Questions 1-3

Due problems 3.1, 3.2, and 3.4 in Gurnett and Bhattacharjee, p. 71.

Question 4

Consider a charged particles within a uniform magnetic field. Imagine that all of the particles initially have the same speed, $v_0$, but isotropic velocity distribution. In other words, the kinetic energy of each particle is proportional to $v_0^2/2$, but the particle velocity vector may point in any direction.

If the particles have mass, $m$, and density, $n$, then the kinetic energy density of the charged particles are

$$K.E. = \int_0^\infty 4\pi v^2 dv \frac{1}{2}mv^2 f(v)$$

$$= \frac{1}{2}nmv_0^2$$

where $n = \int 4\pi v^2 dv f$.

Now, consider the following (non-adiabatic) sequence of events.

1. The magnetic field strength is (uniformly) increased slowly by a factor $\lambda$.

2. At fixed field strength, the distribution is allowed to become isotropic due to, for example, collisions.

3. Now, the magnetic field is slowly returned to it’s original value.

4. Finally, the distribution becomes isotropic.

Part A

Express the final energy density in terms of the initial energy density and the factor $\lambda$.

Part B

Show that the final energy is larger than the initial energy. (That is, “work” has been done to the plasma, and it’s become “hotter”.)

Part C

At the end, the particles no longer have the same speed. There are a range, or distribution, of speeds. What is the minimum and maximum particle energies?