3) means that $Z \approx A/(1 + 1.4)$ and $N \approx 1.4A/(1 + 1.4)$. The expression for $B(Z = A/2.4, N = 0.58A)$ is

$$B(Z = A/2.4, N = 0.58A) \approx a_V A - a_s A^{2/3} - 0.17a_c A^{5/3} - a_0 0.027A$$

The mass difference that appears above is $2(m_H + m_n) - m_\alpha = 28.8 \text{ MeV}/c^2$.

When $\Delta_\alpha M > 0$, then the nucleus is unstable to alpha decay. When $\Delta_\alpha M < 0$, then the nucleus is stable.

[Please note: you need only estimate the maximum $\alpha$-stable value of $A$.]
Figure 1: The vector $\vec{l}$ precesses rapidly around the $z$ axis, so that $l_z$ stays constant, but $l_x$ and $l_y$ are variable.

**Question 5**

Fig. 1 (above) illustrates a model of the convention in quantum mechanics. The $z$ component of the angular momentum is quantized with the $m_l$ quantum number. $m_l$ can take the integer values $m_l = 0, \pm 1, \pm 2, \pm 3, \ldots, \pm l$.

What are the possible angles between the angular momentum vector, $\vec{l}$, and the $z$-axis for $l = 1$ and $l = 2$?

**Question 6**

For a two-dimensional box (with infinite-potential boundaries), how do the energy levels and level degeneracies change when the box dimensions change from $a$ by $a$ (i.e. square) to $a$ by $2a$ (i.e. rectangular)?