Supplementary Appendix for
"Contests for Experimentation"

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SA.1. Intermittently-public contests

This section provides details for Section 4.4 of the paper. We consider the class of contests with *intermittently-public* information disclosure: the principal specifies a set of times at which the entire history of successes to date is publicly disclosed; at other times, there is no information disclosed. Formally, an intermittently-public disclosure policy is a sequence $(M_t, \mu_t)_{t\in[0,T]}$ where, for an arbitrary set $\mathcal{T} \subseteq [0, T]$, $M_t = O^t$ and $\mu_t(\boldsymbol{o}^t) = \boldsymbol{o}^t$ if $t \in \mathcal{T}$, and $|M_t| = 1$ if $t \notin \mathcal{T}$.[1] Define a *mixture contest* (with switching time t) as one that implements public winner-takes-all (WTA) from time 0 to some time $t \in [0, T]$ and then hidden equal-sharing (ES) from time t to T.

Proposition SA.1. *Among intermittently-public contests, an optimal contest is a mixture contest with some deadline T and switching time $t_S \in [0, T]$. Moreover,*

1. *If $\lambda\overline{w}/2 < c$, then $t_S = T = T^{PW}$ (so the contest is public winner-takes-all),*
2. *If $\lambda\overline{w}/N > c$, then $t_S = 0$ and $T = T^{HS}$ (so the contest is hidden equal-sharing).*

It bears highlighting that the above result is logically incomparable with Proposition 4 of the paper: Proposition SA.1 assumes a restricted set of information disclosure policies but does not require rank monotonicity of the prize scheme.

An intuition for the form of the optimal mixture contest stems from our discussion in Section 4.3 of the paper about how changes in the prior alter the principal's choice between public WTA and hidden ES: the latter is more beneficial when the agents' beliefs are lower. The formal proof of Proposition SA.1, presented at the end of this section, is

[1] The analysis in this section also holds if the principal randomizes over disclosure times, so long as randomization is independent of the history.

constructive: given any intermittently-public contest $C = (w(\cdot), \mathcal{T}, T)$, with $t_C := \sup \mathcal{T}$ and public belief p_{t_C} at t_C, we construct a mixture contest that implements public WTA until the public belief reaches p_{t_C} and hidden ES from then on, and we show that this mixture contest weakly improves on the original contest C.

Similar to Corollary 1 in the paper, Proposition SA.1 provides simple sufficient conditions for either public WTA or hidden ES to be optimal among intermittently-public contests; these conditions are intuitive given our discussion of Proposition 3 and Corollary 1 in the paper. Plainly, for $N = 2$ it is always optimal to use either public WTA or hidden ES. When $\lambda\overline{w}/c \in (2, N)$, the optimal mixture contest can have a deadline T and a strictly interior switching time, $t_S \in (0, T)$. The logic turns on the tradeoff between increasing an agent's expected reward from success versus increasing his belief that he can succeed: as t_S increases, the agent's belief about the innovation's feasibility from t_S on decreases, but his expected reward for success after t_S increases because in expectation the prize is shared with a smaller number of agents. Figure SA.1 presents an example in which the optimal mixture contest has a strictly interior switching time.

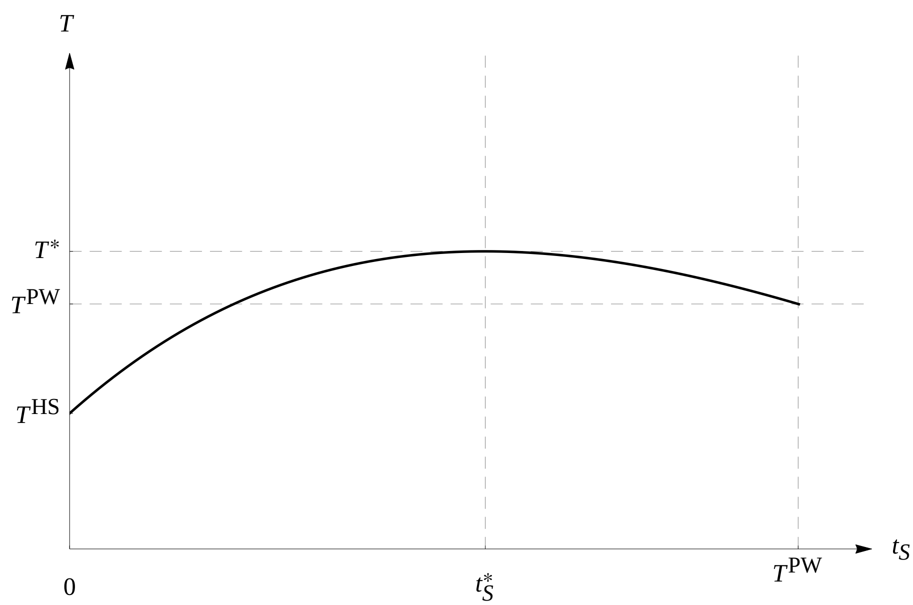

Figure SA.1 – Stopping time T as a function of switching time $t_S \in [0, T]$ in a mixture contest. Parameters are $p_0 = 0.9$, $c = 0.18$, $\overline{w} = 1$, $\lambda = 0.4$, and $N = 3$.

Proposition SA.1 shows that an optimal intermittently-public contest has a prize scheme that is rank-monotonic: if agent i succeeds earlier than agent j in a mixture contest, then i receives a weakly larger share of the prize than j. As mentioned in the paper, it therefore follows from Proposition 4 that no intermittently-public contest can improve on the optimal cutoff-disclosure ES contest.

Proof of Proposition SA.1. We first show that a mixture contest is optimal among intermittently-public contests and then verify the sufficient conditions for the optimality of public WTA and hidden ES.

Step 1. Consider an arbitrary contest C with prize scheme $w(s_i, \boldsymbol{s}_{-i})$, intermittently-public information disclosure policy $\mathcal{T}$, and deadline T. We focus on symmetric equilibria and, without loss of generality, define the deadline T so that agents exert positive effort at T given no success by T. The aggregate cumulative effort up to T induced by contest C (given no success by T) is A^T. We want to show that there exist $t_S^* \geq 0$, $T^* \geq t_S^*$, and a mixture contest with deadline T^* and switching time t_S^* such that the aggregate cumulative effort by T^* (given no success by T^*) induced by this contest is $A^* \geq A^T$. This mixture contest will therefore weakly improve on C.

Suppose, towards contradiction, that $A^* < A^T$ for all $t_S^* \geq 0$ and $T^* \geq t_S^*$. Then $\max\{A^{PW}, A^{HS}\} < A^T$, where A^{PW} and A^{HS} are the aggregate cumulative efforts induced respectively in an optimal public WTA and optimal hidden ES contest.

Let t_C be the last time at which information is disclosed in contest C; more precisely, $t_C := \sup \mathcal{T}$. By Proposition 2 in the paper, $t_C > 0$. Consider a history of no success. The agents' belief at t_C is equal to the public belief;[2] denote this belief by p_{t_C}. Since agents are willing to exert positive effort at some point $t \in [t_C, T]$ (recall that, without loss, T is defined so that agents exert positive effort at T given no success by T), and an agent's reward for success at any such point cannot be strictly larger than the prize $\overline{w}$, we must have $p_{t_C} \geq \frac{c}{\lambda \overline{w}}$. It follows that there is a public WTA contest that induces full effort by each agent until the public belief reaches $p^{PW} \leq p_{t_C}$. Let t^* be the time at which the belief reaches p_{t_C} in a public WTA contest and denote by $\tilde{A}^{PW}$ the aggregate cumulative effort induced by this contest over $[0, t^*]$.

Next, note that given the public belief p_{t_C} at t_C, the continuation game in contest C has hidden disclosure. Consider an optimal hidden ES contest starting with a prior belief $p_0 = p_{t_C}$. Denote by $\tilde{T}^{HS}$ and $\tilde{A}^{HS}$ respectively the stopping time and the aggregate cumulative effort induced by such a contest. By Proposition 2 in the paper, $\tilde{A}^{HS}$ is weakly larger than the aggregate cumulative effort induced by contest C over $[t_C, T]$. Let $T^* := t^* + \tilde{T}^{HS}$.

We will show that a mixture contest with switching time t^* and deadline T^* induces aggregate cumulative effort $A^* := \tilde{A}^{PW} + \tilde{A}^{HS}$, which implies that it weakly improves on contest C, contradicting the hypothesis that $A^* < A^T$. Let $p_{i,t}$ be agent i's belief at time

[2] This is immediate if $\sup \mathcal{T} \in \mathcal{T}$. Otherwise, letting $\Omega_{i,z}$ denote all information an agent i has at time z and $p(\Omega_{i,z})$ the agent's belief given this information, the claim follows from the fact that the definition of t_C implies $p(\Omega_{i,z}) \to p_{t_C}$ as $z \to t_C$.

t, where this belief is updated given public disclosure from time 0 until t^* and hidden disclosure from t^* until T^*. Denote by w the agent's expected reward for success at any time $t \geq t^*$ given no success by t^*, and let $A_{-i,z}$ denote (i's conjecture of) the aggregate effort exerted by i's opponents at time z so long as they have not succeeded by z. The agent's problem is:

$$\max_{(a_{i,t})_{t\in[0,T^*]}} \int_0^{t^*} (\overline{w} p_{i,t}\lambda - c)\, a_{i,t} e^{-\int_0^t p_{i,z}\lambda(a_{i,z}+A_{-i,z})dz} dt$$
$$+ e^{-\int_0^{t^*} p_{i,z}\lambda(a_{i,z}+A_{-i,z})dz} \int_{t^*}^{T^*} (w p_{i,t}\lambda - c)\, a_{i,t} e^{-\int_{t^*}^t p_{i,z}\lambda a_{i,z} dz} dt.$$

The belief $p_{i,t}$ is decreasing and the expected reward for success is also (weakly) decreasing because $w \leq \overline{w}$. Hence, an optimal strategy for agent i is a stopping strategy: for $t \leq t^*$, $a_{i,t} = 1$ if $\overline{w} p_{i,t}\lambda \geq c$ and $a_{i,t} = 0$ otherwise; for $t > t^*$, $a_{i,t} = 1$ if $w p_{i,t}\lambda \geq c$ and $a_{i,t} = 0$ otherwise. It follows that if a public WTA contest induces $\tilde{A}^{PW}$ until time t^* given no continuation game, it also induces $\tilde{A}^{PW}$ until time t^* when the continuation game is a hidden ES contest. Finally, starting from t^*, the continuation game is the same as that under a hidden ES contest starting at time 0 with prior belief $p_0 = p_{t_C}$. Therefore, this mixture contest induces $A^* = \tilde{A}^{PW} + \tilde{A}^{HS}$.

Step 2. We now verify the sufficient conditions for the optimality of hidden ES and public WTA given in the proposition. Consider a mixture contest with switching time t_S. The stopping time T for any agent i is given by:

$$\frac{c}{\lambda\overline{w}} = \underbrace{\Pr[\text{some } j \neq i \text{ succ. in } [t_S, T] \mid \text{no one did by } t_S, i \text{ didn't by } T]}_{\alpha(t_S)} L_{[t_S,T]}$$
$$+ \underbrace{\Pr[\text{no } j \neq i \text{ succ. in } [t_S, T] \mid \text{no one did by } t_S, i \text{ didn't by } T]}_{1-\alpha(t_S)} \Pr[G \mid \text{no succ. by } T],$$

where

$$L_{[t_S,T]} := \sum_{n=1}^{N-1} \Pr[n \text{ opponents succ. btw } t_S \text{ and } T \mid \text{at least one did, no succ. by } t_S] \left(\frac{1}{n+1}\right).$$

We can rewrite this condition as

$$\frac{c}{\lambda\overline{w}} = \underbrace{\frac{p_0 e^{-N\lambda t_S} e^{-\lambda(T-t_S)}\left(1 - e^{-(N-1)\lambda(T-t_S)}\right)}{p_0 e^{-N\lambda t_S} e^{-\lambda(T-t_S)} + (1-p_0)}}_{\alpha(t_S)} \underbrace{\frac{\frac{1-e^{-\lambda(T-t_S)N}}{\left(1-e^{-\lambda(T-t_S)}\right)N} - e^{-\lambda(N-1)(T-t_S)}}{1 - e^{-\lambda(T-t_S)(N-1)}}}_{L_{[t_S,T]}}$$

$$+\underbrace{\left[1-\frac{p_0 e^{-N\lambda t_S}e^{-\lambda(T-t_S)}\left(1-e^{-(N-1)\lambda(T-t_S)}\right)}{p_0 e^{-N\lambda t_S}e^{-\lambda(T-t_S)}+(1-p_0)}\right]}_{1-\alpha(t_S)}\underbrace{\frac{p_0 e^{-N\lambda T}}{p_0 e^{-N\lambda T}+(1-p_0)}}_{\Pr[G\mid \text{no success by } T]}. \tag{SA.1}$$

Observe that $\alpha(t_S)$ is decreasing in t_S; $L_{[t_S,T]}$ is decreasing in T and increasing in t_S; $\Pr[G \mid \text{no success by } T]$ is decreasing in T; and the RHS of (SA.1) is decreasing in T.

Suppose first that $\frac{\lambda\overline{w}}{2} < c$. Then for any t_S and T, $L_{[t_S,T]} < \frac{c}{\lambda\overline{w}}$. Given T, it follows that $\Pr[G \mid \text{no success by } T] > \frac{c}{\lambda\overline{w}}$, as otherwise (SA.1) would not hold with equality. Consequently, if t_S increases, $(1-\alpha(t_S))$ and $L_{[t_S,T]}$ increase and thus the RHS of (SA.1) increases. This implies that T must increase when t_S increases (so that the RHS decreases and remains equal to the LHS), and therefore setting $t_S = T$ is optimal.

Suppose next that $\frac{\lambda\overline{w}}{N} > c$. Then for any t_S and T, $L_{[t_S,T]} > \frac{c}{\lambda\overline{w}}$. Given T, it follows that $\Pr[G \mid \text{no success by } T] < \frac{c}{\lambda\overline{w}}$, as otherwise (SA.1) would not hold with equality. Note that a change in t_S now causes two opposing effects on the RHS of (SA.1): on the one hand, reducing t_S increases $\alpha(t_S)$, which increases the RHS of (SA.1), but on the other hand it reduces $L_{[t_S,T]}$, which reduces the RHS of (SA.1). We show that if $\frac{\lambda\overline{w}}{N} > c$, the net effect of reducing t_S to zero on the RHS of (SA.1) is positive, which implies that T must increase and therefore setting $t_S = 0$ is optimal.

To show this, note that by (SA.1), $(1-\alpha(t_S))\Pr[G \mid \text{no success by } T] = \frac{c}{\lambda\overline{w}} - \alpha(t_S)L_{[t_S,T]}$, and hence $\alpha(0)L_{[0,T]} + (1-\alpha(0))\Pr[G|\text{no success by } T]$ is equal to

$$\alpha(0)L_{[0,T]} + \frac{1-\alpha(0)}{1-\alpha(t_S)}\left(\frac{c}{\lambda\overline{w}} - \alpha(t_S)L_{[t_S,T]}\right). \tag{SA.2}$$

We need to show that (SA.2) is greater than $\frac{c}{\lambda\overline{w}}$. (SA.2) can be rewritten as

$$\begin{aligned}&\frac{p_0 e^{-\lambda T}}{p_0 e^{-\lambda T}+1-p_0}\left(\frac{1-e^{-\lambda TN}}{(1-e^{-\lambda T})N} - e^{-\lambda T(N-1)}\right) + \frac{p_0 e^{-\lambda T-\lambda t(N-1)}+(1-p_0)}{p_0 e^{-\lambda T}+1-p_0}\frac{c}{\lambda\overline{w}}\\ &-\frac{p_0 e^{-\lambda T-\lambda t(N-1)}+(1-p_0)}{p_0 e^{-\lambda T}+1-p_0}\frac{p_0 e^{-\lambda T-\lambda t(N-1)}}{p_0 e^{-\lambda T-\lambda t(N-1)}+(1-p_0)}\left(\frac{1-e^{-\lambda N(T-t)}}{(1-e^{-\lambda(T-t)})N} - e^{-\lambda(T-t)(N-1)}\right).\end{aligned}$$

Some algebra then shows that (SA.2) is greater than $\frac{c}{\lambda\overline{w}}$ if and only if

$$\frac{1-e^{-\lambda TN}}{(1-e^{-\lambda T})N} - \frac{e^{-\lambda t(N-1)}(1-e^{-\lambda N(T-t)})}{(1-e^{-\lambda(T-t)})N} - \frac{c}{\lambda\overline{w}}\left(1-e^{-\lambda t(N-1)}\right) \geq 0.$$

By assumption, $\frac{c}{\lambda\overline{w}} < \frac{1}{N}$; thus, again doing some algebra, it suffices to show that

$$\frac{1-e^{-\lambda TN}}{(1-e^{-\lambda T})} - \frac{e^{-\lambda t(N-1)} - e^{-\lambda NT+\lambda t}}{(1-e^{-\lambda(T-t)})} - 1 + e^{-\lambda t(N-1)} \geq 0.$$

This inequality holds with equality when $t = 0$, and a routine computation verifies that the derivative of the LHS with respect to t is non-negative for all $N \geq 2$. *Q.E.D.*

SA.2. Discounting

This section provides details for the claims about discounting in Section 5.4 of the paper. Suppose that the principal and the agents discount future payoffs at rate $r \geq 0$. We first study public WTA and hidden ES contests and then compare the two.

Public WTA. In a public WTA contest, agent i's problem can be written as

$$\max_{(a_{i,t})_{t\in[0,T]}} \int_0^T (p_{i,t}\lambda\overline{w} - c)\, a_{i,t} e^{-rt} e^{-\int_0^t p_{i,z}\lambda(a_{i,z}+A_{-i,z})dz} dt. \tag{SA.3}$$

This formulation assumes that the principal pays out the prize money to the first successful agent immediately after he succeeds; it is clear that this is optimal for the principal. Since $p_{i,t}$ is decreasing, the unique solution to (SA.3) is $a_{i,t} = 1$ if $p_{i,t} \geq p^{PW}$ and $a_{i,t} = 0$ otherwise, where $p^{PW} = \frac{c}{\lambda\overline{w}}$ as defined in equation (10) in the paper. Therefore, the aggregate cumulative effort induced by a public WTA contest is invariant to discounting, and is given by NT^{PW} in equation (11) of the paper.

Hidden ES. Consider a hidden ES contest with deadline T. We begin by showing that it is without loss of optimality to focus on symmetric equilibria in which each agent i exerts zero effort from time 0 until some time $\tilde{t} \in [0, T]$ and full effort from $\tilde{t}$ until T. In turn, it is then without loss of optimality to focus on hidden ES contests in which the deadline is such that all agents exert full effort at all times in the contest.

Agent i's payoff from a strategy $a_i := (a_{i,t})_{t\in[0,T]}$ in a hidden ES contest is

$$U(a_i) = -(1-p_0)\int_0^T c a_{i,t} e^{-rt} dt + p_0 \int_0^T \left(\lambda w^{HS} e^{-rT} - c e^{-rt}\right) a_{i,t} e^{-\lambda a_i^t} dt,$$

where $\int_0^T c a_{i,t} e^{-rt} dt$ is agent i's discounted cost of effort conditional on the bad state, $w^{HS} e^{-rT}$ is the discounted expected reward for success, and $e^{-\lambda a_i^t}$ is the agent's belief that he will not succeed by time t (recall, $a_i^t := \int_0^t a_{i,z} dz$). Rearranging terms yields

$$U(a_i) = -(1-p_0)\, c \int_0^T a_{i,t} e^{-rt} dt + p_0 w^{HS} e^{-rT} \int_0^T \lambda a_{i,t} e^{-\lambda a_i^t} dt \\ - p_0 c \int_0^T e^{-rt} a_{i,t} e^{-\lambda a_i^t} dt. \tag{SA.4}$$

Let $\tilde{t} := T - a_i^T$. We construct the following strategy for i:

$$\widetilde{a}_{i,t} := \begin{cases} 0 & \text{if } t < \tilde{t}, \\ 1 & \text{otherwise.} \end{cases}$$

By construction, $a_i^t \geq \widetilde{a}_i^t$ for $t \in [0, T]$ with equality at $t = T$. Moreover, for any $k \in [0, T]$,

$$\int_0^k \lambda a_{i,t} e^{-\lambda a_i^t} dt = \int_0^k e^{-\lambda a_i^t} d\left(\lambda a_i^t\right) = 1 - e^{-\lambda a_i^k} \geq 1 - e^{-\lambda \widetilde{a}_i^k} = \int_0^k \lambda \widetilde{a}_{i,t} e^{-\lambda \widetilde{a}_i^t} dt,$$

where the inequality holds with equality if $k = T$. Hence,

$$\int_0^T a_{i,t} e^{-rt} dt \geq \int_0^T \widetilde{a}_{i,t} e^{-rt} dt,$$
$$\int_0^T e^{-rt} a_{i,t} e^{-\lambda a_i^t} dt \geq \int_0^T e^{-rt} \widetilde{a}_{i,t} e^{-\lambda \widetilde{a}_i^t} dt,$$
$$\int_0^T \lambda a_{i,t} e^{-\lambda a_i^t} dt = \int_0^T \lambda \widetilde{a}_{i,t} e^{-\lambda \widetilde{a}_i^t} dt.$$

It follows from (SA.4) that $U(a_i) \leq U(\widetilde{a}_i)$. It is thus without loss of optimality to focus on a hidden ES contest in which agents exert full effort from time 0 until some deadline T.

We next solve for the amount of experimentation induced by a hidden ES contest in the presence of discounting. Given the result above, take a hidden ES contest with deadline T in which $a_{i,t} = 1$ for all $t \in [0, T]$ and $i \in \mathcal{N}$. Agent i's expected payoff is

$$U(a_i) = -(1-p_0) \int_0^T c e^{-rt} dt + p_0 \int_0^T \left(\lambda w^{HS} e^{-rT} - c e^{-rt}\right) e^{-\lambda t} dt.$$

For any $x \in [0,T)$ and $\varepsilon > 0$ such that $x+\varepsilon < T$, consider the following deviation for i:[3]

$$a_{i,t}^x := \begin{cases} 1 & \text{if } t \in [0,x] \cup [x+\varepsilon, T], \\ 0 & \text{if } t \in (x, x+\varepsilon). \end{cases}$$

Under this deviation, agent i's expected payoff is

$$\begin{aligned} U\left(a_i^x\right) &= -\left(1-p_0\right)\left(\int_0^x ce^{-rt}dt + \int_{x+\varepsilon}^T ce^{-rt}dt\right) + p_0 \int_0^x \left(\lambda w^{HS} e^{-rT} - ce^{-rt}\right) e^{-\lambda t} dt \\ &\quad + p_0 \int_{x+\varepsilon}^T \left(\lambda w^{HS} e^{-rT} - ce^{-rt}\right) e^{-\lambda(t-\varepsilon)} dt. \end{aligned}$$

The optimality of a_i requires $\frac{d}{d\varepsilon} U\left(a_i^x\right)|_{\varepsilon=0} \leq 0$ for all $x \in [0,T)$. Note that

$$\begin{aligned} \frac{d}{d\varepsilon} U\left(a_i^x\right)|_{\varepsilon=0} &= \left(1-p_0\right) ce^{-rx} - p_0 \left(\lambda w^{HS} e^{-rT} - ce^{-rx}\right) e^{-\lambda x} \\ &\quad + p_0 \int_x^T \left(\lambda w^{HS} e^{-rT} - ce^{-rt}\right) \lambda e^{-\lambda t} dt, \\ \frac{d}{dx}\frac{d}{d\varepsilon} U\left(a_i^x\right)|_{\varepsilon=0} &= -r\left(1-p_0\right) ce^{-rx} - rp_0 ce^{-(r+\lambda)x} < 0. \end{aligned}$$

Consequently, $\frac{d}{d\varepsilon} U\left(a_i^x\right)|_{\varepsilon=0} \leq 0$ for all $x \in [0,T)$ if and only if $\frac{d}{d\varepsilon} U\left(a_i^x\right)|_{\varepsilon=0,x=0} \leq 0$. We compute

$$\begin{aligned} \frac{d}{d\varepsilon} U\left(a_i^x\right)|_{\varepsilon=0,x=0} &= \left(1-p_0\right) c - p_0 \left(\lambda w^{HS} e^{-rT} - c\right) + p_0 \int_0^T \left(\lambda w^{HS} e^{-rT} - ce^{-rt}\right) \lambda e^{-\lambda t} dt \\ &= c - p_0 \lambda w^{HS} e^{-(\lambda+r)T} - p_0 c \frac{\lambda}{\lambda+r} \left(1 - e^{-(\lambda+r)T}\right). \end{aligned}$$

The condition $\frac{d}{d\varepsilon} U\left(a_i^x\right)|_{\varepsilon=0,x=0} \leq 0$ is thus equivalent to

$$\lambda w^{HS} \frac{p_0 e^{-(\lambda+r)T}}{1 - p_0 \frac{\lambda}{\lambda+r} + p_0 \frac{\lambda}{\lambda+r} e^{-(\lambda+r)T}} \geq c.$$

Recall that $w^{HS} = \overline{w} \frac{1-e^{-\lambda NT}}{\left(1-e^{-\lambda T}\right)N}$. The above inequality further reduces to

$$\frac{1-e^{-\lambda NT}}{\left(1-e^{-\lambda T}\right)N} \frac{p_0 e^{-(\lambda+r)T}}{1 - p_0 \frac{\lambda}{\lambda+r} + p_0 \frac{\lambda}{\lambda+r} e^{-(\lambda+r)T}} \geq \frac{c}{\lambda \overline{w}}. \tag{SA.5}$$

[3] In what follows, we implicitly presume that if a_i is suboptimal, then there must exist a profitable such deviation. This is analogous to the one-step deviation principle in discrete-time games. Our approach can be justified by studying a sequence of discrete-time games with period length $\Delta \to 0$, and applying the one-step deviation principle along the sequence of games.

The LHS of (SA.5) is continuous and strictly decreasing in T. Thus, the aggregate cumulative effort induced in a hidden ES contest with discount rate r is NT_r^{HS}, where T_r^{HS} is the unique solution to

$$\frac{1-e^{-\lambda NT_r^{HS}}}{\left(1-e^{-\lambda T_r^{HS}}\right)N}\frac{p_0e^{-(\lambda+r)T_r^{HS}}}{1-p_0\frac{\lambda}{\lambda+r}+p_0\frac{\lambda}{\lambda+r}e^{-(\lambda+r)T_r^{HS}}}=\frac{c}{\lambda\overline{w}}. \tag{SA.6}$$

Plainly, T_r^{HS} is continuous in r. When $r=0$, equation (SA.6) reduces to equation (16) in the paper: $T_0^{HS}=T^{HS}$.

Public WTA versus Hidden ES. Analogous to our analysis in Section 4.3 in the paper, we can compare the amount of experimentation induced by public WTA and hidden ES in the presence of discounting by comparing the induced stopping times, T^{PW} and T_r^{HS}. Since the LHS of (SA.6) is strictly decreasing in T_r^{HS}, it holds that for any given discount rate r, a hidden ES contest induces a strictly larger aggregate cumulative effort (and thus a strictly higher probability of success) than a public WTA contest if and only if

$$\frac{1-e^{-\lambda NT^{PW}}}{\left(1-e^{-\lambda T^{PW}}\right)N}\frac{p_0e^{-(\lambda+r)T^{PW}}}{1-p_0\frac{\lambda}{\lambda+r}+p_0\frac{\lambda}{\lambda+r}e^{-(\lambda+r)T^{PW}}}>\frac{c}{\lambda\overline{w}}. \tag{SA.7}$$

This condition is continuous in r and reduces to condition (17) in the paper when $r=0$. As discussed in the paper, discounting also affects the computation of ex-ante payoffs in a way that makes public WTA more beneficial than hidden ES: the principal can profit from an agent's innovation immediately following success in a public WTA contest, whereas she must wait until the deadline in a hidden ES contest. Yet, if (SA.7) holds, a tradeoff arises, and at least for a small discount rate hidden ES will still dominate public WTA.

SA.3. Convex effort costs

This section provides details for the claims about convex effort costs in Section 5.4 of the paper. Let $C:[0,1]\to\mathbb{R}_+$ be a differentiable instantaneous effort cost function with $C'(0)=c>0$. We assume C is convex, which, given $C'(0)=c$, implies $C(a_{i,t})\geq ca_{i,t}$.[4] We will show that the amount of experimentation induced in a public WTA contest and in a hidden ES contest is the same as under a linear effort cost.

[4] We could alternatively consider a sequence of cost functions that converge pointwise to the linear cost function and analyze the corresponding sequence of Nash equilibria. For the purpose of finding the stopping beliefs, it is convenient to directly assume $C'(0)=c$.

Public WTA. In a public WTA contest, agent i's problem can be written as

$$\max_{(a_{i,t})_{t\in[0,T]}} \int_0^T \left(p_{i,t}\lambda \overline{w} a_{i,t} - C\left(a_{i,t}\right)\right) e^{-\int_0^t p_{i,z}\lambda(a_{i,z}+A_{-i,z})dz} dt. \tag{SA.8}$$

Since $p_{i,t}$ is decreasing, the solution to (SA.8) has $a_{i,t} > 0$ if and only if $p_{i,t} \geq \frac{C'(0)}{\lambda \overline{w}} = p^{PW}$. Therefore, the stopping belief is the same as with the linear effort cost studied in the paper, where $C\left(a_{i,t}\right) = ca_{i,t}$. Under a convex cost, instantaneous equilibrium effort will generally be interior and evolving over time. Nevertheless, the aggregate cumulative effort induced by a public WTA contest, A^T, is uniquely determined by the stopping belief via

$$p^{PW} = \frac{p_0 e^{-\lambda A^T}}{p_0 e^{-\lambda A^T} + (1-p_0)},$$

and hence is the same as under a linear effort cost.

Hidden ES. Consider a symmetric equilibrium of a hidden ES contest. Agent i's problem is

$$\max_{(a_{i,t})_{t\in[0,T]}} \int_0^T \left(p_{i,t}^{(1)}\lambda w^{HS} a_{i,t} - C\left(a_{i,t}\right)\right) e^{-\int_0^t p_{i,z}^{(1)}\lambda a_{i,z}dz} dt,$$

where w^{HS} is the expected reward for success. Let A^T be the aggregate cumulative effort in this equilibrium. Then

$$\begin{aligned} w^{HS} &= \overline{w}\sum_{n=0}^{N-1} \frac{1}{n+1}\binom{N-1}{n}\left(1-e^{-\frac{\lambda A^T}{N}}\right)^n e^{-\left(\frac{N-1-n}{N}\right)\lambda A^T} \\ &= \overline{w}\frac{1-e^{-\lambda A^T}}{\left(1-e^{-\frac{\lambda A^T}{N}}\right)N}, \end{aligned} \tag{SA.9}$$

where the second equality follows from the proof of Proposition 2 in the paper. In a symmetric equilibrium, $a_{i,t} > 0$ if and only if $p_{i,t}^{(1)} \geq \frac{C'(0)}{\lambda w^{HS}}$; therefore, the stopping belief satisfies

$$p_T^{(1)} = \frac{c}{\lambda w^{HS}}, \tag{SA.10}$$

where

$$p_T^{(1)} = \frac{p_0 e^{-\frac{\lambda A^T}{N}}}{p_0 e^{-\frac{\lambda A^T}{N}} + 1 - p_0}. \tag{SA.11}$$

Combining (SA.9), (SA.10), and (SA.11) yields that the aggregate cumulative effort in a hidden ES contest, A^T, satisfies

$$\frac{1-e^{-\lambda A^T}}{\left(1-e^{-\frac{\lambda A^T}{N}}\right)N}\frac{p_0 e^{-\frac{\lambda A^T}{N}}}{p_0 e^{-\frac{\lambda A^T}{N}}+1-p_0}=\frac{c}{\lambda\overline{w}}. \tag{SA.12}$$

Recall that under a linear effort cost, the aggregate cumulative effort in a hidden ES contest is NT^{HS}, where T^{HS} is given by equation (16) in the paper. Since the LHS of (SA.12) and that of (16) are each decreasing functions of A^T and NT^{HS} respectively, it holds that $A^T = NT^{HS}$, i.e., the aggregate cumulative effort in a hidden ES contest is unaffected by the introduction of a convex cost.

SA.4. Multistage contests

Suppose that innovation requires obtaining a success in each of two stages, $k \in \{I, F\}$, where I stands for "intermediate" and F stands for "final". Analogous to our baseline model, whether a success in stage k is feasible depends on the stage-specific state $\theta^k \in \{G, B\}$, which is fully persistent and unobservable. We assume that the states are independently distributed and denote their respective prior probabilities by $p_0^k := \Pr(\theta^k = G)$. We allow agents' ability λ to be stage-specific, so that agent i succeeds with instantaneous probability $\lambda^k a_{i,t}$ at time t in stage k if $\theta^k = G$ and the agent exerts effort $a_{i,t}$.

Consider the principal's problem of designing an optimal contest given an arbitrary total prize $\overline{w} < v$, where v is the principal's value of innovation (i.e., from having one agent succeed in both stages). We assume $p_0^k \lambda^k \overline{w} > c$ for $k \in \{I, F\}$, as otherwise no contest can induce innovation. As in our baseline analysis in the paper, the principal must pay the entire prize $\overline{w}$ if innovation obtains; given $\overline{w}$, her objective is then to maximize the probability of innovation. As detailed further below, the principal will set a prize for each stage. We extend rank-monotonicity to this multistage setting by assuming that if agent i succeeds earlier than agent j in stage k, then i receives a weakly larger share of the prize in stage k than j; moreover, if agent i participates in stage k, his share of the prize in this stage is independent of agents' outcomes in stage $\ell \neq k$.[5] More precisely, the principal chooses (i) an "intermediate prize" $\overline{w}^I$ and a sharing scheme $(w_i^I(\boldsymbol{s^I}))_{i\in\mathcal{N}}$ that allocates $\overline{w}^I$ as a function of agents' success times in stage I; (ii) a "final prize" $\overline{w}^F$ and a sharing scheme $(w_i^F(\boldsymbol{s^F}))_{i\in\mathcal{N}}$ that allocates $\overline{w}^F$ as a function of agents' success times in stage F; and (iii) an "advancement rule" $\rho(\boldsymbol{s^I}) \in \Delta(2^{\mathcal{N}})$ that determines (possibly stochastically)

[5] While such a strong independence assumption is not needed for our analysis, it makes our points more transparent and it also appears consistent with multistage contests often observed in practice.

which agents advance to stage F as a function of agents' success times in stage I. The prizes must satisfy $\overline{w}^I + \overline{w}^F = \overline{w}$ and we require anonymity and rank-monotonicity on $(w_i^I(\boldsymbol{s^I}))_{i \in \mathcal{N}}$ and $(w_i^F(\boldsymbol{s^F}))_{i \in \mathcal{N}}$ as well as $\rho(\boldsymbol{s^I})$ (so that if agent i succeeds earlier than agent j in stage I, then i advances to stage F with a weakly higher probability than j). In addition, the principal chooses (iv) deadlines T^I and T^F and (v) information disclosure policies for each stage.

To illustrate how our results can be applied to this multistage setting, we focus on a simple case in which all the uncertainty pertains to the intermediate stage, i.e. $p_0^I < 1$ while $p_0^F = 1$.[6] We can show that the following multistage contest is optimal: in stage I, the principal runs a cutoff-disclosure ES contest with a finite deadline T^I and $\overline{w}^I = 0$; one agent among those who succeeded by T^I is selected via uniform randomization to advance to stage F; in stage F, the agent selected at T^I (if any) is awarded the entire prize $\overline{w}$ upon success in this stage, with deadline $T^F = \infty$.

The argument is as follows. Consider first stage F. Since $p_0^F = 1$, a public WTA contest maximizes the probability of success in stage F for any prize $\overline{w}^F$ allocated to this stage. Moreover, recall from Proposition 1 in the paper that the probability of success in a public WTA contest is independent of the number of agents and increasing in the prize. Hence, having only one agent work on stage F maximizes the probability of success on F for any given prize $\overline{w}^F$, and given only one agent working on stage F, the probability of success is increasing in $\overline{w}^F$.

Consider next stage I. The total prize agents receive for success in this stage is given by the intermediate prize $\overline{w}^I$ plus the value of advancing to stage F. Call the value of that total prize W^I. For any given W^I, Proposition 4 in the paper implies that a cutoff-disclosure ES contest (with a cutoff n^* that depends on λ^I, W^I, and c) maximizes the probability of success in stage I. Moreover, recall that an ES prize scheme can be implemented by a lottery, and the probability of success in an ES contest is increasing in the prize. Hence, running a cutoff-disclosure ES contest in which each successful agent has an equal probability of receiving the prize maximizes the probability of success in stage I for any given prize W^I, and given such a cutoff-disclosure ES contest, the probability of success is increasing in W^I.

The preceding arguments imply that to maximize innovation, the principal can run a cutoff disclosure ES contest in stage I, with the ES scheme implemented via lottery, and have only one agent work on stage F. All that remains to be shown is that it is optimal for the principal to set $\overline{w}^I = 0$, that is, to allocate the entire prize $\overline{w}$ to stage F and have

[6] Bimpikis, Ehsani, and Mostagir (2014) also assume no uncertainty in the second stage of their setting.

the value of succeeding in stage I be simply the continuation value of working on stage F. To see why this is the case, note that the continuation value from being selected at T^I to advance to stage F is equal to $\overline{w}^F - c/\lambda^F$, and therefore an agent's expected reward for success in the cutoff disclosure ES contest of stage I is $W^I \mathbb{E}_n \left[\frac{1}{n} \middle| n \geq 1, T^I\right]$ with $W^I = \overline{w}^I + \overline{w}^F - c/\lambda^F$. It is immediate that reducing $\overline{w}^F$ by any amount $\Delta > 0$ and increasing $\overline{w}^I$ by the same amount would reduce the prize in stage F, and hence (weakly) reduce the probability of success in that stage, while leaving unchanged an agent's expected reward for success and thus the probability of success in stage I.

References

Bimpikis, Kostas, Shayan Ehsani, and Mohamed Mostagir. 2014. "Information Provision in Dynamic Innovation Tournaments." Manuscript, Graduate School of Business, Stanford University.