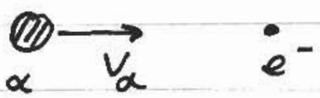
AP 4010 INTRO NUC SCIENCE
HOMEWORK #2 SOLUTIONS

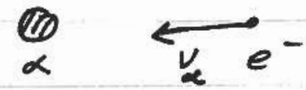
PROBLEM 5.1 USE CONSERVATION OF MOMENTUM. THIS WILL BE EASY IN THE FRAME OF REFERENCE OF THE α-PARTICLE INSTEAD OF THE e^- (LAB FRAME).

SINCE THE α-PARTICLE IS NON-RELATIVISTIC, THIS TRANSFORMATION IS EASY:

LAB FRAME

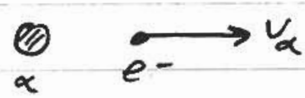


α REST FRAME



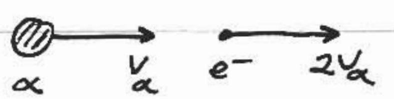
IN THE α REST FRAME, IT'S EASY TO SEE THAT THE MAXIMUM ENERGY WILL BE A "HEAD-ON" COLLISION: THE ELECTRON WILL RECOIL WITH A REVERSAL OF IT'S MOTION

α - REST FRAME
(AFTER HEAD-ON COLLISION)



BACK IN THE LAB FRAME, THIS LOOKS LIKE

LAB FRAME
(AFTER HEAD-ON COLLISION)



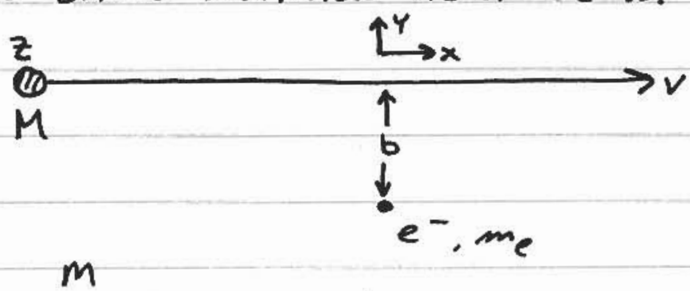
PROBLEM 5.1 (CONTINUED)

SO THE ELECTRON ENERGY IS

$$E_{max} = \frac{1}{2} m_e (2v_\alpha)^2 = 4\left(\frac{m_e}{M_\alpha}\right) \frac{1}{2} M_\alpha v_\alpha^2$$

$$= 4\left(\frac{m_e}{M_\alpha}\right) E_\alpha \sim 0.2\% \, E_\alpha$$

PROBLEM 5.2 THIS IS THE SAME PROBLEM DISCUSSED IN CLASS. IN THE SOLUTION, I USE THE SAME NOTATION AS IN CLASS.



$$\Delta P_x \simeq 0$$

$$\Delta P_y = \int_{-\infty}^{\infty} dt \, F_\perp$$

$$F_\perp = eZ E_\perp$$

WITH $E_\perp = E_y$ = PERPENDIC. ELECTRIC FIELD

IF b IS "LARGE", THEN $V \simeq$ CONSTANT. WE CAN WRITE

$$\Delta P_y \simeq \frac{eZ}{V} \int_{-\infty}^{\infty} dx \, E_\perp$$

BUT THE INTEGRAL FORM OF GAUSS'S LAW IS

$$2\pi b \int_{-\infty}^{\infty} dx \, E_\perp = e/\epsilon_0$$

THUS

$$\Delta P_y = \frac{e^2 Z}{2\pi b \epsilon_0 V} = \frac{2A}{Vb} \quad \left(\text{WITH } A \equiv \frac{e^2 Z}{4\pi\epsilon_0}\right)$$

THE ENERGY LOST TO THE ELECTRON IS

$$\frac{(\Delta P_y)^2}{2m_e} = \frac{1}{2m_e} \frac{4A^2}{V^2 b^2} = \frac{A^2}{Eb^2}\left(\frac{M}{m_e}\right)$$

$$= \left(\frac{e^2 Z}{4\pi\epsilon_0}\right)^2 \frac{M/m_e}{Eb^2}$$

PROBLEM 5.3

JUST KEEPING TRACK OF THE ADDITIONAL MASS AND CHARGE...

$$\frac{\text{ENERGY LOST TO } ^{12}C}{\text{ENERGY LOST TO } m_e} = \underbrace{\left(\frac{m_e}{M_c}\right)}_{\text{MASS RATIO}} \underbrace{\left(\frac{12}{1}\right)^2}_{Z^2} = \frac{36}{12 \times 1823} << 1$$

$\sim 0.16\%$

PROBLEM 5.4

GIVEN: (5 MeV PROTON IN Si) $-\frac{1}{\rho}\frac{dE}{dx} = 59 \frac{keV\,cm^2}{mg}$

$$\rho R = 50 \frac{mg}{cm^2}$$

BUT $\frac{dE}{dx} \propto Z^2$ AND $R \propto M/Z^2$ AT SAME VELOCITY!

"RELATIVE" PARAMETERS

	E	Z	M	$\frac{dE}{dx}$	R
p	1	1	1	1	1
d	2	1	2	1	2
T	3	1	3	1	3
^3He	3	2	3	4	0.75
^4He	4	2	4	4	1

THUS ^3He ^4He LOOSE ENERGY 4 TIMES FASTER ALONG PATH (AT SAME VELOCITY) AND TRITONS PENETRATE 3 TIMES FARTHER INTO Si.

PROBLEM 5.5

PART A) $-\frac{dE}{dx} \cong -\frac{K}{E^m}$ (K ~ CONSTANT)

$$R = \int_E^0 \frac{dE}{(dE/dx)} = \int_0^E \frac{dE}{K} E^m = \frac{1}{K} \frac{E^{m+1}}{(1+m)}$$

$$\therefore \quad \left(-\frac{dE}{dx}\right) \times R = \frac{E}{1+m} = 59 \ \frac{\text{keV cm}^2}{\text{mg}} \times 50 \ \frac{\text{mg}}{\text{cm}^2}$$

OR

$$(1+m) = \frac{5000 \text{ keV}}{59 \times 50 \text{ keV}}$$

$$m = 0.694$$

PART B)

$$R(E) = \int_E^0 \frac{dE}{(dE/dx)}$$

$$R(E/2) = \int_{E/2}^0 \frac{dE}{(dE/dx)}$$

$$\therefore \quad R(E) - R(E/2) = \text{DISTANCE TRAVELED TO } \tfrac{1}{2} \text{ ENERGY}$$

$$= R(E)\left(1 - R(E/2)/R(E)\right)$$

$$\cong R(E)\left(1 - (E/2)^{m+1}/E^{m+1}\right)$$

$$\approx R(E)\left(1 - \left(\tfrac{1}{2}\right)^{m+1}\right) = 0.69\, R(E)$$

PROBLEM 5.6 IN THIS PROBLEM, ALL PARTICLES ARE NON-RELATIVISTIC. THUS $\beta \ll 1$

BLOCH'S FORMULA IS

$$-\frac{dE}{dx} \approx 2\pi \frac{(mz^2) A^2}{E} \left(\frac{M}{m_e}\right) \ln\left(\frac{4E\left(\frac{m_e}{M}\right)}{I}\right)$$

WITH $A \equiv Z\, 1.44 \text{ MeV fm}$ $\quad I \sim 11 Z$ (OR SO...)

PROBLEM 5.6 (CONTINUED)

NOTICE THAT $(nZ') \sim$ DENSITY OF ELECTRONS IN MATERIAL

$E/m \sim v^2 \sim$ VELOCITY^2 OF PARTICLE

$A^2 \sim z^2 \sim$ CHARGE^2 OF PARTICLE

IN THIS PROBLEM, WE CONSIDER DIFFERENT VELOCITIES AND DIFFERENT ELECTRON DENSITIES.

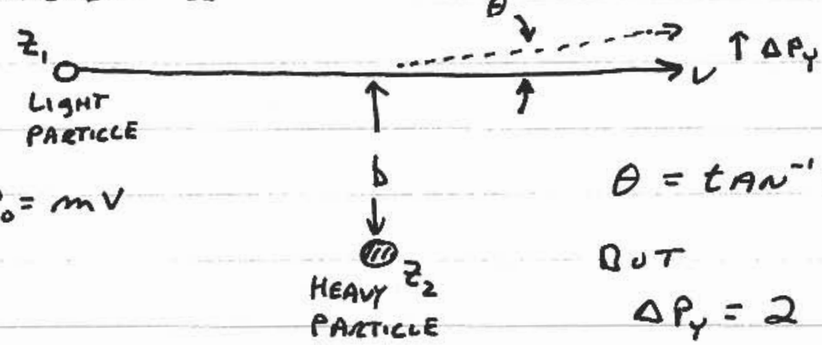
THE RELATIVE STOPPING POWERS ARE PROPORTIONAL TO

$$-\rho \frac{dE}{dx} \propto \frac{z^2}{(E/m)} \ln\left(\frac{4 \cdot E\left(\frac{m_e}{M}\right)}{11\, Z} \right) \quad \text{IN Si}$$

RELATIVE PARAMETERS

	$E/M = v^2$	z	$\frac{dE}{dx}$	$\ln\left(\frac{4E\left(\frac{m_e}{M}\right)}{11 Z}\right)$
α	1	2	1	5.52
p	4	1	0.0860	7.60
d	2	1	0.156	6.91
^3He	4/3	2	0.789	5.81

PROBLEM 2



$P_0 = mV$

$$\theta = \tan^{-1}\left(\frac{\Delta P_Y}{P_0}\right) \approx \frac{\Delta P_Y}{P_0}$$

BUT

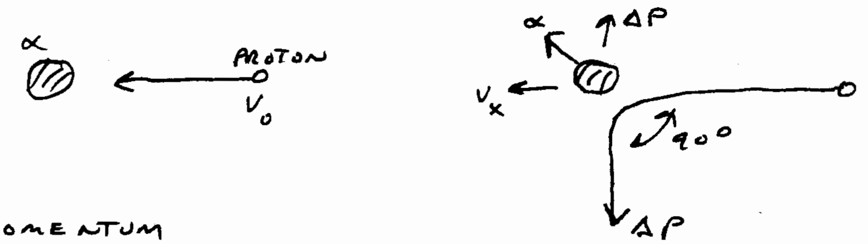
$$\Delta P_Y = 2\,\frac{A}{bV} \quad \therefore$$

$$\theta \approx \frac{2A}{bV\,mV} \sim \frac{A}{bE} \equiv \frac{b^*}{b}$$

PROBLEM 3 THERE ARE TWO WAYS TO SOLVE THIS PROBLEM. FIRST, LIKE PROBLEM 5.1, IT IS USEFUL TO EXAMINE THE α REST FRAME.

α - REST - FRAME HERE, THE LARGEST DEFLECTION WILL OCCUR WHEN THE PROTON IS SCATTERED AT $90°$

BEFORE COLLISION AFTER COLLISION



MOMENTUM X-COMPONENT	Y-COMPONENT	ENERGY
$M_\alpha V_x = m_p V_0$	$\Delta p = \Delta p$	$M_\alpha\left(\frac{m_p}{M_\alpha}V_0\right)^2 + \frac{(\Delta P)^2}{M_\alpha} + \frac{(\Delta P)^2}{m_p} = m_p V_0^2$
$\therefore \quad V_x = \frac{m_p}{M_\alpha} V_0$		

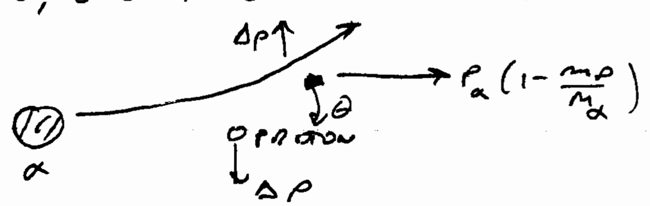
SOLVING FOR ΔP IN TERMS OF V_0 ...

$$m_p v_0^2\left(1 - \frac{m_p}{M_\alpha}\right) = (\Delta P)^2 \frac{M_\alpha + m_p}{M_\alpha m_p}$$

OR

$$\Delta p = P_\alpha\left(\frac{m_p}{M_\alpha}\right)\sqrt{\frac{M_\alpha - m_p}{M_\alpha + m_p}}$$

NOW, BACK INTO THE LAB FRAME ...



$$\tan\theta = \frac{\Delta P}{P_\alpha\left(1 - \frac{m_p}{M_\alpha}\right)} = \frac{m_p}{\sqrt{M_\alpha^2 - m_p^2}}$$

ANOTHER (EQUIVALENT) WAY TO SEE THE ANSWER IS TO LOOK AT THE DIFFERENTIAL CROSS-SECTION. NOTICE THAT WHEN

$$\left(\frac{M_\alpha}{m_p}\right)^2 \sin^2\theta > 1 ,$$

THEN THE CROSS-SECTION IS NOT DEFINED!!

THUS, WHEN $\sin\theta \leq \frac{m_p}{m_\alpha}$ WE HAVE MAXIMUM SCATTERING

OR $\theta_{max} = \sin^{-1}\left(\frac{m_p}{m_\alpha}\right) = 14.5°$ AS ABOVE

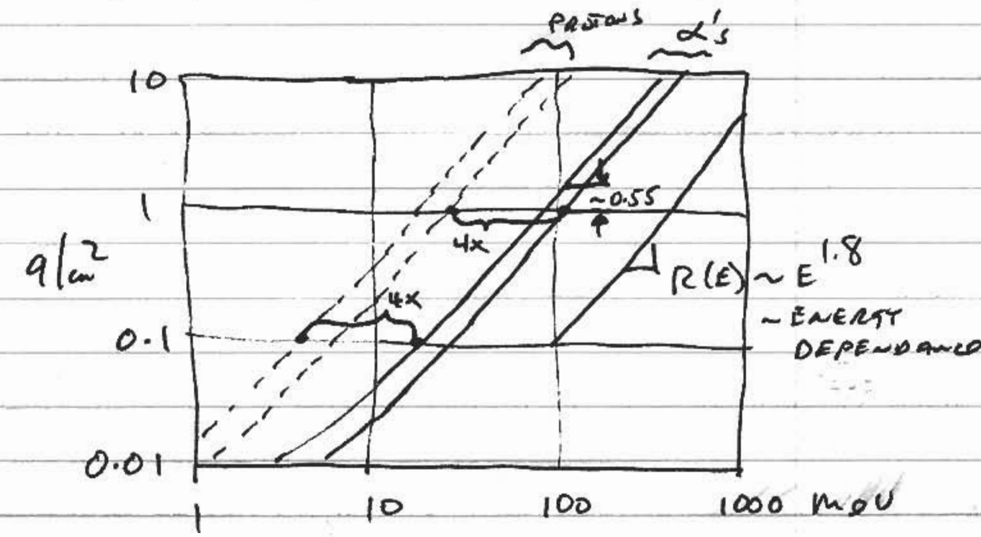
PROBLEM #4

PART A

- FROM EQS. 5.3 + 5.4, THE RANGE SCALES WITH ENERGY AS $\propto E^{1+k} \approx E^{1.8}$ FOR BOTH PROTONS AND α'S WITH E > 1 MeV THIS IS TRUE FOR Pb AND Cu
- FOR PARTICLES WITH THE SAME ENERGY PER AMU, i.e. THE SAME v^2, THE RANGE SCALES LIKE m/z^2.

$$R(\text{ALPHA}) \sim \frac{4}{2^2} R(\text{PROTON WITH } \tfrac{1}{4} \text{ ENERGY})$$

THUS, $R(\alpha) \sim R(\text{PROTON @ } E/4)$



- FINALLY, THE EMPIRICAL BRAGG-KLEEMAN FORMULA SAYS THAT AT THE SAME ENERGY AND PARTICLE, RANGE SCALES LIKE $R(Cu) = \sqrt{\frac{A_{Cu}}{A_{Pb}}}\, R(Pb) \approx 0.55\, R(Pb)$
R(Cu) IS SMALLER LIKE RATIO (BUT NOT EXACTLY)

PART B

$$R(D, 30\,\text{MeV}) = R(P, 15\,\text{MeV}) \times \frac{m}{z^2} = R(P, 15\,\text{MeV}) \times 2$$

	R (P, 15 MeV)	R (D, 30 MeV)
Cu	0.44	0.88
Pb	0.68	1.36