Problem 5.1

Use conservation of momentum. This will be easy in the frame of reference of the $\alpha$-particle instead of the $e^-$ (Lag frame).

Since the $\alpha$-particle is non-relativistic, this transformation is easy:

Lag frame | $\alpha$ rest frame
---|---

\[ \alpha \rightarrow v^+_x \quad e^- \rightarrow e^- \]

In the $\alpha$ rest frame, it's easy to see that the maximum energy will be a "head-on" collision. The electron will recoil with a reversal of its motion.

$\alpha$-rest frame (after head-on collision)

\[ \alpha \rightarrow v^+ \quad e^- \rightarrow e^- \]

Back in the Lag frame, this looks like

Lag frame (after head-on collision)

\[ \alpha \rightarrow v^+_x \quad e^- \rightarrow 2v_x \]
Problem 5.1 (continued)

So the electron energy is

\[ E_{\text{max}} = \frac{1}{2} m_e (2V_x)^2 = 4 \left( \frac{m_e}{m} \right) \frac{1}{2} m_e V_x^2 \]

\[ = 4 \left( \frac{m_e}{m} \right) E_x \sim 0.27 E_0 \]

Problem 5.2 This is the same problem discussed in class. In this solution, I use the same notation as in class.

\[ \Delta P_x = D \]

\[ \Delta P_y = \int_{-\infty}^{\infty} \Delta F \]

\[ F_x = e \frac{E_y}{c} \]

\[ F_y = e \frac{E_x}{c} \]

If \( b \) is "large", then with \( E_x = E_0 \) electric field \( \vec{E} \) is parallel. \( V \) is constant. We can write

\[ \Delta P_y \approx \frac{e^2}{c} \int_{-\infty}^{\infty} \Delta x E_x \]

But the integral form of Gauss's Law is

\[ 2\pi b \int_{-\infty}^{\infty} \Delta x E_x = \frac{e}{c} \]

Thus

\[ \Delta P_y = \frac{e^2}{2\pi b E_0 c} = \frac{2A}{V b} \left( \text{with } A = \frac{e^2}{4\pi \varepsilon_0} \right) \]

The energy lost to the electron is

\[ \left( \frac{\Delta P_y}{2m_e} \right)^2 = \frac{1}{2} \frac{4}{m_e} \frac{A^2}{V b^2} \frac{m}{E_b} \]

\[ = \left( \frac{e^2}{4\pi \varepsilon_0} \right)^2 \frac{M}{E_b} \]
Problem 5.3

Just keeping track of the additional mass and charge...

\[
\text{Energy lost to } \text{He} = \left( \frac{m_p}{M} \right) \left( \frac{12}{1} \right)^2 = \frac{36}{12 \times 1823} \approx 0.16\%\text{.}
\]

Problem 5.4

Given: (5 MeV proton in Si) \(-\frac{dE}{dx} = 5.9 \text{ keV cm}^2 \text{ mg}^{-1}\)

But \(\frac{dE}{dx} \propto Z^2\) and \(R \propto M^{1/3}\) at same velocity!

"Relative" Parameters

<table>
<thead>
<tr>
<th></th>
<th>E</th>
<th>P</th>
<th>Z</th>
<th>M</th>
<th>\frac{dE}{dx}</th>
<th>R</th>
</tr>
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<tr>
<td>p</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td></td>
</tr>
<tr>
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<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td></td>
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<tr>
<td>T</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>(^3\text{He})</td>
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<td>3</td>
<td>4</td>
<td>0.75</td>
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<tr>
<td>(^4\text{He})</td>
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<td>2</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Thus \(^3\text{He}\) \(^4\text{He}\) lose energy 4 times faster along path (at same velocity) and tritons penetrate 2 times further into Si.
Problem 5.5

Part A)
\[-\frac{dE}{dx} = -\frac{K}{E} \quad (K = \text{constant})\]

\[R = \int_0^E \frac{dE}{(E/E_0)} = \int_0^E \frac{dE}{K} E^m = \frac{E^{m+1}}{m+1} \]

\[\Rightarrow (-\frac{dE}{dx}) R = \frac{E}{1+m} = 59 \quad \text{keV/m}^2 \times 5000 \quad \text{keV} \]

or

\[m = \frac{5000 \text{ keV}}{59 \times 50 \text{ keV}} \]

\[m = 0.674 \]

Part B)

\[R(E) = \int_E^0 \frac{dE}{(E/E_0)} \]

\[R(E/2) = \int_{E/2}^0 \frac{dE}{(E/E_0)} \]

\[\therefore R(E) - R(E/2) = \text{distance traveled to } E \text{ energy} \]

\[= R(E) \left(1 - \frac{R(E/2)}{R(E)}\right) \]

\[= R(E) \left(1 - \frac{(E/2)^{m+1}/E^{m+1}}{E^{m+1}}\right) \]

\[= R(E) \left(1 - \left(\frac{1}{2}\right)^{m+1}\right) = 0.69 R(E) \]

Problem 5.6

In this problem, all particles are non-relativistic, thus cork.

Bloch's formula is

\[-\frac{dE}{dx} = 2\pi \frac{(m^2) A^2}{E} \left(\frac{M}{m_0}\right)^2 \ln \left(\frac{4E (m_0/m)}{T}\right)\]

with \[A = 2, 4.4 \text{ MeV } \text{fme, } T = 11.2 \text{ (or so...)}\]
Problem 5:6 (continued)

Note that \((m^2)\) is density of electrons in material.

\[ E/m \propto u^2 \propto \text{velocity}^2 \text{ of particle} \]

\[ A^2 \propto \Delta \text{ charge}^2 \text{ of particle} \]

In this problem, we consider different velocities and different electron densities.

The relative stopping powers are proportional to

\[-p \frac{dE}{dx} \propto \frac{Z^2}{(E/m)} \times \frac{q,E,\left(\frac{e^2}{m}\right)}{112} \text{ in Si} \]

<table>
<thead>
<tr>
<th>Relative Parameters</th>
<th>(E/m = u^2)</th>
<th>(Z)</th>
<th>(\frac{dE}{dx})</th>
<th>(-p \frac{dE}{dx} \times \frac{q,E,\left(\frac{e^2}{m}\right)}{112})</th>
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<tr>
<td>(\alpha)</td>
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<td>5.52</td>
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<tr>
<td>(P)</td>
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<td>1</td>
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<td>(d)</td>
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<td>1</td>
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<tr>
<td>(H)</td>
<td>(\frac{1}{3})</td>
<td>2</td>
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</tbody>
</table>

Problem 2

\[ \theta = \tan^{-1} \left( \frac{\Delta P_y}{P_0} \right) = \frac{\Delta P_y}{P_0} \]

\[ \theta \propto \frac{2A}{Bv \mu v} \sim \frac{A}{b} = \frac{b^*}{b} \]
Problem 3: There are two ways to solve this problem. First, like Problem 5.1, it is useful to examine the x rest frame.

X-REST-FRAME Here, the largest deflection will occur when the motion is scattered by 90°.

Before Collision

\[ \Psi \rightarrow \text{Motion} \rightarrow \Psi' \]

\[ V_k \rightarrow \Delta P \rightarrow \Psi' \]

MOMENTUM

\[ \begin{align*}
M_x V_x &= \frac{m_0}{m} V_0 \\
M_y V_y &= \Delta P \\
M_z V_z &= \Delta P
\end{align*} \]

\[ V_k = \frac{m_0}{m} V_0 \]

Solving for \( \Delta P \) in terms of \( V_0 \)

\[ m_0 V_0 \left( 1 - \frac{m_0}{m} \right) = \Delta P \frac{M_x + M_y}{m_{y} m_{z}} \]

\[ \Delta P = \rho \frac{m_0}{m_{x}} \sqrt{m_{z} m_{y}} \]

\[ \rho = \frac{\Delta P}{\frac{m_0}{m_{x}}} \]

Now, back into the Lab frame...

\[ \Delta P = \rho \frac{m_0}{m_{z}} \left( 1 - \frac{m_0}{m} \right) \]

\[ \tan \theta = \frac{\rho}{\frac{m_0}{m_{z}}} \]

\[ \rho = \sqrt{\frac{m_0^2}{m^2} - m_0^2} \]

Another (equivalent) way to see the answer is to look at the differential cross-section. Notice that when

\[ \left( \frac{m_0}{m_{z}} \right) \sin \theta > 1, \frac{\rho}{\frac{m_0}{m_{z}}} \text{ is not defined!} \]

Thus, when \( \sin \theta \leq \frac{m_0}{m_{z}} \), we have maximum scattering

\[ \theta_{\max} = \sin^{-1} \left( \frac{m_0}{m_{z}} \right) = 14.5^\circ \]

As above.
Problem #4

Part A

* From Eqs. 5.3 + 5.4, the range scales with energy as \( R \propto E^{0.5} \) for both protons and 
  neutrons. This is true for Pb and Cu.

* For particles with the same energy per nucleon, i.e., the same \( A/Z \), the range scales like \( \frac{A}{Z^2} \).

\[
R(\text{Xe/Ne}) \propto \frac{4}{2^2} R(\text{Pb/Ne}) \propto \frac{A}{Z^2} \propto \frac{1}{E^{0.5}}
\]

Thus, \( R(A) \propto R(\text{Pb/Ne}) \times E^{0.5} \).

* Finally, the empirical GRABS-KLEMMW formula

  says that at the same energy and particle type, the range scales like \( R(\text{Cu}) = \frac{R(A)}{R(Pb)} \times 0.55 \times R(Pb) \)

  and \( R(Cu) \) is smaller like ratio (not \( \frac{Pb}{Cu} \) and not exactly).

Part B

\[
R(D, 20\text{MeV}) = R(P, 15\text{MeV}) \times \frac{0.44}{2^2} = R(P, 15\text{MeV}) \times 0.68
\]

\[
R(D, 20\text{MeV}) = R(P, 15\text{MeV}) \times 0.68
\]

\[
Pb \quad 0.68 \quad 1.36
\]